Introduction

1.1 Standard Cosmology

Cosmology as a whole is the scientific study of the large scale properties of the universe. It endeavors to use the scientific method to understand the origin, evolution and ultimate fate of the entire universe. About 13 billion years ago at the creation of the universe, there must have been an initial singularity from which the space time abruptly started evolving. Ever since the Universe has more or less gone through a process of expansion and cooling from a tremendously hot and intense state to the present day cool state. In the foremost few seconds or so there was a very quick expansion, known as inflation which is responsible for the present homogeneous and isotropic Universe. Following this inflationary stage, further expansion cooled down the Universe and matter was formed in the course of action called baryogenesis. An assortment of light elements similar to deuterium, helium, lithium-7 were created in a process called Big Bang Nucleosynthesis. The universe was still very hot for the nuclei to combine electrons and form atoms, as a result causing the Universe to be opaque to photons and other electromagnetic radiation. In due course the temperature drops enough for free nuclei and electrons to merge into atoms in a process called recombination. After the formation of atoms photons could travel freely without being scattered. This caused the emission of Cosmic Microwave Background Radiation (CMBR) which gives us the information about the Universe at that time. Galaxies and stars began to form after a few hundred million of years as a result of gravitational collapse. Like any field of science, cosmology involves the formation of theories or hypothesis about the universe which makes specific predictions for phenomena that can be tested with observations. Depending on the outcome of the observations, the theories will need to be abandoned, revised or extended to accommodate the data. The prevailing theory about the origin and evolution of our universe is the so-called Big Bang theory. The central themes of modern cosmology is the idea that universe is expanding, and that this implies that at some time in the distant past it was incredibly dense and hot. Some of the most important problems in cosmology are associated with understanding how galaxies and clusters of galaxies formed, and determining the nature of the mass of the universe.

Modern Cosmology with the help of observational evidences has within its reach billions of galaxies and all the heavenly bodies spread all across the vast distances. Advanced observational techniques have strengthened the particular branch of science, sometimes supporting the conventional theories and sometimes producing reverse results. As a outcome of these observational advances cosmology turn out to be more or less data driven, so that all the theories require to be fitted with array of observations, although there are still doubts and debates about the reliability and interpretation of such data. The standard cosmological model which is very successful in describing the evolution of the Universe, is based on homogeneous and isotropic Friedmann-Robertson-Walker (FRW) spacetime.

The law of gravitation for space, filled with matter has been achieved in the gravitational field equations of general relativity. The universe assumed to consist of a vast swarm of electrically neutral galaxies whose mutual interaction is solely gravitational. Cosmology studies the global properties of this galactic multitude by solving Einstein field equation for the whole and deciding which solutions best satisfy known observational and assumed theoretical constraints. There is no unambiguous solution to this problem because there is quite a bit about the universe that is only well-informed conjecture, such as the existence of a big bang, and quite a bit that is plainly unknown, such as the extent of dark matter in the universe. But inspite of these uncertainties, there are strong limitations on the possibilities. In fact, these are surprisingly few solutions to the field equations that are taken seriously as models of the cosmological order.

1.2 Cosmological Principle

The cosmological principle consists of the basic belief that the fundamental laws of science are universal and two other simplifying assumptions about the nature of the universe when considered as a single entiry.

(a) The Homogeneity of Space : It is the belief that over the largest distance scales, matter and energy are distributed approximately uniformly. There is no preferred location in space.

(b) The Isotropy of Space : It states that there is no preferred direction in space.

So, the hypothesis that the universe is isotropic and homogeneous is known as the "cosmological principle". It means that the universe does not possess any privileged positions or directions. (This idea of such importance in cosmology that it has been elevated to the status of cosmological principle).The ordinary meaning of the assumption is that on a sufficiently large scale the universe has no preferred position or direction i.e. the picture of the universe as seen from different locations would be essentially the same and also there would be no observable difference between different directions. Obviously, in view of the manifest irregularities one should give a precise definition of what it meant by a sufficiently large scale but this is not easy. In any case, in order to be meaningful this "large scale" must be small compared to the universe as a whole or that portion of it which is the theoretically observable at a particular epoch. This idea is of such importance in cosmology that it has been elevated to the status of a principle.

There are many approaches one can take to the cosmological principle. One is philosophical approach in which the cosmological principle is quite satisfying – it is the antithesis of the earlier dogmatic view that the earth is at the centre of the universe. The philosophical approach is characterized by the work of Milne in the 1930s and later by Bondi, Gold and Hoyle in 1948. This line of reasoning is based, to a large extent, on the aesthetic appeal of the cosmological principle. Ultimately this appeal stems from the fact that it would indeed be very difficult for us to understand the universe of physical conditions, or even the laws of physics themselves, were to vary dramatically from place to place. These thoughts have been further leading to the perfect cosmological principle in which the universe is the same not only in all places and in all directions but also at all times. This stronger version of the cosmological principle was formulated by Bondi & Gold (1948) and it subsequently led Hoyle (1948) and Hoyle & Narlikar (1963, 1964) to develop the steady state cosmology. This theory implies, amongst other things, the continuous creation of matter to keep the density of the expanding universe constant. The steady state universe was abandoned in the 1960s because of the properties of the cosmic microwave background, radio sources and the cosmological helium abundance which are more readily explained in a Big Bang model than in a steady state. Nowadays the latter is only of historical interest.

Many attempts have also been made to justify the cosmological principle on more direct physical grounds. As we shall see homogeneous and isotropic universe described by the theory of General Relativity posses what is known as "cosmological horizon". The mystery is this: if two regions of the universe have never been able to communicate with each other by means of light signa physical conditions pertaining to each other. If they cannot know this how is it that they evolve in such a way that these conditions are the same in each of the regions. On either has to suppose that casual physics is not responsible of this homogeneity or that the calculation of the horizon is not correct. This conundrum is usually called the Cosmological Horizon Problem.

Several attempts have been made to avoid this problem. For example, particular models of the universe such as some that are homogeneous but not isotropic, do not possess the required particle horizon. These models can also isotropise in the cause of their evolution. A famous example is the "mix-master" universe of Misner (1968) in which isotropisation is effected by viscous dissipation involving neutrinos in the early universe. Another way to isotropise an initially anisotropic universe is by creating particles at the earliest stage of all the Plan era. Guth (1981) proposed an idea which could resolve the horizon problem : the inflationary universe which is of great contemporary interest in cosmology.

In any case the most appropriate approach to this problem is an empirical one. We accept the Cosmological Principle because it agrees with observations. Regarding this point the data concerning radio galaxies, clusters of galaxies quasars and microwave background all demonstrate that the level of anisotropy of the universe on large scale is very less.

1.3 Robertson-Walker Metric

The large-scale distribution of extragalactic nebular clusters in space is roughly isotropic around our own galaxy and that the number of cluster in a given volume seems to remain constant all over. To simplify the mathematical description, we therefore make the physical assumption that matter is distributed homogeneously in the world. Furthermore, we craving that the geometry of space will be determined by the matter distribution. This general prerequisite is known as Mach's principle. It is the name given to it by Einstein (1918) when generalizing Mach's hypothesis that the inertia of one body is due to the presence of all other bodies in the universe (Mach, 1883). Einstein's equations are one possible particularization of this principle.

In 1932, Einstein and de Sitter presented the Standard Cosmological Model of the Universe, which has been the most favourite among the cosmologists till 1980. Initially Einstein assumed homogeneity and isotropy in his cosmological problem. He choose a time coordinate t such that the line element of static space-time could be described by [Narlikar, An Introduction to Cosmology],

$$ds^2 = c^2 dt^2 - g_{\mu\nu} dx^\mu dx^\nu \tag{1.1}$$

where $g_{\mu\nu}$ are functions of space coordinates $x^\mu (\mu, \nu = 1, 2, 3)$.

We can now construct the homogeneous and isotropic closed space of three dimensions that Einstein wanted for his model of the Universe. The equation of such a 3-surface of a four dimensional hypersphere of radius a is given in Cartesian coordinates x_1, x_2, x_3, x_4 by

$$x_1^2 + x_1^2 + x_1^2 + x_1^2 = R^2 \tag{1.2}$$

Therefore the spatial line element on the surface is given by

$$d\sigma^2 = (dx_1)^2 + (dx_2)^2 + (dx_3)^2 + (dx_4)^2 = R^2[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)] \tag{1.3}$$

where $x_1 = R\sin\chi\cos\theta$, $x_2 = R\sin\chi\sin\theta\cos\phi$, $x_3 = R\sin\chi\sin\theta\sin\phi$, $x_4 = Ra\cos\chi$ and the ranges of θ, ϕ, and χ are given by $0 \leq \chi \leq \pi$, $0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$.

