CHAPTER 2

METHODOLOGY

2.1 Introduction

Fuzzy set theory was discovered by Zadeh [86] in 1965. The theory of fuzzy sets actually has been a generalization of the classical theory of sets in the sense that the theory of sets should have been a special case of the theory of fuzzy sets. But unfortunately it has been accepted that for fuzzy set A and its complement A^C, neither $A \cap A^C$ is empty set nor $A \cup A^C$ is the universal set. Whereas the operations of union and intersection of crisp sets are indeed special cases of the corresponding operation of two fuzzy sets, they end up giving peculiar results while defining $A \cap A^C$ and $A \cup A^C$.

Also from usual definition of complement of fuzzy sets, it can be visualized that the complement of a fuzzy set can remain included in the set itself which is not desirable from mathematical point of view. After a considerable period of time, the requirement of a new definition of complement of fuzzy set is realized by many researchers. Baruah ([8, 9, 10, 11, 12, 13, 14]) also expressed some dissatisfaction regarding the existing definition of complementation of fuzzy set as well as probability-possibility consistency principles. To overcome the drawbacks that exists in the conventional fuzzy set theory, Baruah ([8, 9, 10, 11, 12, 13, 14]) realize the way towards

developing of new definition of complement of fuzzy sets by introducing the notion of reference function.

The most standard method considered in finding the membership function of fuzzy numbers is the method of α-cut. In this chapter, an alternative method has been proposed to find the membership function of a fuzzy number. Dubois and Prade[30, 31, 32] and Kaufmann and Gupta[38] have defined a fuzzy number $X = [a,b,c]$ with membership function

$$\mu_N(x) = L(x), \text{ if } a \leq x \leq b,$$
$$= R(x), \text{ if } b \leq x \leq c, \text{ and} \quad \ldots\ldots\ldots\ldots\ldots\ldots.(2.1.1)$$
$$= 0, \text{ otherwise,}$$

where $L(x)$ being a continuous non-decreasing function in the interval $[a, b]$, and $R(x)$ being a continuous non-increasing function in the interval $[b, c]$, with $L(a) = R(c) = 0$ and $L(b) = R(b) = 1$. Dubois and Prade named $L(x)$ as left reference function and $R(x)$ as right reference function of the concerned fuzzy number. A continuous non-decreasing function of this type is also called a distribution function with reference to a Lebesgue-Stieltjes measure.

The basic problem in constructing normal fuzzy number was the misunderstanding in defining the partial presence of an element in an interval. Indeed various explanations regarding the possible relationship between probability and fuzziness have come up, and no concrete conclusion could be arrived at. Baruah ([8, 9, 10, 11, 12, 13, 14]) has recently shown that two laws of randomness can define a normal law of fuzziness. This has led to a proper measure

theoretic explanation of partial presence, and construction of fuzzy numbers can be based on that. Accordingly, the Dubois-Prade's left reference function $L(x)$ is the distribution function in the interval $a \leq x \leq b$ and the right reference function $R(x)$ is the complementary distribution function in $b \leq x \leq c$. Thus the functions $L(x)$ and $(1- R(x))$ would have to be associated with densities $\frac{d}{dx}(L(x))$ and $\frac{d}{dx}(1 - R(x))$ in $[a, b]$ and $[b, c]$ respectively. Thus two laws of randomness are necessary and sufficient to construct a normal fuzzy number, which leads to the *Randomness-Fuzziness Consistency Principle* based on the superimposition of sets. This chapter presents the discussion about the proposed methods, a description about the procedures of the methods, the advantages and applications.

