Generalized Cosmic Chaplygin Gas Model Interacting In Bianchi Type-I Universe

6.1 Introduction

In the last decades in understanding the Physics behind the accelerated expansion of the universe have taken considerable interest by the cosmologist. Recent cosmological observations like Type Ia Supernovae, cosmic microwave background (CMB) radiation, large scale structure (LSS) have strongly indicate that our universe is not only expanding but also going through a phase of accelerated expansion. The fact behind the accelerated expansion of the universe is known as Dark Energy (DE). Cosmologists have given many theoretical models describing dark energy. The expansion of the universe will be accelerating only when the pressure p and energy density ρ of the universe will violate the strong energy condition i.e. when the pressure is negative. The dark energy models describing the accelerated expansion of the universe with negative pressure are cosmological constant, quintessence, phantom, quintom, tachyon, holographic dark energy, K-essence and various models of Chaplygin gas. The matter component in most of the dark energy models are considered as an invisible cosmic fluid. The Chaplygin gas is also used as an exotic type of fluid. The lifting force on a wing of an airplane in aerodynamics is taken as the base of the equation of state of

This Chapter is published in **International Journal of Applied Mathematics and Statistical Sciences**, Volume 5(1), pp 63-70 (2016)

the Chaplygin gas. The Chaplygin gas equation of state for a homogeneous model is given by

$$p = -A/\rho \tag{6.1}$$

where p and $\rho$are respectively pressure and energy density in commoving reference frame, with $\rho > 0$; $A$is a positive constant. The above equation is connected to string theory and can be achieved by the D-branes Nambu-Goto action which is moving in a (D+2)-dimensional space-time in the light-cone parametrization.

From the relativistic energy conservation equation using the equation of state (6.1) the density is given by

$$\rho_\Lambda = \sqrt{A + B/V^2} \tag{6.2}$$

where B is an integration constant.
Bento *et al.* [2002] generalized the equation of state (6.1) to

$$p = -A/\rho^\alpha, 0 \leq \alpha \leq 1 \tag{6.3}$$

which is known as generalized Chaplygin gas.

This equation of state leads to a density evolution as

$$\rho_\Lambda = \left[A + \frac{B}{V^{1+\alpha}}\right]^{1+\alpha} \tag{6.4}$$

Benaoum *et al*. [2012] within the framework of FRW introduced the modified Chaplygin gas whose equation of state is given by

$$p = A\rho - \frac{B}{\rho^\alpha},\ \alpha \geq 1 \tag{6.5}$$

where ρ and p are energy density and pressure respectively and A and B are positive constant.

The density evolving from the above equation of state is

$$\rho = \left[\frac{B}{1+A} + \frac{C}{V^{(1+A)(1+\alpha)}}\right]^{\frac{1}{\alpha+1}} \tag{6.6}$$

Chaubey et al. [2011] considering the generalized Chaplygin gas as a dark energy model studied the generalized Chaplygin gas to obtain the equation of state for the generalized Chaplygin gas energy density in anisotropic Bianchi Type I cosmological model. In this present paper, we obtain the equation of state for interacting Chaplygin gas energy density in Bianchi Type I cosmological model using Generalized Cosmic Chaplygin gas.

6.2 Generalized Cosmic Chaplygin Gas Interacting Dark Energy

When Generalized Cosmic Chaplygin gas energy density ρ_Λ interacts with Cold Dark Matter (CDM) we try to obtain the equation of state along with $w_m = 0$. The energy conservation equations interacting dark energy and CDM are given by

$$\dot{\rho}_\Lambda + 3H(1+w_\Lambda)\rho_\Lambda = -Q \tag{6.7}$$

$$\dot{\rho}_m + 3H\rho_m = Q \tag{6.8}$$

The interaction between the two is given by the quantity $Q = \Gamma\rho_\Lambda$, which is a decaying of the Chaplygin gas component with the decay rate Γ into cold dark matter.