Another way to express $d\sigma^2$ through coordinates r, θ, ϕ with $r = \sin\chi$, $(0 \leq r \leq 1)$ is

$$d\sigma^2 = R^2\left[\frac{dr^2}{1 - r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{1.4}$$

The line element for the Einstein Universe is therefore given by

$$ds^2 = c^2dt^2 - d\sigma^2 = c^2dt^2 - R^2\left[\frac{dr^2}{1 - r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{1.5}$$

This line element is for positive curvature only.

In general we have,

$$ds^2 = c^2dt^2 - R^2\left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{1.6}$$

where, $k = 0$,$+1$,-1 for zero, positive, negative curvatures respectively and are also known as flat, closed, open models and $R(t)$ is known as the scale factor or expansion factor.

Thus for $c = 1$, FRW line element reduces to,

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{1.7}$$

The energy-momentum tensor describing the material contents of the Universe is given by

$$T_{\mu\nu} = (\rho c^2 + p)u_\mu u_\nu - p g_{\mu\nu} \tag{1.8}$$

where, $\rho = T_{00}$ is mean energy density of matter, $p = T_{ii}$ is the pressure, and $u_\mu = (c, 0, 0, 0)$ is fluid four velocity and $\Omega_i = \frac{\rho_i}{\rho_c}$, where $\rho_c = \frac{3H^2}{8\pi G}$, is called the critical energy density.

Also the equation of motion describing the Universe, known as Einstein field equations in general relativity are

$$G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R = \frac{8\pi G}{c^4}T_{ij} \tag{1.9}$$

where, G_{ij} = Einstein Tensor, R_{ij} = Ricci Tensor, R = Ricci Scalar.

Thus for a static $(\dot{R})$, dust filled $(p = o)$ and closed $(k = +1)$ model of the universe taking $8\pi G = c = 1$, the field equations become

$$\frac{3}{R(t)^2} = \rho, \frac{1}{R(t)^2} = 0 \tag{1.10}$$

From these equations no feasible solution is possible which suggest that no static homogeneous isotropic and dense model of the Universe is possible under the regime of Einstein equations stated above.

For this reason Einstein later modified his field equation as

$$G_{ij} = \frac{8\pi G}{c^4}T_{ij} + \Lambda g_{ij} \tag{1.11}$$

So he introduced the famous Λ-term, known as cosmological constant.

Einstein first proposed the cosmological constant as a mathematical fix to the theory

$$\frac{\dot{R(t)}^2}{R(t)^2} + \frac{k}{R(t)^2} = \frac{1}{3}\rho \tag{1.12}$$

$$\frac{\ddot{R(t)}}{R(t)} = -\frac{1}{6}(\rho + 3p) \tag{1.13}$$

in general relativity. In its simplest form, general relativity predicted that the universe must either expand or contract. Einstein thought the universe was static, so he added this new term to stop expansion. Friedmann, a Russian mathematician, realized that this was an unstable fix and proposed an expanding universe model called Friedmann model of the Universe.

For expanding Universe, $\dot{R} > 0$. But since for normal matter $\rho > 0$, $p \geq 0$, hence the second equation gives $\ddot{R} < 0$. So that, $\dot{R}$ is decreasing, i.e., expansion of Universe is decelerated. This model is known as Standard Cosmological Model (SCM).

Teams of prominent American and European Scientists using both Hubble Space Telescope (HST) and Earth based Telescopes had announced in 1998 the results of their many years observation and measurement of the expansion of the Universe. Their collective announcement was that the Universe is not just expanding, it is in fact, expanding with ever increasing speed. The SCM states that Universe is decelerating but recent high redshift type Ia Supernovae (explosion) observation suggests the Universe is accelerating [Perlmutter et al, 1998, 1999; Riess et al, 1998; Garnavich et al, 1998]. So there must be some matter field, either neglected or unknown, which is responsible for accelerating Universe.

For accelerating Universe, $\ddot{R} > 0$, i.e., $\rho + 3p < 0$, i.e., $p < -\frac{\rho}{3}$. Hence, the matter has the property of negative pressure. This type of matter is called Quintessence matter (Q-matter) and the problem is called Quintessence problem. The missing energy in Quintessence problem can be associated to a dynamical time dependent and spatially homogeneous or inhomogeneous scalar field evolving slowly down its potential $V(\phi)$.

These types of cosmological models are known as quintessence models. In this models the scalar field can be seen as a perfect fluid with a negative pressure given by $p = \omega\rho$, $(-1 \leq \omega \leq 1)$.

Introducing Λ term,

$$\frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = \frac{1}{3}(\rho + \Lambda) \tag{1.14}$$

$$\frac{\ddot{R}}{R} = -\frac{1}{6}(\rho + 3p) + \frac{1}{3}\Lambda = -\frac{1}{6}[(\rho + \Lambda) + 3(p - \Lambda)] \tag{1.15}$$

If Λ dominates, $\ddot{R} > 0$, i.e, Universe will be accelerating. For normal fluid, relation between p and ρ is given by, $p = \omega\rho$, $(0 \leq \omega \leq 1)$ which is called equation of state. For dust, $\omega = 0$; for radiation, $\omega = \frac{1}{3}$.

Here Λ satisfies an equation of state $p = -\rho$, so pressure is negative. Therefore for accelerating Universe we need such type of fluid which generates negative pressure. The most puzzling questions that remain to be explained in cosmology are the questions about the nature of these types of matter or the mystery of “missing mass”, that is, the “dark energy” and “dark matter” problem.

1.4 Bianchi Universes

Bianchi universes are the set of cosmological models that are homogeneous but not necessarily isotropic on spatial slices, named after Luigi Bianchi who classified the relevant 3-dimensional spaces. They contain, as a subclass, the standard isotropic models known as Friedmann-Lemaître-Robertson-Walker (FLRW) universes. Calculations of nucleosynthesis and microwave background anisotropies in Bianchi models have been compared against figures from the actual Universe, classically giving void results which can be into upper limits on anisotropy. Tentative detections of non-zero anisotropic shear by Jaffe et al. [2005] are currently believed to be inconsistent with other known cosmological parameters and with polarisation of the microwave background. Though the models stay widely-studied for their didactic value: homogeneity in space implies that the Einstein equations reduce from partial to ordinary differential equations in time, making them tractable exact solutions of Einstein's field equation. Early studies of the Bianchi models in the context of Einstein's equations were presented by Taub (1951), Heckmann & Schücking (1962) and Ellis and MacCallum (1969). The general phenomenology is extremely composite and the theme of a noteworthy body of work in mathematical physics. From an observational standpoint, one of the most important results is that of Wald's theorem (1983), which (alongside later works that expand the applicability) shows that universes with accelerating expansion tend towards isotropy. At countenance value this implies that, if the universe underwent an near the beginning period of inflation our present-day universe will appear highly isotropic; and, furthermore, since the universe is now starting to accelerate again, any anisotropy will remain small into the far future.

Cosmology is the study of the universe in its totality which deals with origin, structure and spacetime relationships with the universe and by extension, humanity's place in it. There has been considerable interest in the study of spatially homogeneous and anisotropic cosmological models of Bianchi type I-IX. The simplicity of the field equations made Bianchi space-time useful in constructing models of spatially homogeneous and anisotropic cosmologies. Bianchi models are spatially homogeneous and anisotropic universe models. According to Di Pierto and Demaret these models are nine in number but their classification permits to split them into two classes. The Bianchi type space time are generally defined by the following metric

$$ds^2 = -dt^2 + dl^2 \tag{1.16}$$

where $dl^2 = g_{ab}dx^a dx^b$.