2.2 COMPLEMENTATION OF FUZZY SET ON THE BASIS OF REFERENCE FUNCTION

According to the Zadehian definition, if a normal fuzzy number $N = [\alpha, \beta, \gamma]$ is associated with a membership function $\mu_N(x)$, where

$$\mu_N(x) = \Psi_1(x), \text{ if } \alpha \leq x \leq \beta,$$

$$= \Psi_2(x), \text{ if } \beta \leq x \leq \gamma,$$

$$= 0, \text{ otherwise.}$$

The complement N^C will have the membership function $\mu_N{}^C(x)$, where

$$\mu_N{}^C(x) = 1 - \Psi_1(x), \text{ if } \alpha \leq x \leq \beta,$$

$$= 1 - \Psi_2(x), \text{ if } \beta \leq x \leq \gamma, \text{ and}$$

$$= 1, \text{ otherwise.}$$

We first cite two counterexamples that this definition is defective.

Counterexample - 1: First, we would like to ask the readers a simple question. Can a statement and its complement ever be the same? Common sense says that the answer is negative. Consider now the set of real numbers with constant fuzzy membership function equal to 1/2 everywhere. Therefore according to the Zadehian definition, its complement too will have the constant fuzzy membership function equal to $(1 − 1//2) = 1/2$ everywhere.

In other words, here is an example of a statement defining a fuzzy number, which is exactly the same as the statement defining its complement! Accordingly, if the Zadehian definition of the complement of a fuzzy set is true, we have arrived at a contradiction that a statement and its complement can be the same. Some people might still argue that a half truth is half wrong too, and therefore they are equivalent! We therefore proceed to cite a second counterexample.

Counterexample - 2: We would like to ask the readers another simple question. Can a statement ever include its complement? Once again, common sense says that the answer is negative. Consider now the set of real numbers with constant fuzzy membership function equal to 3/4 everywhere. Therefore according to the Zadehian definition, its complement will have constant fuzzy membership function equal to $(1 − 3/4) = 1/4$ everywhere.

In other words, here is an example of a statement that actually includes its complement! Accordingly, if the Zadehian definition of the complement of a fuzzy set is true, we have arrived at a contradiction that a statement can include its complement.

Our counterexamples are based on the fact that if a glass is partially filled with water, then the height of empty portion is to be counted from the height upto which the glass is partially full. These two counterexamples should be enough to establish that the very definition of complement of a fuzzy set is wrong. In fact, here logic has been forced to follow mathematics. Where is the error then? Fuzzy membership function and fuzzy membership value are two different things. In the Zadehian definition of complementation, these two things have been taken to be the same, and that is where the error lies.

Baruah ([8, 9, 10, 11, 12, 13, 14]) gave an idea of definition of fuzzy set on the basis of reference function. According to Baruah ([8, 9, 10, 11, 12, 13, 14]) to define a fuzzy set, two functions namely fuzzy membership function and fuzzy reference function are necessary. Fuzzy membership value is the difference between fuzzy membership function and fuzzy reference function.

Let $\mu_1(x)$ and $\mu_2(x)$ be two functions such that $0\leq \mu_2(x) \leq \mu_1(x) \leq 1$. For fuzzy number denoted by $\{x, \mu_1(x), \mu_2(x) ; x\epsilon X\}$, we call $\mu_1(x)$ as fuzzy membership function and $\mu_2(x)$ a reference function such that $(\mu_1(x) - \mu_2(x))$ is the fuzzy membership value for any x in X.

The meaning of the definition is as follows. In the definition of complement of fuzzy set, the fuzzy membership value and fuzzy membership function have to be different, in the sense that for a usual fuzzy set the membership value and the membership function are of course equivalent. However, our definition is not to contradict with others definition of fuzzy set and we do not really need our definition to describe the usual fuzzy sets. In defining complement of a fuzzy set, the use of reference function is very essential to get proper result.

