Chapter 2

Whether Lyra's Manifold itself is a Hidden Source of Dark Energy

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2.1   Introduction

We have been established and believed that our universe is accelerating at the present epoch instead of showing down as predicted by the Big Bang theory Silk (1989). However, till today we are not in a state to provide an exactly clear statement about the origin and evolution of our universe. When we study different literatures and philosophical point of views in this regard we found that different mines provide different opinions about our universe. Actually the universe is full of mysterious elements and numerous effects of interactions which cannot be detected and difficult to explain even with advanced technology. So the most challenging problems in Astrophysics and modem cosmology are to understand the late time acceleration of the universe. In recent years many researchers and Scientists are putting huge effort to explain the dynamics of the universe and to understand the future evolution of the universe with the attention in the context of dark energy and modified theories of gravity. On the other hand many prominent results of the cosmological observations like Type SNeIa supernovae [Riess et al. 1988, 2004; Perlmutter et al. 1999; Amanullah et al. 2010; Astier et al. 2006; Suzuki et al. 2012], CMB (Cosmic Microwave Background) anisotropies [Bennett et al. 2003; Spergel et al. 2003; Bernardis et al. 2000], the large scale galaxies structures of universe [Daniel et al. 2008; Allen et al. 2004], BAO (Baryon Acoustic Oscillations) [Anderson et al. 2013; Eisenstein et al. 2005], WMAP (Wilkinson

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Microwave Anisotropy Probe) [Komatsu et al. 2009; Hinshaw et al. 2013] Sachs-Wolfe effects [Nishizawa 2014] and SDSS (Sloan digital sky survey) [Seljak et al. 2005; Tegmark et al. 2004; Abazajian et al. 2004; Adelman McCarthy et al. 2008] are noticed to us for the cosmic acceleration with direct and indirect evidence. Not only the above mentioned observations and surveys, but some other new cosmological results and data sets like Planck [Ade et al. 2014, 2016], Atacama Cosmology Telescope (ACT) [Sievers et al. 2013], South Pole Telescope Sunyaev-Zel'dovice (SPT-SZ) survey [Story et al. 2013] have measured the temperature and polarization of CMB to exquisite precision, are also supporting this fact. These results are more acceptable to the community and beneficial to the researchers to understand about the universe in this modern era.

From the recent literatures and findings we know that dark energy dominates the universe with positive energy density and negative pressure, responsible to produce sufficient acceleration in late time evolution of the Universe. So the investigation of dark energy becomes a major outstanding issue in physics and cosmology today. There are a number of useful reviews of dark energy that are focus on theory [Yoo and Watanabe 2012; Ziaeepour 2014; Copeland et al. 2006; Linder et al. 2016; Tsujikawa 2010], on probes of dark energy [Frieman et al. 2008] and on the cosmological constant [Carroll 2001; Martin 2012]. Also,many prominent researchers like Yoo and Watanabe (2012), Bahrehbakhsh et al. (2013) , Joyce et al. (2016), Pasechnik (2016), Wang et al. (2016), El-Nabulsi (2011a, 2013a,b,c,d , 2015a,b , 2016a,b) , Motloch et al. (2015), Silbergleit (2016), ; Linder (2015), Josset et al. (2016), Nojiri et al. (2006b), Nojiri and Odintsov (2007, 2014a), Felice and Tsujikawa (2010) , Faraoni (2008), Boehmer et al.(2012), Tsujikawa [Chapter Dark Matter and Dark Energy, volume 370 of astrophys. Space Sci.] discussed different models about the dark energy in different context with the comparisons of observational findings. Some of the important claimants of dark energy are tachyons [Padmanabhan et al. 2002], chaplygin gas [Elmardi et al. 2016], phantom [Dabrowski 2008], k-essence and quintessence [Armendariz-Pecon et al. 2000a,b; de Putter et al. 2007] along with other four elements i.e. dark matter, baryons, radiation and neutrinos. But so far there is no direct detection of such exotic fluids. Although the literature is now flooded with hundreds of model for dark energy what we lack is precise cosmological data coming from variety of observations involving both background and inhomogeneous universe that can discriminate among these models. In this connection if we accept that Einstein was correct with his general relativity theory to explain accelerated expansion of the universe could also be explained by negative pressure working against gravity. The belief of Einstein to the static universe made him to think about negative pressure which will stop the attraction of the gravity. However, we know that we have non

