Bianchi Type-III Spacetime and Generalized Cosmic Chaplygin Gas

7.1 Introduction

The most recent notable observational discoveries have shown that our universe is presently accelerating. In order to explain why the cosmic acceleration happens, many theories have been proposed. The main stream explanation for this is known as theories of dark energy. The existence of dark energy fluids appear from the observations of the accelerated expansion of the universe. The isotropic-pressure cosmological models give the best fitting of the observations. However, some authors have suggested a cosmological model with anisotropic and viscous dark energy in order to explain an anomalous cosmological observation, in the cosmic microwave background (CMB) at the largest angles. The Bianchi universe anisotropies give rise to CMB anisotropies depending on the model type. The isotropization of the Bianchi metrics is due to the implicit assumption that the dark energy is isotropic in nature. It is well known that the exact solution of general theory of relativity for the homogeneous space time belongs to either Bianchi type or Kantowski-Sachs spacetime. Singh and Singh [1991a, 1991b] have presented Bianchi type-I,III,V,VI_0 and Kantowski-Sachs cosmological model with time-dependent displacement field and have presented a comparative study of Robertson-Walker models with constant deceleration parameter in the presence of

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cosmological term in Einstein theory with the cosmological theory based on Lyra geometry. Bali et al. [2010] have described Bianchi type-III cosmological models for barotropic perfect fluid distribution with variable G and Λ in general relativity. Yadav and Yadav [2011] have investigated Bianchi type-III bulk viscous and barotropic perfect fluid cosmological models in Lyra geometry. Bianchi type-III cosmological model in f(R,T) theory of gravity have been discussed by Reddy et al. [2012] Pradhan et al. [2012] discussed anisotropic Bianchi type-III string cosmological models in normal gauge for Lyra's manifold with electromagnetic filed. Adhav et al. [2015] discussed the Bianchi type-III cosmological with quadratic equation of state. Singh and Rani [2015] studied Bianchi type-III cosmological model with modified Chaplygin gas in Lyra geometry. Adhav et al. [2014] discussed the Bianchi type-III cosmological model with linear equation of state. Behera et al. discussed bulk viscous Bianchi type III models with time dependent G and Λ in the framework of Einstein's general relativity. Vidyasagar et al. have discussed Bianchi type III bulk viscous fluid in presence of one dimensional cosmic string in Saez-Ballester theory. Sahoo and Mishra studied Bianchi type III viscous fluid models in bimetric theory of gravitation.

In this chapter we consider the generalized cosmic Chaplygin gas in Bianchi type-III universe for perfect fluid. The generalized cosmic Chaplygin gas is

$$p = -\rho^{-\alpha}\left[C + \left(\rho^{1+\alpha} - C\right)^{-\omega}\right] \tag{7.1}$$

This is the generalization of Chaplygin gas equation of state and is suitable for representing dark energy. The statefinder parameters are given by

$$r = \frac{\dddot{R}}{RH^3}, \quad \text{and} \quad s = \frac{r-1}{3(q-1/2)} \tag{7.2}$$

7.2 Metric and field equations:

The homogeneous Bianchi type III spacetime is

$$ds^2 = dt^2 - a_1^2 dx^2 - a_2^2 e^{-2mx} dy^2 - a_3^2 dz^2 \tag{7.3}$$

where $a_1(t)$, $a_2(t)$ and $a_3(t)$ are function of cosmic time t only and $m \neq 0$ is a constant.

The energy momentum tensor is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{7.4}$$

The Einstein's field equations are

$$G_{ij} = R_{ij} - \frac{1}{2}R g_{ij} = -T_{ij} \tag{7.5}$$

where

$$g_{ij}u^i u^j = 1 \tag{7.6}$$

The Einstein's field equations for the metric (7.2) using equations (7.3) and (7.4) can be written as

$$\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_1\dot{a}_3}{a_1a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} - \frac{m^2}{a_1^2} = \rho \tag{7.7}$$

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} = -p \tag{7.8}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1\dot{a}_3}{a_1a_3} = -p \tag{7.9}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1\dot{a}_3}{a_1a_3} - \frac{m^2}{a_1^2} = -p \tag{7.10}$$

$$m\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right) = 0 \tag{7.11}$$

where dot denotes the derivative with respect to time.