Here dl^2 is the three dimensional line element. According to MacCallum [1979], spatially homogeneous cosmological models plays an important role in attempts to understand the structure and properties of the space of all cosmolocical solutions of Einstein field equations. Moreover from the theoretical point of view anisotropic universes have a greater generality than isotropic models. Although the present day universe is satisfactorily described by homogeneous and isotropic models given by the FRW space-time but as we know the universe in a smaller scale is neither homogeneous nor isotropic nor do we expect the universe in its early stages to have these properties. In fact to get physically realistic description of the universe one has to consider inhomogeneous models. In this case, the solutions of Einstein's field equations become more complicated or may be sometimes impossible. Therefore, many theoretical cosmologists are trying to use the spatially homogeneous and anisotropic Bianchi type models instead of inhomogeneous models. These types of space-times present a "middle way" between FRW models and inhomogeneous and anisotropic universes and hence play an important role in modern cosmology. The choices of anisotropic cosmological models in Einstein system of field equations lead to the cosmological models more general than Robertson-Walker model. Considerable works have been done in obtaining various Bianchi type cosmological models and their inhomogeneous generalization.

1.5 Dark Energy and Dark Matter

In most regions of the universe due to their mutual gravitational attraction dark matter and visible matter are found together. Non-baryonic dark matter is supposed to be made of one or more elementary particles other than the usual electrons, protons and neutrons. However, the vast majority of the dark matter in the universe is believed to be non-baryonic and thus not formed out of atoms. It is moreover thought not to intermingle with ordinary matter via electromagnetic forces. Non-baryonic dark matter is classified in terms of the mass of the particles that is assumed to make it up and the velocity dispersion of those particles. Based on these there are three prominent hypothesis on non-baryonic dark matter, called Hot dark matter (HDM), Warm dark matter (WDM) and Cold dark matter (CDM). Hot dark matter has zero or near zero mass particles allowing it to move with a velocity approximately over $0.95C$ which is very close to the velocity of light C. Thus HDM are particles that travel at ultra-relativistic velocities. This high velocity causes a high energy state that is not conductive to building structure.

There are a lot of experiments at present in succession or planned aiming to distinguish dark matter. These can be divided into two classes: direct detection experiment and indirect detection experiment. Direct detection experiment search for the scattering of dark matter particles of atomic nuclei within a detector and operate in deep underground laboratories to reduce the background from cosmic rays. The majority of present experiments use one of the two detector technologies: Cryogenic detectors and Noble liquid detectors. Indirect detection experiment search for the products of WIMP (weakly interacting massive particles) annihilations.

Existence of dark energy has been driven by the recently revealed accelerated expansion of the Universe. Acceleration of the Universe is to be expected when pressure is sufficiently negative and expansion of the Universe should decelerate if it is dominated by baryonic matter and CDM. Until the late 1990's cosmologists took it for approved that the expansion of the universe was slowing down under the influence of gravitation. A spectacular breakthrough happened in 1998 when two independent teams of astronomers, one led by S. Perlmutter and the other by A. G. Riess, were searching for distant supernovae hoping to measure the rate at which the expansion of the universe was slowing down. This discovery roundabout at a new negative pressure contributing to the mass-energy of the Universe. They traced the expansion of the universe over the past five billion years and were in a shock to find that the cosmic expansion is not slowing down but speeding up. This discovery has created a bewildering situation among the cosmologists because even though the standard cosmological models have been confirmed by data from Wilkinson Microwave Anisotropy Probe (WMAP) and by other telescope surveys of the large-scale structure of the universe, it was not known why the cosmic expansion is accelerating. To unveil the truth, intensive search is going on both in the theoretical and observational level. Many researchers suggested modifications and changes to Einstein's General Theory of Relativity. Some others expected a conventional explanation for the accelerating expansion of the universe based on a kind of repulsive force which acts as anti-gravity is responsible for gearing up the universe some five billion years ago. This hitherto unknown physical entity is dubbed as “dark energy” which has negative pressure and makes up about three quarters of the total present cosmic density. In some models of the dark energy, it is identified with cosmological constant. However, particle physics looks at cosmological constant as an energy density of the vacuum. Moreover, the energy scale of should be much larger than that of the present Hubble constant H_0, if it originates from the vacuum energy density. This is the cosmological constant problem [Weinberg, 1989]. Thus cosmological constant with equation of state may play the role to drive the recent cosmic acceleration. Another possibility is that there exists a universal evolving scalar field with equation of state , called quintessence field. Also a few more models have been projected recently in support of dark energy investigation. We discuss a few of these models below.

1.5.1 Cosmological Constant

The simplest form of dark energy is Cosmological constant. The cosmological constant is an additional expression in Einstein’s equations of general relativity which physically represents the prospect that there is a density and pressure connected with “empty space”. It represents the energy density of the vacuum and a potentially significant provider to the dynamical narration of the universe. Contrasting, ordinary matter, which can tuft simultaneously or diffuse as it evolves, the energy density in a cosmological constant is a property of space-time itself, and under ordinary state of affairs is identical all over the place. Quite the reverse to the penchant of ordinary forms of energy to slow down the collapse of distant objects, a adequately huge cosmological constant will grounds galaxies to emerge to accelerate away from us. The insertion of this vacuum energy expression can significantly affect cosmological theories.

Modern cosmology is largely built upon Einstein’s theory of general relativity in which the universe can be pictured as a finite entity with no boundaries rather like the surface of a ball. Einstein’s theory does not imply that the universe must necessarily be finite one of the perceived problems with a finite universe is that with only attracting masses in the universe there seemed to be no way to build a model so that the universe was not continually a model so that the universe was not continually contracting under its gravity. In order to solve his problem Einstein invented the cosmological constant, which acted like negative mass uniformly distributed throughout space acting in opposition to matter and so keeping the universe static.

The cosmological constant is denoted by the Greek capital letter lambda (Λ) occurs in Einstein's theory of general relativity as

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = - K T_{\mu\nu} , \qquad where \quad K = \frac{8\pi G}{C^4}$$

The constant is proportional to the energy-density of the vacuum ρ given by

$$\Lambda = \frac{8\pi G}{3C^2} \rho$$

whereπ is pi, G is the gravitational constant and C is the speed of light in vacuum.

The unit of Λ are 1/second^2.

Einstein field equations of general relativity consisting cosmological constant is

$$G_{ij} + \Lambda g_{ij} = 8\pi G T_{ij} \tag{1.17}$$

Friedmann equations then becomes,

$$H^2 + \frac{k}{R^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \tag{1.18}$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \tag{1.19}$$

and the conservation of energy equation,

$$\dot{\rho} + 3\frac{\dot{R}}{R}(\rho + p) \tag{1.20}$$

Now we define [Carroll et al., 1992],

$$\Omega_\rho = \frac{8\pi G}{3H_0^2}\rho_0, \Omega_\Lambda = \frac{\Lambda}{3H_0^2}, \Omega_k = -\frac{k}{H_0^2 R_0^2} \tag{1.21}$$

From equations (1.17) and (1.21) we get,

$$\Omega_\rho + \Omega_\Lambda + \Omega_k = 1 \tag{1.22}$$

We assume,

$$\Omega_\rho + \Omega_\Lambda = \Omega_{tot} \tag{1.23}$$

so that $\omega_{tot} < 1 (> 1)$ implies a spatially open (closed) Universe. Now as we have seen before,

$$p_{vac} = -\rho_{vac} \tag{1.24}$$

and thus the relation between Λ and vacuum energy density is,

$$\Lambda = -8\pi G \rho_{vac} \tag{1.25}$$

Thus the deceleration parameter ($q = -\frac{R\ddot{R}}{\dot{R}^2}$) becomes,

$$q = \frac{(1+3\omega)}{2}\Omega_\rho - \Omega_\Lambda \tag{1.26}$$

so that, for the present universe consisting of pressureless dust (dark matter) and Λ, the deceleration parameter takes the form,

$$q_0 = \frac{1}{2}\Omega_\rho - \Omega_\Lambda \tag{1.27}$$

The value of Λ can be positive, negative or zero. Since it is the energy density of empty space, it can be thought of as the "cost" of having space. According to general relativity a positive cosmological constant which means empty space has positive energy causes the expansion of empty space to accelerate because the cosmological constant has negative pressure.