static universe, moreover we know that we have accelerated expansion. According to the above observational data analysis, it can be estimated the amount of the negative pressure in our universe, which we call it as dark energy. The simple question about the nature of the dark energy is still one of the intriguing questions and left free space for new speculations.

Many Physicists have been investigating about gravitation in different contexts after Einstein. Hermann Weyl (1918) attempted to generalize the idea of geometrizing the gravitation and electromagnetism by applying different techniques and methods. He described both gravitation and electromagnetism geometrically by formulating a new kind of gauge theory involving metric tensor with an intrinsic geometrical significance. With the concept of Einstein's general theory of relativity Lyra (1951) suggested a modification by introducing a gauge function into the structure less manifold which removes the non-integrability condition of the length of a vector under parallel transport. Such theories are commonly known as the modified theories of the gravitation or alternate theory of gravitation. Some important modified theories of gravitation are Brans-Dicke theory [Brans and Dicke 1961], Scalar-tensor theories [Barker 1978], Vector-tensor theory [Bekenstein 2004], Weyl's theory [Weyl 1918], F(R) Gravity [Nojiri et al. 2003; Sotiriou 2010; Singh, Bishi and Sahoo 2016b; Nojiri, S. et al. 2011; Buchdahl 1970], Mimetic Gravity [Chamseddine et al. 2013], Mimetic F(R) gravity [Nojiri et al. 2014b ; Myrzakulov et al. 2015], Lyra geometry [Scheibe 1952] and many more. We can explain about the accelerating expansion of the universe in the context of these modified theories of gravitation. Out of these modified theories of gravitation, here we will discuss about the Lyra geometry.

It is common to us that Lyra geometry is a modification of Riemanian geometry by introducing a gauge function into the structure less manifold which removes the non-integrability condition of the length of a vector under parallel transport. Lyra geometry along with constant gauge vector ϕ^i will either play the role of cosmological constant or creation field (equal to Hoyle's creation field [Hoyle 1948; Hoyle and Narlikar 1963; Hoyle and Narlikar 1964]) which is discussed by Soleng (1987). Many researchers proposed different cosmological models in different context of Lyra geometry. Sen (1957) formulated a new scalar-tensor theory of gravitation based on Lyra geometry in which he found that static model with finite density in Lyra geometry is similar to the Einstein's static model, but it exhibited red shift which is a significant difference of the model. Later, Sen and Dunn (1971) and Rosen (1983), suggested that this theory was based on non-integrability of length transfer so that it had some unsatisfactory features and hence this theory did not gain general acceptance. Halford (1970, 1972) pointed out that in the normal general relativistic treatment the constant

displacement vector field ϕ_k in Lyra's geometry plays the role of cosmological constant. Also, as in the Einstein's theory of relativity the scalar-tensor treatment based on Lyra's geometry predicts the same effect, within observational limits, as far as the classical solar system test are concerned . Many authors like Rahaman et al. (2003, 2005), Casana et al. (2005, 2006); Mohanty et al. (2007, 2009a); Mahanta et al. (2012); Asgar et al. (2014b); Mollah et al. (2015); Mollah and Singh (2016) attempted to solve Einstein's field equations in the framework of Lyra's geometry and successfully find their solutions under different circumstances.