The conservation law for the energy momentum tensor gives

$$\dot{\rho} + \left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right)(\rho + p) = 0 \tag{7.12}$$

The average scale factor and the spatial volume are defined as

$$R = \sqrt[3]{a_1a_2a_3}, V = R^3 = a_1a_2a_3 \tag{7.13}$$

The generalized mean Hubble parameter H is given by
$H = \frac{1}{3}(H_x + H_y + H_z)$, where, $H_x = \frac{\dot{a}_1}{a_1}, H_y = \frac{\dot{a}_2}{a_2}, H_x = \frac{\dot{a}_3}{a_3}$

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\dot{R}}{R} = \frac{1}{3}(H_x + H_y + H_z) \tag{7.14}$$

Deceleration parameter in cosmology is the measure of the cosmic acceleration of the universe expansion and is defined as

$$q = -\frac{\ddot{R}R}{\dot{R}^2} \tag{7.15}$$

The relation between average Hubble's parameter and average scale factor is given by

$$H = DR^{-m} \tag{7.16}$$

where $D > 0$ and $m \geq 0$.

From (7.13) and (7.15) we get,

$$\dot{R} = DR^{-m+1} \tag{7.17}$$

Integrating equation (7.16) we get,

$$R = l_0 e^{Dt}, m = 0 \tag{7.18}$$

$$R = (mDt + l_1)^{\frac{1}{m}}, m \neq 0 \tag{7.19}$$

where l_0 and l_1 are constants of integration. The two values of the average scale factors correspond to two different models of the universe.

7.3 GCCG model of the universe when $m = 0$

The average scale factor of the universe when $m = 0$ is $R = l_0 e^{Dt}$.

From equations (7.13) and (7.17) we get,

$$V = R^3 = a_1 a_2 a_3 = l_0^3 e^{3Dt} = c_1 e^{3Dt} \tag{7.20}$$

where $c_1 = l_0^3$.

Since the shear scalar σ is proportional to scalar expansion θ, which gives

$$a_3 = a_1^n \tag{7.21}$$

where, $n > 1$ is a constant. Integrating equation (7.10) we get,

$$a_2 = d_1 a_1 \tag{7.22}$$

where d_1 is the constant of integration and let $d_1 = 1$ then $a_2 = a_1$

From equations $(7.19) - (7.22)$ we get,
$a_1 = c_2 e^{\frac{3Dt}{n+2}}$, where $c_2 = (c_1)^{\frac{1}{n+2}}$
where c_1 is an arbitrary integration constant.

From equations (7.1), (7.11), (7.12) and (7.13) we get,

$$\rho = \left[C + \left\{ 1 + \frac{B}{V^{(1+\alpha)(1+\omega)}} \right\}^{\frac{1}{1+\omega}} \right]^{\frac{1}{1+\alpha}} \tag{7.23}$$

where *B* is an arbitrary integration constant.

Case(i): For small values of the scale factors $a_1(t)$, $a_2(t)$ and $a_3(t)$ we have

$$\rho \cong \frac{B^{(1+\alpha)(1+w)}}{V} \tag{7.24}$$

which is a very large value and corresponds to the universe dominated by an equation of state

$$p = -\frac{1}{\rho^{\alpha(1+w)+w}} \tag{7.25}$$

From (7.19) and (7.25) we get,

$$\rho \cong \frac{B^{(1+\alpha)(1+w)}}{c_1 e^{3Dt}} \tag{7.26}$$

$$p = -\frac{1}{\left[\frac{B^{(1+\alpha)(1+w)}}{c_1 e^{3Dt}} \right]^{\alpha(1+w)+w}} \tag{7.27}$$

The physical parameters expansion scalar θ, mean anisotropy parameter Δ and shear scalar σ^2 are

$$\theta = 3H = 3D \tag{7.28}$$

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = \frac{3(n^2+2)}{(n+2)^2} - 1 \tag{7.29}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right) = \frac{3}{2}D^2\left(\frac{n-1}{n+2}\right)^2 \tag{7.30}$$

The values of statefinder parameters using (7.2) and (7.17) are given by $r = 1$ and $s = 0$.

Case(ii): For large values of the scale factors we have

$$\rho \cong (C+1)^{\frac{1}{(1+\alpha)}} \tag{7.31}$$

For this value of ρ, the value of p is $p = -\rho$

$$p = -(C+1)^{\frac{1}{(1+\alpha)}} \tag{7.32}$$

The expansion scalar θ, mean anisotropy parameter Δ and shear scalar σ^2 remain same.