Let us consider $r = \rho_m/\rho_\Lambda$ which is the ratio of the two energy densities. Then the above equations lead to

$$\dot{r} = 3Hr\left[w_\Lambda + \frac{1+r}{r}\frac{\Gamma}{3H}\right] \tag{6.9}$$

After following Kim et al. [2006], if we consider

$$w_{\Lambda}^{eff} = w_{\Lambda} + \frac{\Gamma}{3H}, w_{m}^{eff} = -\frac{1}{r}\frac{\Gamma}{3H} \tag{6.10}$$

Then in the standard form the continuity equations can be written as

$$\dot{\rho}_{\Lambda} + 3H(1 + w_{\Lambda}^{eff})\rho_{\Lambda} = 0 \tag{6.11}$$

$$\dot{\rho}_{m} + 3H(1 + w_{m}^{eff})\rho_{m} = 0 \tag{6.12}$$

The space-time metric of the spatially homogenous and anisotropic Bianchi type-I cosmological model is

$$ds^2 = dt^2 - a_1^2(t)\, dx^2 - a_2^2(t)\, dy^2 - a_3^2(t)\, dz^2 \tag{6.13}$$

where $a_1(t)$, $a_2(t)$ and $a_3(t)$ are functions of cosmic time t only.

With the time-dependent G and Λ the Einstein's field equations are given by

$$R_{ij} - \frac{1}{2} g_{ij} R = 8\pi G T_{ij} + \Lambda g_{ij} \tag{6.14}$$

The stress-energy-momentum tensor T_{ij} for a perfect fluid is given by

$$T_{ij} = (\rho_{\Lambda} + p_{\Lambda})\, u_i u_j - p_{\Lambda} g_{ij} \tag{6.15}$$

where u^i is the fluid four-velocity vector of the fluid satisfying the condition

$$u^i u_i = 1 \tag{6.16}$$

For the metric (6.13) the Einstein's field equations $(6.14$ with T_{ij} given by equation (6.15) in a co-moving system of coordinates are

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} = -8\pi G p_{\Lambda} + \Lambda \tag{6.17}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} = -8\pi G p_{\Lambda} + \Lambda \tag{6.18}$$

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} = -8\pi G p_{\Lambda} + \Lambda \tag{6.19}$$

$$\frac{\dot{a}_1\dot{a}_2}{a_1a_2}+\frac{\dot{a}_2\dot{a}_3}{a_2a_3}+\frac{\dot{a}_3\dot{a}_1}{a_3a_1}=8\pi G\rho_\Lambda+\Lambda \tag{6.20}$$

For the Bianchi type-I model the spatial volume is given by

$$V=a_1a_2a_3 \tag{6.21}$$

The average scale factor $'R'$ of anisotropic model is defined by

$$R=(a_1a_2a_3)^{\frac{1}{3}}=V^{\frac{1}{3}} \tag{6.22}$$

The generalized Hubble parameter H is defined as

$$H=\frac{1}{3}\left(H_x+H_y+H_z\right) \tag{6.23}$$

where $H_x=\frac{\dot{a}_1}{a_1}$, $H_y=\frac{\dot{a}_2}{a_2}$, $H_z=\frac{\dot{a}_3}{a_3}$ are the directional Hubble parameters in the direction of x, y and z axes respectively and an over dot denotes the differentiation with respect to cosmic time t.

From equations (6.22) and (6.23) it is obtained that

$$H=\frac{\dot{R}}{R}=\frac{1}{3}\left(\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}\right)=\frac{1}{3}\frac{\dot{V}}{V} \tag{6.24}$$

Following Saha et al. [2001] we obtain

$$a_1(t)=l_1V^{1/3}\exp\left(m_1\int\frac{dt}{V(t)}\right) \tag{6.25}$$

$$a_2(t)=l_2V^{1/3}\exp\left(m_2\int\frac{dt}{V(t)}\right) \tag{6.26}$$

$$a_3(t)=l_3V^{1/3}\exp\left(m_3\int\frac{dt}{V(t)}\right) \tag{6.27}$$

where $l_i(i=1,2,3)$ and $m_i(i=1,2,3)$are constants which satisfies the relations $l_1l_2l_3=1$ and $m_1+m_2+m_3=0$.

Now adding equations (6.17), (6.18) (6.19) and three times equation (6.20), we get

$$\frac{\ddot{V}}{V} = 12\pi G\left(\rho_{\Lambda} - p_{\Lambda}\right) + 3\Lambda \tag{6.28}$$

The critical density and the density parameters for matter and cosmological constant are, respectively, defined as

$$\rho_{cr} = \frac{3H^2}{8\pi G}; \Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{8\pi G\rho_m}{3H^2}; \Omega_{\Lambda} = \frac{\rho_{\Lambda}}{\rho_{cr}} = \frac{8\pi G\rho_m}{3H^2} \tag{6.29}$$