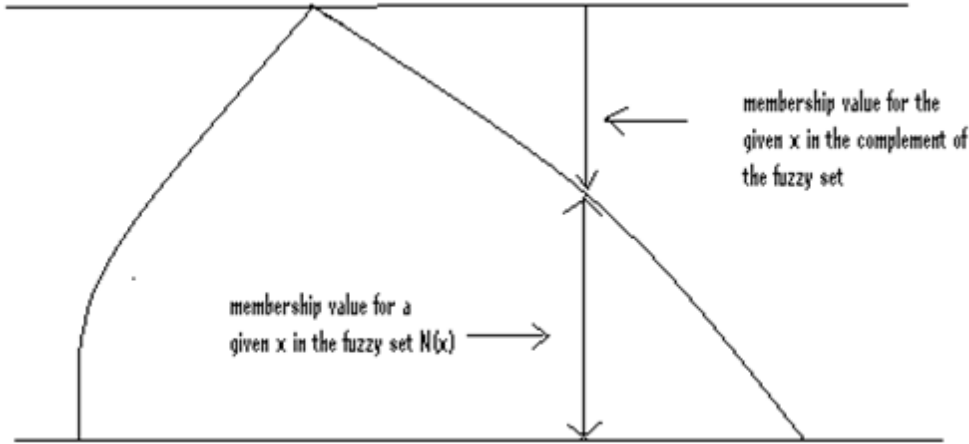Now let us discussed with the help of following diagram

Figure 2: Complement of a fuzzy set on the basis of reference function

Now if A and B are two fuzzy sets with new definition of fuzzy sets and let these are

$A(\mu_1, \mu_2)=\{x, \mu_1(x), \mu_2(x); x\epsilon X\}$ and $B(\mu_3, \mu_4)=\{x, \mu_3(x), \mu_4(x); x\epsilon X\}$.

Then the operations of intersection and union are defined as

$A(\mu_1, \mu_2)\cup B(\mu_3, \mu_4) =\{x, \max(\mu_1(x), \mu_3(x)), \min(\mu_2(x), \mu_4(x)); x\epsilon X\}$.

$A(\mu_1, \mu_2)\cap B(\mu_3, \mu_4) =\{x, \min(\mu_1(x), \mu_3(x)), \max(\mu_2(x), \mu_4(x)); x\epsilon X\}$.

Two fuzzy sets $C= \{x, \mu_C(x); x\epsilon X\}$ and $D= \{x, \mu_D(x); x\epsilon X\}$ in the usual definition would be

expressed as $C(\mu_C, 0)= \{x, \mu_C(x), 0; x\epsilon X\}$ and $D(\mu_D, 0)= \{x, \mu_D(x), 0; x\epsilon X\}$ in our way.

$C(\mu_C, 0)\cup D(\mu_D, 0) =\{x, \max(\mu_C(x), \mu_D(x)), \min(0, 0); x\epsilon X\}$.

$$=\{x, \mu_C(x)\vee\mu_D(x); x\epsilon X\}.$$

Which in our in the usual definition is nothing but $C\cup D$.

Similarly, $C(\mu_C, 0) \cap D(\mu_D, 0) = \{x, \mu_C(x) \wedge \mu_D(x); x \in X \}$, Which in usual definition is nothing but $C \cap D$.

Note: It is clearly seen that for usual fuzzy sets $\mu_2(x) = \mu_4(x) = 0$.

Now for two fuzzy sets $A(\mu, 0) = \{x, \mu(x), 0; x \in X\}$ and $B(1, \mu) = \{x, 1, \mu(x); x \in X\}$ defined over the same universe X, we would have

$A(\mu, 0) \cap B(1, \mu) = \{x, \min(\mu(x), 1), \max(0, \mu(x)); x \in X\}$

$$= \{x, \mu(x), \mu(x); x \in X\}$$

which is nothing but the null set φ.

Now taking union of these two fuzzy sets, we see that

$A(\mu_A, 0) \cup B(1, \mu_A) = \{x, \max(\mu_A(x), 1), \min(0, \mu_A(x)); x \in X\}$

$$= \{x, 1, 0; x \in X\}$$

which is nothing but the universal set X.