7.4 GCCG model of the universe when $m \neq 0$:

The average scale factor of the universe when $m \neq 0$ is $R = (mDt + l_1)^{\frac{1}{m}}$.

From equations (7.2) and (7.18) we get,

$$V = R^3 = a_1 a_2 a_3 = (mDt + l_1)^{\frac{3}{m}} \tag{7.33}$$

From equations (7.21), (7.22) and (7.33) we get,

$$a_1 = (mDt + l_1)^{\frac{3}{m(n+2)}} \tag{7.34}$$

Case(i): For small values of the scale factors $a_1(t)$, $a_2(t)$ and $a_3(t)$ we have

$$\rho \cong \frac{B^{(1+\alpha)(1+w)}}{V} \tag{7.35}$$

which is a very large value and corresponds to the universe dominated by an equation of state

$$p = -\frac{1}{\rho^{\alpha(1+w)+w}} \tag{7.36}$$

From (7.33), (7.35) and (7.36) we get,

$$\rho \cong \frac{B^{(1+\alpha)(1+w)}}{(mDt + l_1)^{\frac{3}{m}}} \tag{7.37}$$

$$p = -\frac{1}{\left[\frac{B^{(1+\alpha)(1+w)}}{(mDt+l_1)^{\frac{3}{m}}}\right]^{\alpha(1+w)+w}} \tag{7.38}$$

The physical parameters expansion scalar θ, mean anisotropy parameter Δ and shear scalar σ^2 are

$$\theta = 3H = \frac{3D}{mDt + l_1} \tag{7.39}$$

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = \frac{3(n^2+2)}{(n+2)^2} - 1 \tag{7.40}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right) = \frac{3(n-1)^2D^2}{(n+2)^2(mDt+l_1)^2} \tag{7.41}$$

and $q = m - 1$

The values of statefinder parameters are given by

$$r = (1-m)(1-2m) \tag{7.42}$$

$$s = \frac{(1-m)(1-2m) - 1}{3\left(m - \frac{3}{2}\right)} \tag{7.43}$$

Case(ii): For large values of the scale factors we have

$$\rho \cong (C+1)^{\frac{1}{(1+\alpha)}} \tag{7.44}$$

For this value of ρ, the value of p is $p = -\rho$

$$p = -(C+1)^{\frac{1}{(1+\alpha)}} \tag{7.45}$$

The expansion scalar θ, mean anisotropy parameter Δ and shear scalar σ^2 remain same.

7.5 Stability analysis:

The sound speed is given as

$$C_s^2 = \frac{dp}{d\rho} \tag{7.46}$$

When $C_s^2 \geq 0$, the model becomes physically acceptable. To obtain C_s^2, for generalized cosmic Chaplygin gas model of the universe corresponding to $m = 0$ we use equations (7.1), (7.20) and (7.23) for the model of the universe corresponding to $m \neq 0$ we use equations (7.1), (7.20) and (7.32). We plotted the resulting C_s^2 in terms of time as shown in fig. 1 and fig. 2 and for both the models we get, $0 \leq C_s^2 \leq 1$. Thus we can say that the generalized cosmic Chaplygin gas model is satable.