An initial large value of Λ would explain inflation and galaxy formation, while subsequent slow decay of $\Lambda(t)$ would produce a small present value $\Lambda(t_0)$ to be reconciled with observations suggesting $\Omega_\Lambda \sim 0.7$ [Sahni etal, 2000]. For this purpose a few phenomenological models have been introduced, which Sahni has classified into three categories [Sahni, 2000], viz, (1) Kinematic models where Λ is a function of cosmic time t or the scale factor $R(t)$ in FRW cosmology; (2) Hydrodynamic models where Λ is described by a barotropic fluid with some equation of state and (3) Field-theoretic models where Λ is assumed to be a new physical classical field with some phenomenological Lagrangian.

In late 1990's of distance-redshift relations point towards that the universe is accelerating as observations are made. These annotations can be explained very well by pretentious a very small positive cosmological constant in Einstein's equations. The cosmological constant is in most economical soluition although there exists alternative possible causes of an accelerating universe. Thus, the existing Lambda-CDM model which measured to be on the order of 10^{-35}Sec^{-2} or about 10^{-123} in Planck units includes the cosmological constant. A precise fortitude of its value will be one of the major goals of observational cosmological in the near future.

1.5.2 Quintessence Scalar Field

The fine tuning trouble facing dark energy models with a constant equation of state can be avoided if the equation of state is tacit to be time reliant. In physics, quintessence is a theoretical form of dark energy, more specifically a scalar field, postulated as an explanation of the observation of an accelerating rate of growth of the universe, to a certain extent than due to a true cosmological constant. The first example of this scenario was proposed by Ratra and Peebles [1988] which slowly rolls down its potential such that the potential term dominates over the kinetic term and thus generates enough negative pressure to make the acceleration. The concept was expanded to more general types of time-varying dark energy and the term "quintessence" was first introduced in a paper by Robert R. Caldwell, Rahul Dave and Paul Steinhardt [1998]. It has been projected by several physicists to be a fifth fundamental force. Quintessence differs from the cosmological constant explanation of dark energy in that it is dynamic; that is, it changes over time, unlike the cosmological constant, which always stays constant. It is recommended that quintessence can be either attractive or repulsive based on the ratio of its kinetic and potential energy. Those working with this postulate believe that quintessence became repulsive about ten billion years ago, about 3.5 billion years after the Big Bang. Quintessence is a homogeneous minimally coupled scalar field ϕ which is a plausible alternative to the cosmological constant Λ. Quintessence model provides a solution to the fine-tuning problem and by means of tracker solutions it provides a solution to the coincidence problem also. But with particular potentials the scalar field ϕ lead to late time inflation. Since Quintessence relics on the potential energy of the scalar fields to lead to the late time acceleration of the universe, therefore it is possible to have a situation where the accelerated expansion arises out of modifications to the kinetic energy of the scalar fields. Quintessence is a canonical scalar field introduced to explain the late-time cosmic acceleration. This Q-field couple minimally to gravity so that the action for this field is given by

$$S = \int d^4x\sqrt{-q}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right) \tag{1.28}$$

where, $V(\phi)$ is the potential energy and the field ϕ is assumed to be spatially homogeneous. The energy-momentum tensor is given by, [Copeland et al. 2006]

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right) \tag{1.29}$$

For a scalar field ϕ, with Lagrangian density $L = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)$ in the background of flat FRW Universe, we have the pressure and energy density are respectively,

$$p = -T^\mu_\mu = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{1.30}$$

$$\rho = T^0_0 = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{1.31}$$

Hence the equation of state (EOS) parameter is,

$$\omega = \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \tag{1.32}$$

where an overdot means derivative with respect to cosmic time and prime denotes differentiation with respect to ϕ. Thus if $\dot{\phi}^2 \ll V(\phi)$, that is, Q-field varies very slowly with time, it behaves as a cosmological constant, as $\omega \approx -1$, with $\rho_{vac} \simeq V[\phi(t)]$. Now the equation of motion for the quintessence field is given by the Klein-Gordon equation,

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0 \tag{1.33}$$

From equation (1.32) we see that ω can take any value between -1 and +1 and varies with time [Frieman et al. 2008]. Also, $\omega < -\frac{1}{3}$ if $\dot{\phi}^2 < V(\phi)$ and $\omega < -\frac{1}{2}$ if $\dot{\phi}^2 < \frac{2}{3}V(\phi)$.

From theoretical point of view a lot of works [Caldwell et al, 1998; Ostriker et al, 1995; Peebles, 1984; Wang et al, 2000; Perlmutter et al, 1999; Dodelson et al, 2000; Faraoni, 2000] have been done to solve the quintessence problem. Scalar fields [Peebles et al, 1988, 2002; Ratra et al, 1988; Ott, 2001; Hwang et al, 2001; Ferreira et al, 1998] with a potential giving rise to a negative pressure at the present epoch, a dissipative fluid with an effective negative stress [Cimento et al, 2000] and more exotic matter like a frustrated network of non-abelian cosmic strings or domain wall [Bucher et al, 1999; Battye et al, 1999], scalar fields with non-linear kinetic term, dubbed K-essence model [Armendariz-Picon etal, 2001], are plausible candidates of Q-matter. Also, there exist models of quintessence in which the quintessence field is coupled to dark matter and/or baryons [Amendola, 2000].

Scalar fields, although, being very well-liked in theory, they have several shortcomings, as they need some appropriate potential to explain the accelerated expansion, they need to believe cosmological constant to be zero [Padmanabhan, 2006]. Also, most of the fields (Q-matter field, tracker field) work only for a spatially flat ($k = 0$) FRW model. Recently, Cimento et al [2000] showed that a combination of dissipative effects such as a bulk viscous stress and a quintessence scalar field gives an accelerated expansion for an open universe ($k = -1$) as well. This model also provides a solution for the 'coincidence problem' as the ratio of the density parameters corresponding to the normal matter and the quintessence field asymptotically approaches a constant value. Bertolami and Martins [2000] obtained an accelerated expansion for the universe in a modified Brans-Dicke (BD) theory by introducing a potential which is a function of the Brans-Dicke scalar field itself. Banerjee and Pavon [2001] have shown that it is possible to have an accelerated universe with BD theory in Friedmann model without any matter.

1.5.3 Chaplygin Gas

In recent years a lot of other models, other than cosmological constant and quintessence scalar fields, have also been proved to provide plausible explanation for dark energy. One of the most popular among these models is Chaplygin gas. To amalgamate the dark sector of the universe an economical and attractive idea is to consider it as a single component which acts as both dark energy and dark matter. Dark energy and dark matter unification is achieved by using the so-called Chaplygin gas (Kamenshchik et al. 2001). Chaplygin gas with negative pressure behaves similar to a pressureless fluid for small values of the scale factor as well as speed up the expansion of the universe for large values of the scale factor. Chaplygin gas gives a unified representation of dark matter and dark energy to confer new cosmological models in different ways. So it is well thought-out as a probable contender for dark energy model (Gorini et al. 2005) which was introduced in aerodynamics in 1904 for calculating the lifting force on a wing of an airplane. The equation of state for Chaplygin gas is

$$p = -\frac{B}{\rho} \tag{1.34}$$

where p and ρ are respectively pressure and energy density and B is a positive constant. The Born-Infeld Lagrangian density

$$\mathcal{L}_{BI} = -\sqrt{A}\sqrt{1 - g^{\mu\nu}\theta_{,\mu}\theta_{,\nu}} \tag{1.35}$$

gives rise to equation (1.34). Kamenshchik et al. [2001] showed that Chaplygin gas cosmology can interpolate between different phases of the Universe, starting from dust dominated phase to a de-Sitter Universe passing through an intermediate phase which is a mixture of a cosmological constant and a stiff matter (given by the EOS $p = \rho$) and thus can explain the evolution of the Universe from a decelerated phase to a stage of cosmic acceleration for a flat or open Universe, i.e., for $k = o$ or $k = -1$ and for a closed Universe with $k = 1$ they obtained a static Einstein Universe.