The relation r for ratio of energy densities from the above equations is obtained as

$$r = \frac{\Omega_m}{\Omega_{\Lambda}} \tag{6.30}$$

In the Generalized Cosmic Chaplygin gas the equation of state is

$$p_{\Lambda} = -\rho_{\Lambda}^{-\alpha}\left[C + \left(\rho_{\Lambda}^{1+\alpha} - C\right)^{-w}\right] \tag{6.31}$$

The density evolution from the above equation of state is

$$\rho_{\Lambda} = \left[C + \left\{1 + \frac{B}{V^{(1+\alpha)(1+w)}}\right\}^{\frac{1}{1+w}}\right]^{\frac{1}{1+\alpha}} \tag{6.32}$$

With respect to cosmic time we take derivatives on both sides of the above equation,

$$\dot{\rho}_{\Lambda} = -BV^{(1+\alpha)(1+w)}\frac{\dot{V}}{V}\left[C + \left\{1 + \frac{B}{V^{(1+\alpha)(1+w)}}\right\}^{\frac{1}{1+w}}\right]^{-\frac{\alpha}{1+\alpha}}\left[1 + \frac{B}{V^{(1+\alpha)(1+w)}}\right]^{-\frac{w}{1+w}} \tag{6.33}$$

Substituting this relation in equation (6.7) and using the definition $Q = \Gamma\rho_{\Lambda}$, we obtain

$$w_{\Lambda} = \frac{B}{V^{(1+\alpha)(1+w)}\left[1 + \frac{B}{V^{(1+\alpha)(1+w)}}\right]^{\frac{w}{1+w}}\left[C + \left\{1 + \frac{B}{V^{(1+\alpha)(1+w)}}\right\}^{\frac{1}{1+w}}\right]} - \frac{\Gamma}{(\dot{V}/V)} - 1 \tag{6.34}$$

We assume the decay rate given by the relation

$$\Gamma = b^2 (1+r) \left(\frac{\dot{V}}{V}\right) \tag{6.35}$$

with coupling constant b^2. Using equation (6.30) the above decay rate takes the following form

$$\Gamma = b^2 \left(\frac{\Omega_\Lambda + \Omega_m}{\Omega_\Lambda}\right)\left(\frac{\dot{V}}{V}\right) \tag{6.36}$$

Substituting the value of this in equation (6.34), then the generalized cosmic Chaplygin gas energy equation of state is given by

$$w_\Lambda = \frac{B}{V^{(1+\alpha)(1+w)}\left[1+\frac{B}{V^{(1+\alpha)(1+w)}}\right]^{\frac{w}{1+w}}\left[C+\left\{1+\frac{B}{V^{(1+\alpha)(1+w)}}\right\}^{\frac{1}{1+w}}\right]} - b^2\left(\frac{\Omega_\Lambda + \Omega_m}{\Omega_\Lambda}\right) - 1 \tag{6.37}$$

Now using the definition of generalized cosmic Chaplygin gas energy ρ_Λ and using Ω_Λ the above equation can be written as

$$w_\Lambda = \frac{B}{V^{(1+\alpha)(1+w)}\left[1+\frac{B}{V^{(1+\alpha)(1+w)}}\right]^{\frac{w}{1+w}}\left[\frac{1}{3}\Omega_\Lambda\left(\frac{\dot{V}}{V}\right)^2\right]^{1+\alpha}} - b^2\left(\frac{\Omega_\Lambda + \Omega_m}{\Omega_\Lambda}\right) - 1 \tag{6.38}$$

From equations (6.10), (6.36), (6.38) and the effective equation of state is given by

$$w_\Lambda^{eff} = \frac{B}{V^{(1+\alpha)(1+w)}\left[1+\frac{B}{V^{(1+\alpha)(1+w)}}\right]^{\frac{w}{1+w}}\left[\frac{1}{3}\Omega_\Lambda\left(\frac{\dot{V}}{V}\right)^2\right]^{1+\alpha}} - 1 \tag{6.39}$$

Now when the value of B is negative we get $w_\Lambda^{eff} < -1$, which corresponds to a phantom energy dominated universe and it corresponds to the effective parameter of state of Chaplygin gas for $\alpha = 1$. The term under the square root in the equation (6.32) for energy density should be positive for $\alpha = 1$, i.e. $V^2 < \left[\frac{B}{-1+(-C)^{1+w}}\right]^{\frac{1}{1+w}}$, then the minimal value for the volume factor is given by

$$V_{\min} = \left[\frac{B}{-1+(-C)^{1+w}}\right]^{\frac{1}{2(1+w)}} \tag{6.40}$$