In other words, $B(1, \mu)$ defined above is nothing but $(A(\mu,0))^C$ in the classical sense of set theory. This means, if we define the fuzzy set $(A(\mu, 0))^C = \{x, 1, \mu(x); x \in X\}$, it can be seen that it should be nothing but the complement of the fuzzy set $A(\mu, 0) = \{x, \mu(x), 0; x \in X\}$.

Now from above discussion it is cleared that, a fuzzy set defined by $A = \{x, \mu(x); x \in X\}$, would be defined as $A = \{x, \mu(x), 0; x \in X\}$, so that complement would become

$A^C = \{x, 1, \mu(x); x \in X\}$.

We therefore conclude that if we express the complement of a fuzzy set $A = \{x, \mu_A(x), 0; x \in X\}$ as $A^C = \{x, 1, \mu_A(x); x \in X\}$, we get $A \cap A^C =$ the null set φ, and $A \cup A^C =$ the universal set X. This would enable us to establish that the fuzzy sets do form a field if we define complementation in our way.

Accordingly, if a normal fuzzy number $N = [\alpha, \beta, \gamma]$ is defined with membership function $\mu_N(x)$, where

$$\mu_N(x) = \Psi_1(x), \text{ if } \alpha \leq x \leq \beta,$$

$$= \Psi_2(x), \text{ if } \beta \leq x \leq \gamma, \text{ and}$$

$$= 0, \text{ otherwise,}$$

such that, $\Psi_1(\alpha) = \Psi_2(\gamma) = 0,$

$$\Psi_1(\beta) = \Psi_2(\beta) = 1,$$

where $\Psi_1(x)$ is the distribution function of a random variable defined in the interval $[\alpha, \beta]$, and $\Psi_2(x)$ is the complementary distribution function of another random variable defined in the interval $[\beta, \gamma]$, with randomness defined in the measure theoretic sense.

The complement N^C will have the membership function $\mu_N{}^C(x)$, where

$$\mu_N{}^C(x) = 1, \ -\infty < x < \infty,$$

where the values of $\mu_N{}^C(x)$ are to be counted from $\Psi_1(x)$, if $\alpha \leq x \leq \beta$, from $\Psi_2(x)$, if $\beta \leq x \leq \gamma$, and from 0, otherwise, so that there happens to be a difference between a fuzzy membership *function* and the corresponding fuzzy membership *values*.

Our definition of complementation of a fuzzy set thus based on the following axiom:

Axiom-1: The fuzzy membership function of the complement of a *normal* fuzzy number is equal to 1 for the entire real line, with the membership values counted not from zero but from the membership function of the fuzzy number concerned.

2.3 PROPERTIES OF FUZZY SETS WHEN FUZZY SET IS EXPRESSED ON THE BASIS OF REFERENCE FUNCTION

Let A, B and C are fuzzy sets, then some properties of are shown below

    (i)   Idempotent: A∩A=A and A∪A=A

    (ii)   Commutative: A∩B= B∩A and A∪B= B∪A

    (iii)   Associative: (A∩B)∩A= A∩(B∩C) and (A∪B)∪C = A∪(B∪C)

    (iv)   Distributive: A∩(B∪C)= (A∩B) ∪ (A∩C) and A∪ (B∩C)= (A∪B) ∩ (A∪C)

    (v)   Double complement: $(A^C)^C$=A

    (vi)   DeMorgan's laws: $(A∩B)^C = A^C ∪ B^C$

    (vii)   Exclusion: $A∪A^C$=X

    (viii) Contradiction: $A∩A^C$=ϕ

(ix)   Identity: A∪φ=A and A∩X=A, where φ and X are empty and universal set

respectively.

In this entire research work, this definition of complementation of fuzzy sets based on reference function is considered for defining fuzzy sets, fuzzy function, fuzzy topology, fuzzy open set, fuzzy closed set, and fuzzy point and most importantly for defining in fuzzy boundary.