It is also learn that the space-time symmetry plays a vital role in the features of space-time that can be described as exhibiting some form of symmetry. The importance of symmetries in relativity and cosmology is to simplify Einstein's field equations and to provide a classification of the space time according to the structure of the corresponding Lie algebra. Some authors like Henriksen et al. (1978), Mohanty ang Samanta (2009a), Rao and Neelima (2013a), Singh et al. (2016a), Cahill and Taub (1971) solved Einstein's field equations for an axially symmetric space-time by imposing certain conditions upon the scale factor of the space time together with some conditions on matter which represents such space-time and some restrictions on its physical properties. To explain the accelerated expansion of the universe many prominent researchers like Shchigolev (2013) , Hova (2013), Ali and Rahman (2013), Megied et al. (2014), Khurshudyan et al. (2014a,b), Saadat (2015), Darabi et al. (2015), Ziaie et al. (2015), Pucheu et al. (2016) have investigated and proposed different cosmological models and ideas of the universe within the framework of Lyra's geometry and other theories of relativity in different context.

Earlier majority of researchers in the field of Astrophysics, Cosmology and Particle Physics were doing their research in this area by considering only 4-dimensional case. But in this modern era peoples are more interested to study higher dimensional case, since the solutions of Einstein field equations in higher dimensional space times are believed to have physical relevance possibly at extremely early times before the universe underwent the compactification transitions. So to fit the observational data in a better way and to study the accelerated expansion of our universe many researchers like Bahrehbakhsh et al. (2011), Rahaman (2003), Chatterjee (1990), Chatterjee and Bhui (1990), Chatterjee and Banerjee (1993), Sil and Chatterjee (1994), Rahaman et al. (2006), El-Nabulsi (2008; 2009b; 2011a,b,d), Rador (2007), Kahil and Harko (2009) discussed about different cosmological models concerning extra dimension. From the critical study we know that by using a suitable scalar field we can show that the phase transitions on the early universe can give rise to such objects which

are nothing but the topological knots in the vacuum expectation value of the scalar field and most of their energy is concentrated in a small region. So it is necessary for us to study the cosmological problems by considering higher-dimensional space-time to unify gravity with other interactions. In cosmology, particularly for study of the early stage of universe the present four dimensional stage of the universe might have been preceded by a multi-dimensional stage. Recently, Singh and Mollah (2016) studied about higher dimensional LRS Bianchi type-I cosmological model universe interacting with perfect fluid in Lyra geometry with different cases. Banerjee et al. (1990) have investigated Bianchi type-I cosmological models with viscous fluid in higher dimensional space time. Also Krori et al. (1994) studied Bianchi type-I string cosmological model in higher dimensions. Many authors [Bali and Kumawat 2010; Pradhan et al. 2015; Accioly 1985; Kandalkar et al. 2009; Adhav et al. 2013; Adhav 2011; Pradhan and Singh 2011b; Asgar and Ansary 2014c] have studied Bianchi type models in order to examine the role of certain anisotropic sources during the formation of the large-scale structures that we see in the present universe and for better understanding the small amount of observed anisotropy in the universe.

Motivated from the above, in this chapter we discussed about cosmic string universes by considering Locally Rotationally Symmetric (LRS) Bianchi Type I metric in five dimensional Lyra manifold with two different cases and find out the realistic solutions which are supporting to the present observational facts. We also found that the geometry itself of Lyra manifold behaves as a new source of dark energy which will be beneficial for further research work.

This chapter is organized as follows: In section 2.2 we are presenting the field equations and their solutions. In Section 2.3, Physical interpretations of the solutions are presented. In the last section, conclusion of our work is presented.

2.2   Field Equation and Their Solutions

The Einstein's field equations based on Lyra's manifold can be written as

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -\chi T_{ij} \qquad (2.1)$$

where ϕ_i is the displacement vector and other symbols have their usual meanings in Riemannian geometry.

Let us consider the five dimensional LRS-Bianchi type - I metric in the form

$$ds^2 = A^2 dx^2 + B^2 dy^2 + B^2 dz^2 + C^2 d\psi^2 - dt^2 \tag{2.2}$$

where A, B and C are functions of cosmic time t only.

The displacement vector ϕ_i is defined as

$$\phi_k = (0,0,0,0,\beta(t)) \tag{2.3}$$

and the energy momentum tensor for cosmic strings is taken as

$$T_{ij} = -\rho u_i u_j - \lambda x_i x_j \tag{2.4}$$

where, $\rho = \rho_p + \lambda$, is the energy density of the cloud of strings with particles attached to them, ρ_p being the rest energy density of particles attached to the strings and λ is the tension density of the strings.