Now the minimal value of the scale factor from equations (6.25), (6.26), (6.27), (6.40) are given by

$$a_1(t) = l_1\left[\frac{B}{-1+(-C)^{1+w}}\right]^{\frac{1}{6(1+w)}} \exp\left(m_1\left[\frac{B}{-1+(-C)^{1+w}}\right]^{-\frac{1}{2(1+w)}} t\right) \tag{6.41}$$

$$a_2(t) = l_2\left[\frac{B}{-1+(-C)^{1+w}}\right]^{\frac{1}{6(1+w)}} \exp\left(m_2\left[\frac{B}{-1+(-C)^{1+w}}\right]^{-\frac{1}{2(1+w)}} t\right) \tag{6.42}$$

$$a_3(t) = l_3\left[\frac{B}{-1+(-C)^{1+w}}\right]^{\frac{1}{6(1+w)}} \exp\left(m_3\left[\frac{B}{-1+(-C)^{1+w}}\right]^{-\frac{1}{2(1+w)}} t\right) \tag{6.43}$$

For this model $C > 0, B < 0$ and $1+\alpha > 0$ which is a bouncing universe. From equation (6.32) it is observed that the value of cosmic scale factor lies in the interval $R_{i\ \min} < R_i < \infty$(for $i = 1, 2, 3$) which yields $0 < \rho < (C+1)^{\frac{1}{1+\alpha}}$, where

$$R_{i\ \min} = l_i\left[\frac{B}{-1+(-C)^{1+w}}\right]^{\frac{1}{3(1+\alpha)(1+w)}} \exp\left(m_i\left[\frac{B}{-1+(-C)^{1+w}}\right]^{-\frac{1}{(1+\alpha)(1+w)}} t\right), \tag{6.44}$$

$i = 1, 2, 3$

From equation (6.2) we observe that the Chaplygin gas interpolates between dust and cosmological model for small and large values of the scale factor R_i. If we consider a homogenous scalar field and potential field to describe the Chaplygin Cosmology, then

$$\dot{\phi}^2 = \left[C + \left\{1 + \frac{B}{R^{3(1+\alpha)(1+\omega)}}\right\}^{\frac{1}{1+\omega}}\right]^{\frac{1}{1+\alpha}}$$
$$\left[1 - \left\{C + \left\{1 + \frac{B}{R^{3(1+\alpha)(1+\omega)}}\right\}^{\frac{1}{1+\omega}}\right\}^{-1}\left\{C + \left\{1 + \frac{B}{R^{3(1+\alpha)(1+\omega)}}\right\}^{\frac{-\omega}{1+\omega}}\right\}\right] \tag{6.45}$$

Now by choosing negative values of B we get $\dot{\phi}^2 < 0$, then $\phi = i\psi$.

The Lagrangian of scalar field $\phi(t)$ in this case is given by

$$L = \frac{1}{2}\dot{\phi}^2 - V(\phi) = -\frac{1}{2}\dot{\psi}^2 - V(i\psi) \tag{6.46}$$

Corresponding to the scalar field $\psi$the energy density and the pressure are respectively

$$\rho_\psi = -\frac{1}{2}\dot{\psi}^2 + V(i\psi) \tag{6.47}$$

$$p_\psi = -\frac{1}{2}\dot{\psi}^2 - V(i\psi) \tag{6.48}$$

Therefore, ψ the scalar field is a phantom field. Thus interacting generalized cosmic Chaplygin gas dark energy model in anisotropic universe generates equation of state which corresponds to phantom energy.

6.3 Conclusion

As a dark energy candidate Chaplygin gas plays an important role in describing the accelerated expansion because it represents different phases of the universe from the early stage to later stage of the universe as a pressure less fluid and as a cosmological constant respectively. In this chapter, we have studied the equation of state of generalized cosmic Chaplygin gas to obtain the Chaplygin gas energy density interacting dark energy in anisotropic Bianchi type I cosmological model. The effective equation of state is obtained as

$$w_\Lambda^{eff} = \frac{B}{V^{(1+\alpha)(1+w)}\left[1 + \frac{B}{V^{(1+\alpha)(1+w)}}\right]^{\frac{w}{1+w}}\left[\frac{1}{3}\Omega_\Lambda\left(\frac{\dot{V}}{V}\right)^2\right]^{1+\alpha}} - 1$$

We get $w_\Lambda^{eff} < -1$ for the homogeneous scalar field $\phi(t)$, by taking a negative value for B, which corresponds to a phantom dark energy dominated universe.