To find a homogeneous scalar field $\phi(t)$ and a self-interacting potential $V(\phi)$ corresponding to the Chaplygin gas cosmology, we consider the Lagrangian of the scalar field as,

$$\mathcal{L}_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{1.36}$$

Then analogous energy density ρ_ϕ and pressure p_ϕ for the scalar field are,

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{1.37}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{1.38}$$

An extension of this equation of state is generalized Chaplygin gas (Bento et al. 2002), an amalgamation of dark energy and dark matter is given by

$$p = -\frac{B}{\rho^\alpha} \tag{1.39}$$

This is obtained from the generalized Born-Infeld Lagrangian density given by,

$$\mathcal{L}_{GBI} = -A^{\frac{1}{1+\alpha}}\left[1 - \left(g^{\mu\nu}\theta_{,\mu}\theta_{,\nu}\right)^{\frac{1+\alpha}{2\alpha}}\right]^{\frac{\alpha}{1+\alpha}} \tag{1.40}$$

It was further extended to modified Chaplygin gas model as at high density the generalized Chaplygin gas match up to dust universe and also the observational data is not consistent with this model. The equation of state for modified Chaplygin gas (Benaoum 2002, 2012) is given by

$$p = A\rho - \frac{B}{\rho^\alpha} \tag{1.41}$$

where $0 \leq \alpha \leq 1$ and A and B are positive constants.

In 2003, P. F. González-Diaz have introduced the generalized cosmic Chaplygin gas (GCCG) model. Even though the vacuum fluid satisfies the phantom energy condition, the generalized cosmic Chaplygin gas model can be made stable which is free from unphysical behaviors. The equation of state of generalized cosmic Chaplygin gas is

$$p = -\rho^{-\alpha}\left[C + \left(\rho^{1+\alpha} - C\right)^{-\omega}\right] \tag{1.42}$$

where p and ρ are respectively the pressure and energy density, $C = \frac{B}{1+\omega} - 1$, with B being a constant that can take on both positive and negative values, and $-L < \omega < 0$, L being a positive definite constant, which can take on values larger than unity.

B. Pourhassan (2013) constructed the viscous modified cosmic Chaplygin gas and its equation of state is given by

$$p = A\rho - \frac{1}{\rho^{\alpha}}\left[\frac{B}{1+\omega} - 1 + \left(\rho^{1+\alpha} - \frac{B}{1+\omega} + 1\right)^{-\omega}\right] \tag{1.43}$$

The equation of state when we include bulk viscosity is given as

$$p = A\rho - \frac{1}{\rho^{\alpha}}\left[\frac{B}{1+\omega} - 1 + \left(\rho^{1+\alpha} - \frac{B}{1+\omega} + 1\right)^{-\omega}\right] - 3\zeta H \tag{1.44}$$

where ζ is the viscosity coefficient.

1.5.4 Tachyonic Field

In recent times rolling tachyon condensate has been premeditated as a source of dark energy. This is a scalar field of non-standard form motivated by string theory as the negative-mass mode of the open string perturbative spectrum. The fundamental idea is that the accustomed open string vacuum is unstable but there subsist a stable vacuum with zero energy density [Gibbons, 2002]. The unstable vacuum corresponds to rolling tachyon. Sen showed that this tachyonic state is connected with the condensation of electric flux tubes of closed strings described by Dirac-Born-Infeld action. Tachyonic field has a potential which rolls down from an unstable maximum to a stable minimum with a steady vacuum expectation value which is known as tachyon condensate [Das etal, 2005]. Sen [2002] has shown that the energy momentum tensor for rolling tachyon solution in D-branes in bosonic string theory is described by a pressureless gas with non-zero energy density, which is stored in open string field, even though there are no open string degrees of freedom around the minimum of tachyonic potential. Thus it represents dust, which can be painstaking as a candidate for CDM. Also the energy-momentum tensor of tachyon condensate can be split into two parts, one with $\omega = 0$ and the other with $\omega = -1$. This has led a lot of authors to construct cosmological models with tachyonic field as a candidate for dark energy, as dark matter and dark energy thus can be described by a single scalar field. For this reason in cosmology rolling of tachyon is equivalent to the spreading out of the Universe [Gibbons, 2002]. Also Sen [2002] presented that the supersymmetry breaking by tachyon matter can be attuned since the total energy of the tachyon matter is changeable and is determined by the early position and velocity of tachyon.

The Lagrangian density of tachyon condensate is given by the Born-Infeld action

$$\mathcal{L}_{tach} = -V(T)\sqrt{1 + g^{\mu\nu}\partial_\mu T \partial_\nu T}\sqrt{-\det(g_{\mu\nu})} = -V(T)\sqrt{-\det(G_{\mu\nu})} \quad (1.45)$$

where T is the tachyonic field, $V(T)$ is the tachyon potential having a local maximum at the origin and a global minimum at $T = \infty$ where the potential vanishes.

Tachyonic field can be treated as dark energy or dark matter depending on the form of the potential associated with it. Tachyon potential is usually assumed to be exponentially decaying or inversely quadratic.

1.6 Some aspects of works related to Chaplygin gas cosmology

In this thesis our investigations pertain to Chaplygin gas as such we have described some relevant work. Mainly in this section we have presented some of the relevant work carried out by various authors related to Chaplygin gas.

Kamenshchik et al. [2001] considered a FRW cosmological model with an exotic fluid known as Chaplygin gas and studied that the resulting evolution of the universe is not in disagreement with the current observation of cosmic acceleration. The model predict an increasing value for the effective cosmological constant.

Bento et al. [2002] have considered a generalization of the Chaplygin equation of state and studied that as in the case of the Chaplygin gas, where $\alpha = 1$, the model admits a d-brane connection as its Lagrangian density corresponds to the Born-Infeld action plus some soft logarithmic corrections. Also, spacetime evolves from a phase that is initially dominated, in the absence of other degrees of freedom on the brane, by non-relativistic matter to a phase that is asymptotically de Sitter.

Avelino et al. [2003] demonstrated that, the dynamics of a generalized Chaplygin gas model is equivalent to that of a standard ΛCDM model to first order in the metric perturbations. The gravity alone cannot distinguish between a generalized Chaplygin gas model with $\alpha = 0$ and a standard ΛCDM scenario.

Gorini et al. [2003] considered two cosmological models representing the flat Friedmann universe filled with a Chaplygin fluid, with or without dust, and analyzed in terms of statefinder parameters.

González-Díaz [2003] studied that if we choose a general equation of state for dark energy which is reasonably free from instabilities and unphysical effects, then a phantom energy can be predicted which does not show any big rip at finite time.

Amendola et al. [2003] compared the WMAP temperature power spectrum and SNeIa data to models with a generalized Chaplygin gas as dark energy. They discussed that generalized Chaplygin gas as a unified dark matter candidate (CDM = 0) appears much less likely than as a dark energy model, although its χ^2 is only two sigma away from the expected value.

Szydłowski et al. [2004] studied that the Friedmann-Robertson-Walker model with the generalized Chaplygin gas is structurally stable. They found the domains of cosmic acceleration as well as conditions for which the horizon problem is solved, defining some general class of fluids which generalize the Chaplygin gas.

Debnath et al. [2004] considered a model of modified Chaplygin gas and its role in the accelerating phase of the universe. They also studied statefinder parameters in characterizing different phases of the universe diagrammatically.

Bouhmadi-L$\acute{o}$pez [2005] studied Friedmann-Robertson-Walker quantum cosmology characterized by two Lorentzian sectors, separated by a classically forbidden region with a generalized Chaplygin gas. Zhai et al. [2006] discussed viscous generalized Chaplygin gas cosmology, assuming that there is bulk viscosity in the linear barotropic fluid and generalized Chaplygin gas.

Zhang et al. [2006] proposed a new model for describing the unification of dark energy and dark matter. This new model is a further generalization of the generalized Chaplygin gas (GCG) model, thus dubbed new generalized Chaplygin gas model. New generalized Chaplygin gas model is totally dual to an interacting XCDM parametrization scenario, in which the interaction between dark energy and dark matter is characterized by the constant α.

Gorini et al. [2006] discussed a possible theoretical basis for the Chaplygin gas in cosmology and also considered the relation with scalar field and tachyon cosmological models. Setare [2007] considered a correspondence between the holographic dark energy density and Chaplygin gas energy density in FRW universe and studied the potential and the dynamics of the scalar field which describe the Chaplygin cosmology.

Puxun Wu and Hongwei Yu [2007] studied model with the interaction between generalized Chaplygin gas and dark matter. They found that the stationary attractor exists and the universe will enter a de Sitter phase in which the energy densities of both the generalized Chaplygin gas and dark matter are constant. They also examined the constraints from the Gold sample Type Ia supernova (SN Ia) data, the Supernova Legacy Survey (SNLS) SN Ia data, and the size of the baryonic acoustic oscillation (BAO) peak found in the Sloan Digital Sky Survey (SDSS) on the generalized Chaplygin gas model, proposed as a candidate for the unified dark matter dark energy scenario (UDME) in the cases of both a spatially flat and a spatially curved universe. The results reveal that the generalized Chaplygin gas model is consistent with a flat universe up to the 68% confidence level, and the model parameters are within the allowed parameter ranges of the generalized Chaplygin gas as a candidate for unified dark matter dark energy.