The five velocity vector u^i of the cloud of particles and the direction of string x^i are given by

$$u^i = (0,0,0,0,1) \quad and \quad x^i = (A^{-1},0,0,0,0) \tag{2.5}$$

The direction of strings satisfies

$$u_i u^i = -x_i x^i = -1 \quad and \quad u_i x^i = 0 \tag{2.6}$$

Using the co - moving coordinate system, the field equation (2.1) gives the following equations

$$-\left(\frac{\dot{B}}{B}\right)^2 - 2\frac{\dot{A}\dot{B}}{AB} - 2\frac{\dot{B}\dot{C}}{BC} - 2\frac{\dot{A}\dot{C}}{AC} + \frac{3}{4}\beta^2 = -\chi\rho \tag{2.7}$$

$$2\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\dot{B}\dot{C}}{BC} + \frac{3}{4}\beta^2 = \chi\lambda \tag{2.8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{3}{4}\beta^2 = 0 \tag{2.9}$$

$$\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} + \frac{3}{4}\beta^2 = 0 \tag{2.10}$$

where dot denotes differentiation with respect to t.

Now subtracting (2.9) from (2.10) we have

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \left(\frac{\dot{B}}{B}\right)^2 + \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC} = 0 \tag{2.11}$$

Case - I :

A set of solution of the equations (2.7)-(2.10) are

$$A = a_0 e^{2b_0 t} [b_0 c_0 t^2 + (b_0 c_1 + 2c_0)t + b_0 c_2 + c_1]^{-1} \tag{2.12}$$

$$B = e^{-b_0 t + b_1} \tag{2.13}$$

$$C = c_0 t^2 + c_1 t + c_2 \tag{2.14}$$

where a_0, b_0, c_0, c_1 and c_2 are constants.

Then from equation (2.10) we have

$$\beta^2 = \frac{4}{3}[4b_0^3 c_0^2 t^3 + 6b_0^3 c_0 c_1 t^2 + 6b_0^2 c_0^2 t^2 + 4b_0^3 c_0 c_2 t + 2b_0^3 c_1^2 t + 6b_0^2 c_0 c_1 t - 4b_0 c_0^2 t + 2b_0^3 c_1 c_2$$
$$+ 6b_0^2 c_0 c_2 - 2b_0 c_0 c_1 - 8c_0^2] \times [b_0 c_0 t^2 + b_0 c_1 t + 2c_0 t + b_0 c_2 + c_1]^{-2} - 4b_0^2$$

i.e.

$$\beta^2 = \frac{4}{3}\frac{\beta_1 t^3 + \beta_2 t^2 + \beta_3 t + \beta_4}{[b_0 c_0 t^2 + b_0 c_1 t + 2c_0 t + b_0 c_2 + c_1]^2} - 4b_0^2 \tag{2.15}$$

where $\beta_1 = 4b_0^3 c_0^2$, $\beta_2 = 6b_0^3 c_0 c_1 + 6b_0^2 c_0^2$, $\beta_3 = 4b_0^3 c_0 c_2 + 2b_0^3 c_1^2 + 6b_0^2 c_0 c_1 - 4b_0 c_0^2$ and $\beta_4 = 2b_0^3 c_1 c_2 + 6b_0^2 c_0 c_2 - 2b_0 c_0 c_1 - 8c_0^2$ are constants.