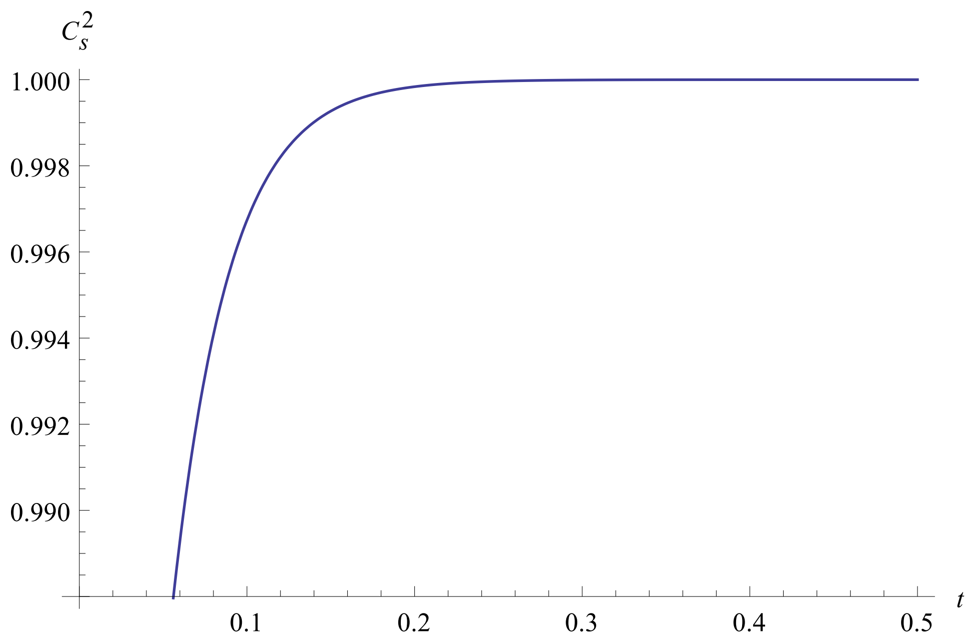

Figure-1 : Variation of C_s^2 vs. time t when $m = 0$

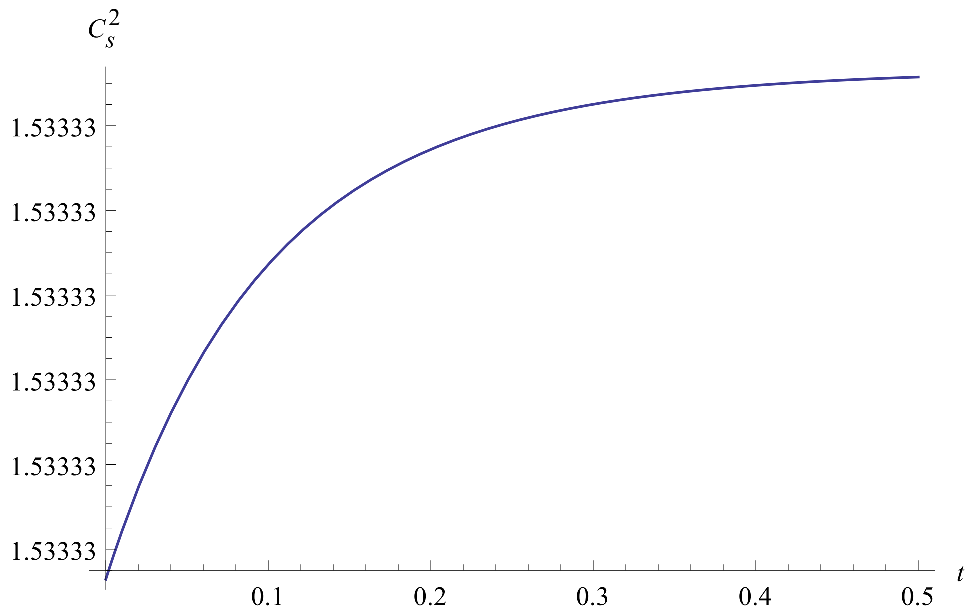

Figure-2 : Variation of C_s^2 vs. time t when $m \neq 0$

7.6 Discussions and Conclusions:

In this chapter we explored the cosmological solutions of Bianchi type-III universe filled with generalized cosmic Chaplygin gas. The exact solutions of this models have been investigated for small and large values of the scale factor and also we examined the stability of the models.

For $m \neq 0$: In this case, at $t = 0$, the energy density almost takes a constant value until it reaches a singularity at time given by where it faces a bounce. After that the density gradually decreases until it (the density) tends to zero as $t \to \infty$, thus undergoing a period of expansion. In this type of universe the expansion gradually decreases with time until at $t \to \infty$ the expansion almost stops ready for a contraction phase. On the other hand, for our model to be a realistic model we know that it must undergo accelerating expansion which imposes that $m < 1$. Though the model here seems to be anisotropic, it gradually becomes isotropic until at $m = 1$ it becomes totally isotropic.

For $m = 0$ **:** For large values of the scale factor our model becomes a pressureless universe if $c = -1$ and if this event is taken to be happened at an early stage of the universe and also since here $\frac{p}{\rho} = -1$ at other times which shows that the Chaplygin gas here behaves as a cosmological constant, we can conveniently take the modified Chaplygin gas in our model as a possible unification of dark energy and dark matter. For small values of the scale factor the density of the universe has a fixed value at $t = 0$ and tends to zero as $t \to \infty$ which shows the behavior of an oscillating universe, ofcourse without a bigbang, but with a bounce. From examining the statefinder parameters this universe seems to be filled with the cosmological constant type of dark energy.