Zong-Kuan Guo and Yuan-Zhong Zhang [2007] considered a new generalized Chaplygin gas model that includes the original Chaplygin gas model as a special case. In such a model the generalized Chaplygin gas evolves as from dust to quiescence or phantom. They found that the background evolution for the model is equivalent to that for a coupled dark energy model with dark matter.

Setare [2007] studied the generalized Chaplygin gas of interacting dark energy to obtain the equation of state for the generalized Chaplygin gas energy density in a non-flat universe. By choosing a negative value for B, he found that $\omega_{\Lambda}^{eff} < -1$, which corresponds to a universe dominated by phantom dark energy.

Chakraborty and Debnath [2007] considered a model of the universe filled with modified Chaplygin gas and another fluid with barotropic equation of state and its role in accelerating phase of the universe where the mixture of these two fluid models is valid from the radiation era to ΛCDM for $-1 \leq \gamma \leq 1$ and the radiation era to quintessence model for $\gamma < -1$.

Pedram et al. [2007] presented Chaplygin gas Friedmann–Robertson–Walker quantum cosmological model and studied the Schutz's variational formalism with positive, negative and zero constant spatial curvature.

Debnath [2007] proposed a model of variable modified Chaplygin gas and discuss its role in accelerating phase of the universe. It is shown that the equation of state of this model valid from the radiation era to quiessence model.

Bouhmadi-L$\acute{o}$pez [2008] studied some types of generalized Chaplygin gas in order to show how different sorts of singularities could appear in such models, either in the future or in the past.

Chattopadhyay and Debnath [2008] considered a new modified Chaplygin gas model which interpolates between radiation at an early stage and ΛCDM at a later stage which is regarded a unification of dark energy and dark matter. They derived the density parameters from the equation of motion for interaction between dark energy and dark matter.

Jamil and Rashid [2008] studied a unified model of dark energy and matter which is presented using the modified variable Chaplygin gas for interacting dark energy in a non-flat universe and derived the effective equations of state corresponding to matter and dark energy in this interacting model. Pun et al. [2008] studied viscous dissipative Chaplygin gas dominated homogenous and isotropic cosmological models.

Wang et al. [2009] proposed generalized Chaplygin gas as a candidate for unification of dark energy and dark matter and investigated constraints on this model with the latest observed data. They test the model with type-Ia supernovae (SNe Ia), cosmic microwave background (CMB) anisotropy, X-ray gas mass fractions in clusters, and gamma-ray bursts (GRBs).

Xing et al. [2009] studied variable modified Chaplygin gas and found that the effective state parameter of dark energy can cross the phantom divide $\omega_\Lambda = -1$ and our universe will not end up with a Big Rip in the future.

Chakraborty and Debnath [2009] studied anisotropic universe along with the model of modified Chaplygin gas and its role in accelerating phase. They assumed that the equation of state of this modified model is valid from the radiation era to ΛCDM model and obtained the possible relation between the hessence and the modified Chaplygin gas.

Lu et al. [2009] investigated observational constraints on the generalized Chaplygin gas model and the best-fit values of the model parameters with their confidence level are $A_s = 0.73^{+0.06}_{-0.06}(1\alpha)^{+0.09}_{-0.09}(2\sigma)$, $\alpha = -0.09^{+0.15}_{-0.12}(1\sigma)^{+0.26}_{-0.19}(2\sigma)$.

Chakraborty and Debnath [2010] by introducing inhomogeneity in the equation of state, presented a new form of the Chaplygin gas model which explains $\omega = -1$ crossing. They studied an interaction of this model with a scalar field by introducing a phenomenological coupling function and showed that the potential decays with time.

Jamil [2010] presented a model in which the new generalized Chaplygin gas interacts with matter where there exists a stable scaling solution at late times in the evolution of the universe.

YaBo [2010] et al. studied modified Chaplygin gas (MCG) as an interacting model of holographic dark energy in which dark energy and dark matter are coupled together. They found the evolution of the universe is from deceleration to acceleration by studying density parameter Ω, equation of state ω, deceleration parameter q and transition redshift Z_T. Also, their present values are consistent with the latest observations.

Roy and Buchert [2010] proposed that the inhomogeneous properties of matter and geometry obey the Chaplygin equation of state and both dark energy and dark matter are manifestations of spatial geometrical properties.

Bouhmadi-L*ó*pez [2011] et al. proposed modified Chpalygin gas model as a phenomenological model for the early universe where there is a smooth transition between an early quintessence phase and a radiation-dominated era. Observationally they mapped the primordial power spectrum of the scalar perturbations to the latest data of WMAP7.

Yadav [2011] studied that if the cosmic dark energy behaves like a fluid with equation of state $p = \omega\rho$; $\omega < -1$ as well as Chaplygin gas simultaneously then the big rip problem does not arise and the scale factor is found to be regular for all time.

Malekjani et al. [2011] in spatially flat universe investigated the generalized Chaplygin gas model and calculated equation os state parameter, deceleration parameter and dimensionless Hubble parameter. Also the dependency of transition from decelerated expansion to accelerated expansion on the parameters of model was investigated.

Jamil and Debnath [2011] investigated the background dynamics when dark energy is coupled to dark matter in the universe described by loop quantum cosmology by considering modified Chaplygin gas as the form of dark energy. A stable scaling solution was obtained by them by solving the dynamical system of equations.

Jamil et al. [2011] in the framework of f-essence cosmology considered the modified Chaplygin gas model which belongs to the class of a unified models of dark energy and dark matter. They constructed an equation connecting the modified Chaplygin gas and the f-essence and solved it to obtain explicitly the pressure and energy density of modified Chaplygin gas.

Adhav [2011] investigated modified Chaplygin gas within the framework of Bianchi Type-V universe which valid from the radiation era to the ΛCDM as well as considered statefinder parameters to characterize different phases of the universe.

Adhav [2011] also investigated variable modified Chaplygin gas having the equation of state $p = A\rho - \frac{B}{\rho^{\alpha}}$, where $0 \leq \alpha \leq 1$, A is a positive constant and B is a positive function of the average scale factor a(t) of the universe [i.e. B = B(a)] within the framework of Bianchi type-V universe. It was observed that the equation of state of the variable modified Chaplygin gas interpolates from radiation dominated era to quintessence dominated era.

Aviles et al. [2012] studied the generalized Chaplygin gas model by using geometrothermodynamics. Rudra [2012] studied the background dynamics of generalized cosmic Chaplygin gas when dark energy is coupled to dark matter with a suitable interaction in the universe described by brane cosmology.

Mazumder et al. [2012] studied Friedmann-Robertson-Walker cosmology with modified Chaplygin gas as the matter contained where the evolution equations are reduced into an autonomous dynamical system with a suitable change of variables and it can be interpreted as the motion of the particle in an one dimensional potential.

Panigrahi and Chatterjee [2012] considered a mixture of generalized Chaplygin gas and ordinary matter field to study the evolution of a universe for a Robertson–Walker type of spacetime. The instant of flip changes depending on the arbitrary constants and a bouncing model appears when the signature of one of the constants changes.

Katore and Shaikh [2012] studied the plane symmetric cosmological model with modified Chaplygin gas where the evolution of the universe from early universe dominated by a dust like matter to de-Sitter inflationary era. The deceleration parameter is found to be positive in the early phase of matter dominated era, which is crucial for the successful nucleosynthesis as well as for the structure formation of the universe.

Pourhassan [2013] studied viscous modified cosmic Chaplygin gas as a model of dark energy and used exponential function method to solve nonlinear equation and obtain time-dependent dark energy density. Hubble expansion parameter and scale factor are fixed by using observational data. Also effect of viscosity to the evolution of universe and stability of this theory was investigated.

Borges et al. [2013] performed a decomposition of Chaplygin gas by separating pressureless dark matter from a cosmological term, both necessarily interacting with each other not only in the background but on the perturbative level as well. Cosmological perturbations in the resulting two-component system are intrinsically non-adiabatic.