Again equation (2.7) gives

$$\chi\rho = [2b_0^2 c_0^3 t^4 + 4b_0^2 c_0^2 c_1 t^3 + 3b_0^2 c_0 c_1^2 t^2 + b_0^2 c_1^3 t + 8b_0 c_0^2 c_2 t - 2b_0 c_0 c_1^2 t + b_0^2 c_1^2 c_2$$
$$- 2b_0^2 c_0 c_2^2 + 4b_0 c_0 c_1 c_2 - b_0 c_1^3 + 8c_0^2 c_2 - 2c_0 c_1^2] \times \left(c_0 t^2 + c_1 t + c_2\right)^{-1} \qquad (2.16)$$
$$\times \left[b_0 c_0 t^2 + b_0 c_1 t + 2c_0 t + b_0 c_2 + c_1\right]^{-2}$$

Also from equation (2.8) we have

$$\chi\lambda = -[8b_0^2 c_0^3 t^4 + 10b_0^2 c_0^2 c_1 t^3 + 12b_0 c_0^3 t^3 + 12b_0^2 c_0 c_1^2 t^2 + 18b_0 c_0^2 c_1 t^2 + 4b_0^2 c_1^3 t +$$
$$10b_0 c_0 c_1^2 t - 4b_0 c_0^2 c_2 t + 4b_0^2 c_1^2 c_2 - 8b_0^2 c_0 c_2^2 + 2b_0 c_1^3 - 2b_0 c_0 c_1 c_2 - 2c_0 c_1^2 + 8c_0^2 c_2]$$
$$\times \left(c_0 t^2 + c_1 t + c_2\right)^{-1} \times \left[b_0 c_0 t^2 + b_0 c_1 t + 2c_0 t + b_0 c_2 + c_1\right]^{-2}$$
$$(2.17)$$

Here the expansion factor θ is given by

$$\theta = \frac{2c_0^2 t^2 + 2c_0 c_1 t + c_1^2 - 2c_0 c_2}{[b_0 c_0 t^2 + (b_0 c_1 + 2c_0)t + b_0 c_2 + c_1](c_0 t^2 + c_1 t + c_2)} \qquad (2.18)$$

And the volume V is obtained as

$$V = \frac{k(c_0 t^2 + c_1 t + c_2)}{b_0 c_0 t^2 + (b_0 c_1 + 2c_0)t + b_0 c_2 + c_1} \qquad (2.19)$$

In this case we get the shear scalar σ in the form

$$\sigma = \frac{1}{\sqrt{18}}[(k_{11}t^4 + k_{12}t^3 + k_{13}t^2 + k_{14}t + k_{15})^2$$
$$+ 2(k_{21}t^4 + k_{22}t^3 + k_{23}t^2 + k_{24}t + k_{25})^2$$
$$+ (k_{31}t^4 + k_{32}t^3 + k_{33}t^2 + k_{34}t + k_{35})^{\frac{1}{2}}] \qquad (2.20)$$
$$\times [b_0 c_0 t^2 + (b_0 c_1 + 2c_0)t + b_0 c_2 + c_1]^{-1} \times (c_0 t^2 + c_1 t + c_2)^{-1}$$

where k_{ij}'s ; $i = 1, 2, 3$ and $j = 1, 2, 3, 4, 5$ are constants.

Also the deceleration parameter q is given by

$$q = 3[4b_0c_0^4t^5 + 10b_0c_0^3c_1t^4 + 4c_0^4t^4 - 8b_0c_0^3c_2t^3 + 12b_0c_0^2c_1^2t^3 + 8c_0^3c_1t^3$$
$$- 12b_0c_0^2c_1c_2t^2 + 8b_0c_0c_1^3t^2 - 16c_0^3c_2t^2 + 10c_0^2c_1^2t^2 - 12b_0c_0^2c_2^2t + 2b_0c_1^4t$$
$$- 16c_0^2c_1c_2t + 6c_0c_1^3t - 6b_0c_0c_1c_2^2 + 2b_0c_1^3c_2 - 4c_0^2c_2^2 - 2c_0c_1^2c_2 + c_1^4]$$
$$\times \left[2c_0^2t^2 + 2c_0c_1t + c_1^2 - 2c_0c_2\right]^{-2} - 1$$

i.e.