2.4 CONTAINMENT OF FUZZY SETS

Let A and B be two fuzzy sets which is expressed on the basis of reference function, then the fuzzy set A is said to be subset of another fuzzy set B if the membership value of the set A is less than or equal to the membership value of the set B.

Thus if ,

A={x, $\mu_1(x)$, $\mu_2(x)$; xϵX} and B={x, $\mu_3(x)$, $\mu_4(x)$; xϵX}.

Be two fuzzy sets on the basis of reference function defined on the same universe then A is said to be subset of B if the following conditions holds

$$\{ \mu_1(x) - \mu_2(x)\} \leq \{ \mu_3(x) - \mu_4(x)\}$$

That is $$\{ \mu_1(x) - \mu_2(x)\}/ \{ \mu_3(x) - \mu_4(x)\} \leq 1$$

For A⊆B and vice versa.

2.5 The operation of set superimposition

We first proceed to define a set operation that we have named superimposition. When we overwrite, the overwritten portion looks darker. Indeed, in the overwritten portion there happens to double representation due to superimposition, which is why that portion looks darker. The operation of union of sets cannot explain this. When two translucent papers with unequal opacities are placed one covering the other partially, the opacity in the portion covered by both the papers would be more than the maximum opacity in comparison with the other parts. This happens due to superimposition. We now proceed to define this mathematically.

The superimposition of sets is defined by Baruah([8, 9, 10, 11, 12, 13, 14]) and later used successfully in recognizing periodic patterns ([8, 9, 10, 11, 12, 13, 14]) the operation of set superimposition is expressed as follows: if the set A is superimposed over the set B, we get

$$A \,(S)\, B = (A\text{-}B) \cup (A \cap B)^{(2)} \cup (B\text{-}A)$$

where S represents the operation of superimposition and $(A \cap B)^{(2)}$ represents the elements of $(A \cap B)$ occurring twice, provided that $(A \cap B)$ is not void. We have defined this operation keeping view that fact that if two line segments A and B of unequal lengths are overdrawn one over the other, this is what we are going to see.

It can be seen that for two intervals $A = [a_1, b_1]$ and $B = [a_2, b_2]$, we should have

Equivalently

$$[a_1, b_1] \,(S)\, [a_2, b_2] = [a_1, a_2] \cup [a_2, b_1]^{(2)} \cup [b_1, b_2], \text{ if } a_1 < a_2 < b_1 < b_2,$$

$$= [a_1, a_2] \cup [a_2, b_2]^{(2)} \cup [b_2, b_1], \text{ if } a_1 < a_2 < b_2 < b_1,$$

$$= [a_2, a_1] \cup [a_1, b_1]^{(2)} \cup [b_1, b_2], \text{ if } a_2 < a_1 < b_1 < b_2,$$

$$= [a_2, a_1] \cup [a_1, b_2]^{(2)} \cup [b_2, b_1], \text{ if } a_2 < a_1 < b_2 < b_1,$$

where $a_1 < a_2 < b_1 < b_2$, $a_1 < a_2 < b_2 < b_1$, $a_2 < a_1 < b_1 < b_2$, and $a_2 < a_1 < b_2 < b_1$ are the four different possibilities in this case. Here we have assumed without loss of any generality that

$[a_1, b_1] \cap [a_2, b_2]$ is not void, or in other words max $(a_i) \leq$ min (b_i), $i = 1, 2$.

We can express this as follows. Indeed

$$[a_1, b_1] (S) [a_2, b_2] = [a_{(1)}, a_{(2)}] \cup [a_{(2)}, b_{(1)}]^{(2)} \cup [b_{(1)}, b_{(2)}]$$

where

$a_{(1)} = $ min (a_1, a_2),

$a_{(2)} = $ max (a_1, a_2),

$b_{(1)} = $ min (b_1, b_2),

and

$b_{(2)} = $ max (b_1, b_2).

This conversion in terms of ordered values is to be noted properly. We would soon see the applicability of this conversion in defining the randomness-fuzziness principle.