Noorbakhsh and Ghominejad [2013] studied the interaction between tachyon dark energy and modified Chaplygin gas which do not evolve separately but interact non-gravitationally with one another.

Sadeghi and Farahani [2013] studied within the framework of Einstein gravity the interaction between the general form of viscous varying modified cosmic Chaplygin gas and the Tachyon fluid.

Herrera et al. [2013] studied an intermediate inflationary universe model in the context of a generalized Chaplygin gas considering two different energy densities; a standard scalar field and a tachyon field, respectively.

Rudra [2013] investigated the role played by dark energy in the form of generalized cosmic Chaplygin gas in an accelerating universe described by FRW cosmology.

Salti [2013] investigated the validity of the second law of gravitational thermodynamics in an expanding Gödel-type universe filled with generalized Chaplygin gas interacting with cold dark matter.

Saadat [2013] studied the behavior of the energy density of modified Chaplygin gas with respect to the constant $\alpha = 0.5$ and time-dependent bulk viscosity. Saadat [2013] also studied viscous generalized Chaplygin gas in the case of non-flat universe and obtain modified Friedmann equations due to viscosity.

Li and Xu [2013] studied viscous generalized Chaplygin gas (VGCG) as a unified dark fluid, which modifies the pressure only by redefining the effective pressure p_{eff}, according to $p_{eff} = p - \sqrt{3}\zeta_0$, where ζ_0 is the newly added model parameter which characterizes the viscous property and can be determined by the cosmic observations.

Amani and Pourhassan [2013] studied generalized Chaplygin gas which has viscosity for the case of arbitrary α and obtained modified time-dependent energy density due to bulk viscosity and generalized Chaplygin gas.

Naji and Saadat [2013] studied new varying modified cosmic Chaplygin gas which has viscosity in presence of cosmological constant and space curvature. The behavior of dark energy density was obtained by using well-known forms of scale factor in Friedmann equation numerically. Saadat and Pourhassan [2013] studied varying generalized Chaplygin gas which has viscosity in presence of cosmological constant and space curvature.

Pourhassan and Kahya [2014] introduced extended Chaplygin gas model as alternative to the dark energy which recovers barotropic fluid with quadratic and higher order equation of state.

JianBo et al. [2014] compared the variable generalized Chaplygin gas model as the unification of dark sectors with observations.

Debnath and Maity [2014] studied generalized cosmic Chaplygin gas and new variable modified Chaplygin gas types of dark energies in the framework of F-essence cosmology and investigated the consequences for their co-existence.

Naji [2014] studied extended Chaplygin gas equation of state with bulk viscosity assumed as power law form of density and shear viscosity considered as a constant.

Saadat [2014] studied interaction between modified cosmic Chaplygin gas and pressureless matter in presence of both bulk and shear viscosities as a model of the Universe.

Karimiyan and Naji [2014] studied viscous modified Chaplygin gas in presence of cosmological constant where bulk viscosity is a function of density as well as considered interaction between modified Chaplygin gas and baryonic matter.

Khurshudyan et al. [2014] studied three different models of dark energy in higher-dimensional spacetime where the first model is a single-component universe including viscous varying modified Chaplygin gas, the second model includes viscous varying modified Chaplygin gas and ghost dark energy and the third model includes viscous modified cosmic Chaplygin gas and ghost dark energy.

Ghose et al. [2014] investigated in the framework of compact Kaluza-Klein cosmology, holographic dark energy correspondence of interacting generalized Chaplygin gas.

Naji et al. [2014] studied variable viscous generalized cosmic Chaplygin gas in the presence of cosmological constant and space curvature and observational data is used to fix solution and stability of model is discussed.

Sadatian [2014] studied different Rip singularity scenarios and bouncing model of the universe in context of modified Chaplygin gas dark energy model.

Sharifa and Saleem [2014] considered an inflationary universe model by taking the matter field as standard and tachyon scalar fields in the context of the generalized cosmic Chaplygin gas.

Yang [2014] investigated both linear and nonlinear growth of the large-scale structure in the superfluid Chaplygin gas model.

Kahya et al. [2015] studied variable modified Chaplygin gas and extended Chaplygin gas in the framework of higher order f(R) modified gravity with varying G and Λ.

Khurshudyan [2015] studied extended Chaplygin gas in Lyra Manifold with a varying Λ-term, which does give us modified field equations.

Sadeghi et al. [2015] studied the holographic dark energy density and interacting extended Chaplygin gas energy density in the Einstein gravity.

Kahya and Pourhassan [2015] studied a universe dominated by the extended Chaplygin gas considering the second-order term which recovers quadratic barotropic fluid equation of state.

Singh and Rani [2015] studied Bianchi Type III cosmological model with modified Chaplygin gas within the framework of Lyra's Geometry.

Kotambkar et al. [2015] discussed Bianchi Type V space-time model with equilibrium pressure and modified generalized Chaplygin gas considering G and Λ.

Ramos [2015] studied two viscous fluids which obey a generalized Chapyglin gas equation of state within the Friedmann–Robertson–Walker formulation and analytical solutions for the density have been obtained for scale factors that are power, exponential, and products of power and exponential functions of time.

Sadeghi et al. [2015] considered varying modified Chaplygin gas in case of variable G and Λ with two different toy models. In the first model, the universe is filled with a phenomenological gas while in the second one, there is the existence of gas and a matter with $P = \omega(t)\rho_m$.

Lu et al. [2015] investigated reduced modified Chaplygin gas and the viabilities of dark cosmological models for different values of model parameter by discussing the evolutions of cosmological quantities and using the currently available cosmic observations.

Ebadi and Moradpour [2015] proposed the thermodynamical interpretation for the modified generalized Chaplygin gas confined to the apparent horizon of FRW universe, whiles dark sectors do not interact with each other.

Santhi et al. [2016] studied in the framework of Brans-Dicke scalar-tensor theory of gravitation, the spatially homogeneous and anisotropic LRS Bianchi type-V universe filled with variable modified Chaplygin gas.

Pourhassan [2016] studied the evolution of the universe using a two-component fluid constituted from extended Chaplygin gas alongside a phantom scalar field extracting solutions for the various cosmological eras, focusing on the behavior of the scale factor, the various density parameters and the equation of state parameter.

Singh et al. [2016] studied the Bianchi type-I space-time in presence of bulk viscosity and Chaplygin gas in the context of Lyra's geometry.

Jawad and Iqbal [2016] studied the spherical top-hat collapse in Einstein gravity and loop quantum cosmology by taking the nonlinear evolution of viscous modified variable Chaplygin gas and viscous generalized cosmic chaplygin gas.

Pourhassan [2016] investigated cosmological models of the extended Chaplygin gas in a universe governed by Horava–Lifshitz gravity.

Singh and Roy Baruah [2016] studied generalized cosmic Chaplygin gas in Bianchi Type-I anisotropic Universe and law of variation of Hubble's parameter.

Singh and Roy Baruah [2016] investigated the universe filled with generalized cosmic Chaplygin gas and barotropic fluid.

Singh and Roy Baruah [2016] investigated the generalized cosmic Chaplygin gas considering perfect fluid in Bianchi Type-I universe for large scale factor.

Kotambkar et al. [2017] studied of generalized Chaplygin gas model with dynamical gravitational and cosmological constants within the framework of anisotropic Bianchi Type I universe.

1.7 Hubble's law and Hubble's constant

Hubble discovered that the galaxies recede from the earth with a velocity that is proportional to their distance i.e. the recession velocity is proportional to the distance.

$\therefore$Recessional velocity = Hubble's constant times distance i.e. V = H D, where V is the observed velocity of the galaxy away from us, usually in km/sec.

H is Hubble's constant in km/sec/MPc and D is the distance to the galaxies in MPc.

The basic entity in this law is however the Hubble's constant H which has to be first calibrated accurately before the law can be used. The calibration of H however contains some inherent uncertainties in it. One has to derive by independent methods the distances to galaxies for which the red-shifts are significant. The recessional speed must largely supersede the random speed of the galaxy. For the purpose the Virgo Cluster of galaxies has so far been considered the most suitably situated. It contains a large number of bright galaxies and at it distance (m-M $\cong$ 31), the photometric method of distance measurement is applicable on the one hand and, on the other hand, red-shifts are significant. But unfortunately, the random velocities of the individual member-galaxies of the Virgo Cluster about the centre of mean recessional motion are of the same order as the mean recessional motion itself.