$$q = \frac{3(a_1t^5 + a_2t^4 + a_3t^3 + a_4t^2 + a_5t + a_6)}{(2c_0^2t^2 + 2c_0c_1t + c_1^2 - 2c_0c_2)^2} - 1 \tag{2.21}$$

where,

$a_1 = -4b_0c_0^4$

$a_2 = 10b_0c_0^3c_1 + 4c_0^4$

$a_3 = -8b_0c_0^3c_2 + 12b_0c_0^2c_1^2 + 8c_0^3c_1$

$a_4 = -12b_0c_0^2c_1c_2 + 8b_0c_0c_1^3 - 16c_0^3c_2 + 10c_0^2c_1^2$

$a_5 = -12b_0c_0^2c_2^2 + 2b_0c_1^4 - 16c_0^2c_1c_2 + 6c_0c_1^3$ *and*

$a_6 = -6b_0c_0c_1c_2^2 + 2b_0c_1^3c_2 - 4c_0^2c_2^2 - 2c_0c_1^2c_2 + c_1^4$

are constants.

Also from (2.16) and (2.17) we have

$$\chi\rho - \chi\lambda = [10b_0^2c_0^3t^4 + 14b_0^2c_0^2c_1t^3 - 12b_0c_0^3t^3 - 9b_0^2c_0c_1^2t^2 - 18b_0c_0^2c_1t^2 + 4b_0c_0^2c_2t -$$
$$10b_0c_0c_1^2t - 4b_0^2c_1^3t + 6b_0^2c_0c_2^2 + 6b_0c_0c_1c_2 - 3b_0^2c_1^2c_2 - 3b_0c_1^3]$$
$$\times \left(c_0t^2 + c_1t + c_2\right)^{-1} \times \left[b_0c_0t^2 + b_0c_1t + 2c_0t + b_0c_2 + c_1\right]^{-2} \tag{2.22}$$

By taking particular values of the integrating constants as $a_0 = b_0 = c_0 = c_1 = c_2 = 1$, the variations of some parameters with respect to time of the **Case-I** are shown in Figures 1-4.
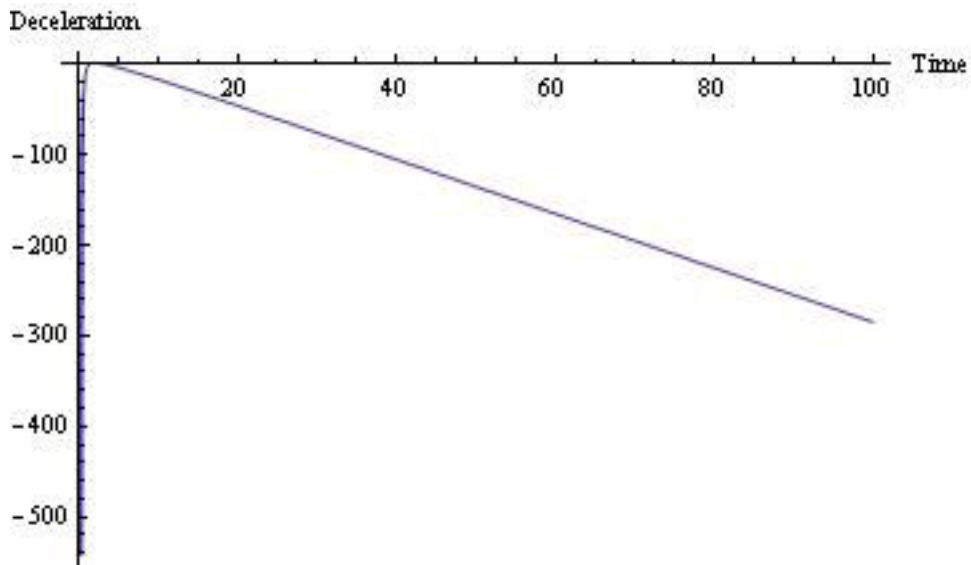
Figure-2.1 : The Variation of Deceleration Parameter q vs. Time t. Here $a_0 = b_0 = c_0 = c_1 = c_2 = 1$.
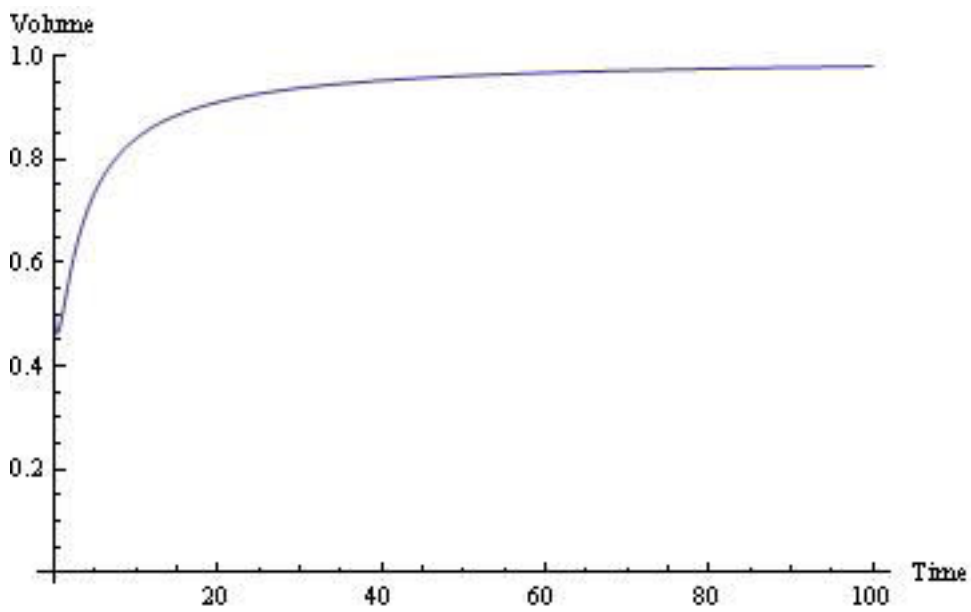