In this way, for n intervals $[a_1, b_1]$, $[a_2, b_2]$, ..., $[a_n, b_n]$, subject to the condition that

$[a_1, b_1] \cap [a_2, b_2] \cap .... \cap [a_n, b_n]$

is not void, we would have $(n!)^2$ different cases that can be in short written as

$[a_1, b_1](S) [a_2, b_2](S) ....(S) [a_n, b_n]$

$$= [a_{(1)}, a_{(2)}] \cup [a_{(2)}, a_{(3)}]^{(2)} \cup .... \cup [a_{(n-1)}, a_{(n)}]^{(n-1)} \cup [a_{(n)}, b_{(1)}]^{(n)} \cup [b_{(1)}, b_{(2)}]^{(n-1)} \cup$$

$$........... \cup [b_{(n-2)}, b_{(n-1)}]^{(2)} \cup [b_{(n-1)}, b_{(n)}],$$

where $a_{(1)}, a_{(2)}, ..., a_{(n)}$ are values of $a_1, a_2, ..., a_n$ arranged in increasing order of

magnitude, and $b_{(1)}$, $b_{(2)}$,........., $b_{(n)}$ also are values of b_1, b_2, ..., b_n arranged in increasing order of magnitude, and for example $[a_{(n-1)}, a_{(n)}]^{(n-1)}$ are elements of $[a_{(n-1)}, a_{(n)}]$ represented (n-1) times. Observe that order statistical matters can now enter into our discussions on superimposition.

Now a random vector $X = (X_1, X_2,..., X_n)$ has been defined as a family of X_k, $k = 1$, $2,...,n$, with every X_k inducing a sub-σ field so that X is measurable. Let $(x_1, x_2... x_n)$ be a particular realization of X, and let $X_{(k)}$ realize the value $x_{(k)}$ where $x_{(1)}, x_{(2)},..., x_{(n)}$ are *ordered* values of $x_1, x_2,..., x_n$ in increasing order of magnitude. Further let the sub-σ fields induced by X_k be independent and identical. Now defining,

$$\phi_n(x) = 0, \text{ if } x < x_{(1)},$$

$$= \frac{r-1}{n}, \text{ if } x_{(r-1)} \leq x \leq x_{(r)}, \text{ r=2, 3,......}$$

$$= 1, x \geq x_{(n)}$$

$\Phi_n(x)$ here is an empirical distribution function of a theoretical distribution function $\Phi(x)$. As there is a one to one correspondence between a Lebesgue-Stieltjes measure and the distribution function, we would have

$$\Pi (a, b) = \Phi (b) - \Phi (a) \text{ ……………………} (2.2.5)$$

where Π is a measure in (Ω, A, Π), A being the σ- field common to every xk

Now the Glivenko-Cantelli theorem (see e.g. Loeve, 1977, pp-20) states that $\Phi n(x)$ converges to $\Phi(x)$ uniformly in x. This means,

$$\sup \left| \Phi n(x) - \Phi(x) \right| \to 0.$$

It has been observed that $(r-1)/n$ in (2.2.5), for $x_{(r-1)} \leq x \leq x_{(r)}$, are membership values of

$[a_{(r-1)}, a_{(r)}]^{((r-1)/n)}$ and $[b_{(n-r+1)}, b_{(n-r)}]^{((r-1)/n)}$, for $r = 2, 3, \ldots, n$. Indeed this fact found from superimposition of uniformly fuzzy sets has led us to look that there is a link between distribution functions and fuzzy membership which leads to the *Randomness-Fuzziness Consistency Principle*.

2.6 Randomness – Fuzziness Consistency Principle

Defining superimposition of sets operations and using the Glivenko-Cantelli Theorem (Loeve, 1977) on Order Statistic, Baruah ([8, 9, 10, 11, 12, 13, 14]) has established the following result which states as a theorem that uncovers the missing link between fuzziness and randomness.