From the above relation we know that the Hubble Parameter or Hubble constant H defines the rate of cosmic expansion. The recession velocity V of an object situated at a distance D given by $H = \frac{V}{D}\cdot$ Also it is the logarithmic derivative of the scale factor R(t) i.e. $H = \frac{\dot{R}(t)}{R(t)}\cdot$

Since the mean recessional speed of the cluster is computed from the motions of these individual members, themselves having a large random motion, large uncertainly may be introduced in the computation of the mean recessional speed. When this speed is used to compute the value of H, that value should be accepted with reservation have devotedly worked for many decades for the correct evaluation of H.

In 1929 Hubble estimated the value of the expansion factor, now called the Hubble constant to about 500 km/sec/MPc. For many decade the controversy rests between two groups of astronomers. Alan Sandage and his co-authors claim on the bases of their observation that H = 50kmS^{-1}MPc^{-1} on the other hand G. de Vauculeurs and his co-authors claim that the value should be around 10 kmS^{-1}MPc^{-1}. But many outsiders thought the geometric mean of theirse value H = 71 kmS^{-1}MPc^{-1} was a good compromise. The controversy persists while authors often work with some intermediate value of H. Much work has been done in sixties and seventies with H = 75 kmS^{-1}MPc^{-1}. Considering various aspects of the problem and inherent uncertainties in the determination. A Dressler has suggested that H = 70 kmS^{-1}MPc^{-1} should be a better acceptable value. Many authors however currently working with the value 50 kmS^{-1}MPc^{-1}. Bret from the latest source the Hubble space Telescope key project team came up with the answer.

H = 75 +/-8 km/sec/MPc.

And finally, WMAP came up with

H = 71 +/-3.5 km/sec/MPc.

where (1 MPc = 3.26 million light years).

1.8 Perfect fluid

In many attractive situations in astrophysical general relativity, the foundation of the gravitational field can be taken to be a perfect fluid as a first approximation. In general, a ‘fluid’ is a special kind of continuum. A continuum is a assortment of particles so abundant that the dynamics of entity particles cannot be followed, leaving only a description of the collection in terms of ‘average’ or ‘bulk’ quantities: number of particles per unit volume, density of energy, density of momentum, pressure, temperature, etc. A perfect fluid is a frictionless, homogeneous and incompressible fluid which is incapable of sustaining any tangential stress or action in the form of a shear but the normal force acts between the adjoining layers of fluid. The pressure at every point of a perfect fluid is equal in all directions, whether the fluid be at rest or in motion. It can be completely characterized by its rest frame energy stresses, viscosity and heat conduction. Perfect fluids are often used in general relativity to model idealized distributions of matter, such as the interior of a star or an isotropic universe.

The energy-momentum tensor describing matter is given by

$$T^{\mu\nu} \;=\; \rho\, u^{\mu}\, u^{\nu} \;+\; S^{\mu\nu} \tag{1.46}$$

where ρ is the mass density, u^{μ} is the four-velocity $u^{\mu} \;=\; \frac{dx^{\mu}}{dS}$ of the individual particles, and S^{μ} is the stress tensor where the speed of the light C = 1. If the matter consists of perfect fluid, namely, one whose pressure is isotropic the stress tensor can be expressed as

$$S^{\mu\nu} \;=\; p\; (u^{\mu} u^{\nu} \;-\; g^{\mu\nu}) \tag{1.47}$$

where p is the pressure.

Thus the energy-momentum-tensor becomes

$$T^{\mu\nu} \;=\; (\rho \;+\; p)\; u^{\mu}\, u^{\nu} \;-\; p\, g^{\mu\nu} \tag{1.48}$$

The only stress they can sustain is the isotropic pressure p :ρ is the mass density. It is generally agreed that except in the early universe, the pressure of the sources can be neglected. The pressure of the dust is obviously 0 in any direction since there is no motion of the particles, i.e. the dust is a pressureless fluid. A perfect fluid with zero pressure is technically referred to as dust. Such that sits still on the substratum. Since any random motion would constitute a pressure. In the early universe, however, uniform radiation is thought to have predominated. This does have pressure, its equation of state is

$$p = \frac{1}{3}\rho \quad (1.49)$$

1.9 Statefinder Diagnostic

Over the years a lot of models have proved to be viable candidates of Dark Energy, thus leading to the problem of discriminating between these models. For this purpose Sahni et al. [2003] proposed a new geometric diagnosis (dimensionless) to characterize the properties of dark energy in a model independent manner. They introduced a pair of parameters called statefinder parameters depending on the scale factor and its derivatives, defined by,

$$r = \frac{\dddot{R}}{RH^3}, \quad \text{and} \quad s = \frac{r-1}{3(q-1/2)} \quad (1.50)$$

where $q = -\frac{R\ddot{R}}{\dot{R}^2}$ is the deceleration parameter. The parameter r forms the next step in the hierarchy of geometrical cosmological parameters after H and q. In fact trajectories in the s, r plane corresponding to different cosmological models demonstrate qualitatively different behaviour. For spatially flat space-time ($k = 0$), considering the Universe to be consisted of non-relativistic matter Ω_m i.e., CDM and baryons, and dark energy $\Omega_x = 1 - \Omega_m$, the statefinder pair r, s takes the form, [Sahni et al. 2003]

$$r = 1 + \frac{9}{2}\Omega_x\omega(1+\omega) - \frac{3}{2}\Omega_x\frac{\dot{\omega}}{H} \quad (1.51)$$

$$s = 1 + \omega - \frac{1}{3}\frac{\dot{\omega}}{\omega H} \quad (1.52)$$

Thus for ΛCDM model with a non-zero Λ ($\omega = -1$), r = 1 and s = 0.

If ω is constant these parameters reduce to

$$r = 1 + \frac{9}{2}\Omega_x\omega(1+\omega) \quad (1.53)$$

$$s = 1 + \omega \tag{1.54}$$

For a quintessence scalar field these parameters take the forms

$$r = 1 + \frac{12\pi G\dot{\phi}^2}{H^2} + \frac{8\pi G\dot{V}}{H^3} \tag{1.55}$$

$$s = \frac{2(\dot{\phi}^2 + 2\dot{V}/3H)}{\dot{\phi}^2 - 2V} \tag{1.56}$$

For pure and generalized Chaplygin Gas, we get respectively, [Gorini et al., 2003]

$$r = 1 - \frac{9}{2}s(1+s) \tag{1.57}$$

$$r = 1 - \frac{9}{2}s(\alpha+s)/\alpha \tag{1.58}$$

For modified Chaplygin gas

$$18(r-1)s^2 + 18\alpha s(r-1) + 4\alpha(r-1)^2 = 9sA(1+\alpha)(2r+9s-2) \tag{1.59}$$

In general, for one fluid model, these r, s can be written as

$$r = 1 + \frac{9}{2}\left(1 + \frac{p}{\rho}\right)\frac{\partial p}{\partial \rho} \tag{1.60}$$

$$r = \left(1 + \frac{\rho}{p}\right)\frac{\partial p}{\partial \rho} \tag{1.61}$$

Gorini et al. [2003] have shown that for pure Chaplygin gas s varies in the interval $[-1, 0]$ and r first increases from $r = 1$ to its maximum value and then decreases to the ΛCDM fixed point $s = 0, r = 1$. For generalized Chaplygin gas, the model becomes identical with the standard ΛCDM model for small values of α from statefinder viewpoint. Debnath etal have shown that in case of modified Chaplygin gas the Universe can be described from radiation era to ΛCDM with statefinder diagnosis.

1.10 Kaluza-Klein Theory

The original Kaluza-Klein theory was one of the first attempts to create an unified field theory i.e. the theory, which would unify all the forces under one fundamental law. In 1921, it was published by German mathematician and physicist Theodor Kaluza and extended in 1926 by Oskar Klein. The fundamental idea of this theory was to postulate one extra compactified space dimension and introduce nothing but pure gravitiy in new (1+4)-dimensional space-time. In our observable (1 + 3)-dimensional space-time, it turns out that the 5-dimensional gravitiy manifests as gravitational, electromagnetic and scalar field. It is considered to be an important precursor to string theory.