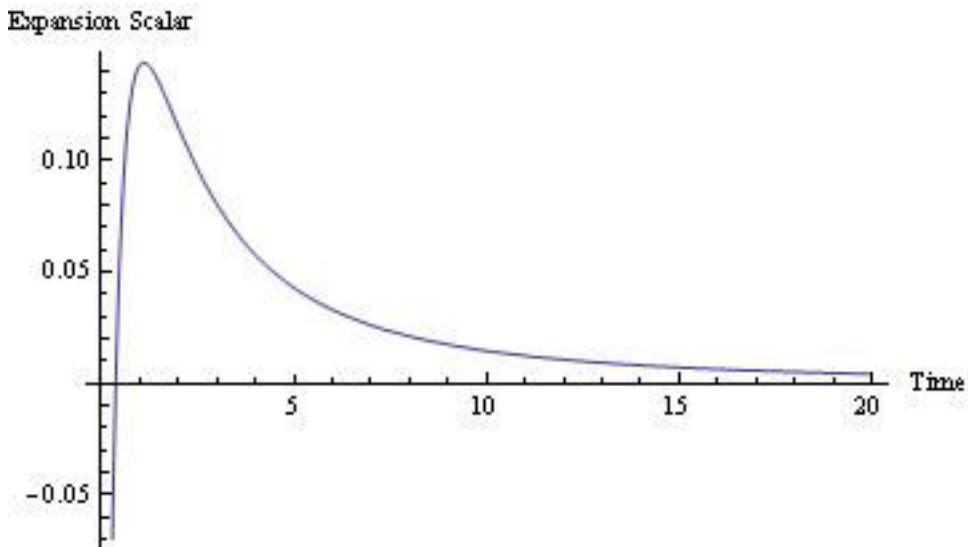**Figure-2.2 :** The Variation of Volume V vs. Time t. Here $a_0 = b_0 = c_0 = c_1 = c_2 = 1$.