For a normal fuzzy number $N = [\alpha, \beta, \gamma]$ with membership function

$$\mu_N(x) = \Psi_1(x), \text{ if } \alpha \leq x \leq \beta,$$

$$= \Psi_2(x), \text{ if } \beta \leq x \leq \gamma, \text{ and}$$

$$= 0, \text{ otherwise,}$$

such that, $\quad\quad\quad\quad \Psi_1(\alpha) = \Psi_2(\gamma) = 0,$

$$\Psi_1(\beta) = \Psi_2(\beta) = 1,$$

where $\Psi_1(x)$ is the distribution function of a random variable defined in the interval $[\alpha, \beta]$, and $\Psi_2(x)$ is the complementary distribution function of another random variable defined in the interval $[\beta, \gamma]$, with randomness defined in the measure theoretic sense, the partial presence of a

value x of the variable X in the interval [α, γ] is expressible as

$\mu_N(x) = \theta$ Prob $[\alpha \leq X \leq x] + (1 - \theta)$ $\{1 - $ Prob $[\beta \leq X \leq x]\}$,

where

Prob $[\alpha \leq X \leq x] = \Psi_1(x)$, if $\alpha \leq x \leq \beta$, with $\theta = 1$

Prob $[\beta \leq X \leq x] = 1 - \Psi_2(x)$, if $\beta \leq x \leq \gamma$, with $\theta = 0$

In other words, the membership function explaining a fuzzy variable taking a particular value is either the distribution function of a random event or the complementary distribution function of another random event. Hence, partial presence of an element in a fuzzy set can actually be expressed either as a distribution function or as a complementary distribution function.

It needs to be mentioned at this point that the Glivenko–Cantelli theorem on convergence of empirical probability distributions can actually be seen as the backbone of mathematical statistics. This theorem is about probability distribution functions, and therefore it will be applicable for distribution functions of random variables with randomness defined in the measure theoretic sense as well ([8, 9, 10, 11, 12, 13, 14])). In the measure theoretic sense, if a variable is probabilistic, it has to be necessarily random, although when a variable is random, it does not have to be probabilistic (Rohatgi and Saleh, 2001).

It is known that a distribution function of a random variable is non-decreasing, and that a complementary distribution function of a random variable is non-increasing. The functions are continuous and differentiable. Differentiation of $\Psi_1(x)$ and $(1 - \Psi_2(x))$ would give two density

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functions. This means, one needs two laws of randomness, one in the interval $[\alpha, \beta]$ and the other in $[\beta, \gamma]$, to construct a normal fuzzy number $[\alpha, \beta, \gamma]$.

For a triangular fuzzy number, differentiation of $\Psi_1(x)$ and $(1 - \Psi_2(x))$ would give two uniform density functions. It is well known that the uniform law of randomness is the simplest of all probability laws. Thus two uniform laws of randomness lead to the simplest fuzzy number. When a normal fuzzy number is of the triangular type, it actually means that the left reference function is a uniform distribution function and the right reference function is a uniform complementary distribution function.

Thus according to this principle, the Dubois-Prade ([30, 31]) left reference function is actually a distribution function by definition and similarly the right reference function is nothing but a complementary distribution function. In other words, two laws of randomness, probabilistic or otherwise, are not only necessary but also sufficient to define a law of fuzziness.

2.7 CONCLUSIONS

Here after having an overview of the evolution of the concepts of complementation of fuzzy sets on the basis of reference function and the *Randomness-Fuzziness* Consistency Principle.

It is observed that in the Zadehian definition of complementation, membership value and membership function had been taken to be of the same meaning. Indeed, fuzzy membership function and fuzzy membership value are two different things with reference to the complement

of normal fuzzy set. The fuzzy membership function of the complement of a normal fuzzy number is one over the entire real line, with the condition that it has to be counted from the membership function of the original fuzzy number.