Figure-2.3 : The Variation of Expansion Scalar θ vs. Time t. Here $a_0 = b_0 = c_0 = c_1 = c_2 = 1$.
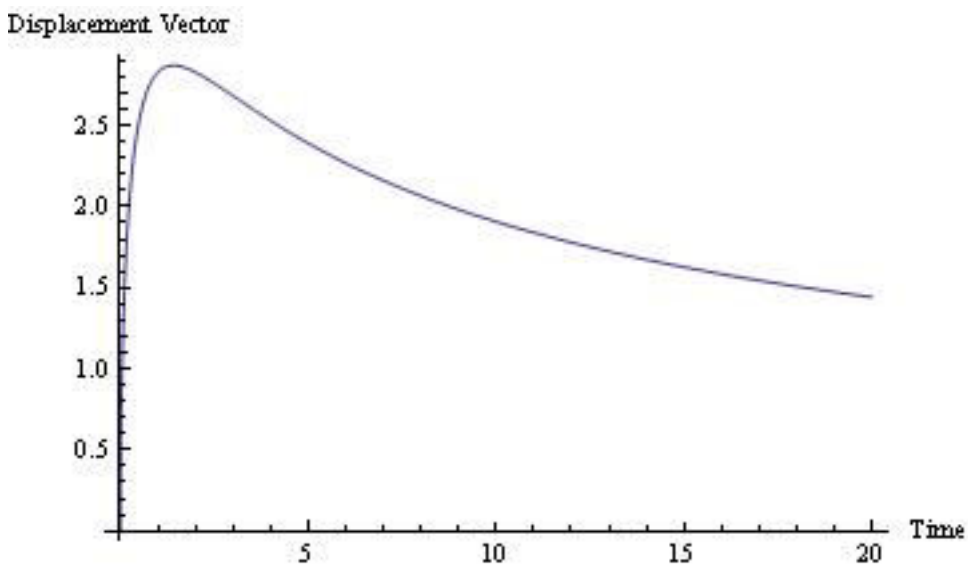

Figure-2.4 : The Variation of Displacement Vector β vs. Time t. Here $a_0 = b_0 = c_0 = c_1 = c_2 = 1$.

Case - II : Again

$$A = e^{ab_0 t + ab_1} \tag{2.23}$$

$$B = e^{b_0 t + b_1} \tag{2.24}$$

and

$$C = c_1 e^{b_0 t} + c_2 e^{-(2+a)b_0 t} \tag{2.25}$$

are another set of solutions of the equations (2.7) to (2.10), where a, b_0, b_1, c_1 and $c_2 = 1$ are constants.

Now using these values of A, B and C we get from (2.7), (2.8) and (2.10),

$$\chi \rho = a^2 + 2a + 3 + (2a+1)b_0^2 + 2(a+1)b_0 \frac{b_0 c_1 e^{b_0 t} - (2+a)b_0 c_2 e^{-(2+a)b_0 t}}{c_1 e^{b_0 t} + c_2 e^{-(2+a)b_0 t}} \tag{2.26}$$

$$\chi \lambda = (3b_0^2 - a^2 - 2a - 3) + \frac{3b_0^2 c_1 e^{b_0 t} + (2+a)ab_0^2 c_2 e^{-(2+a)b_0 t}}{c_1 e^{b_0 t} + c_2 e^{-(2+a)b_0 t}} \tag{2.27}$$

and

$$\beta^2 = -\frac{4}{3}(a^2 + 2a + 3) \tag{2.28}$$

In this case we have

Expansion factor

$$\theta = (a+3)b_0 c_1 e^{b_0 t}[c_1 e^{b_0 t} + c_2 e^{-(2+a)b_0 t}] \tag{2.29}$$

Deceleration parameter

$$q = -[1 + 3\frac{c_2}{c_1} e^{-(3+a)b_0}] \tag{2.30}$$

and the volume

$$V = [c_1 e^{b_0 t} + c_2 e^{-(2+a)b_0 t}]e^{(a+2)b_0 t + (a+1)b_1} \tag{2.31}$$

Also, the shear scalar σ is given by

$$\sigma^2 = \frac{1}{\sqrt{18}}[9(a^2+2)b_0^2 + \frac{9[b_0 c_1 e^{b_0 t} - (2+a)b_0 c_2 e^{-(2+a)b_0 t}]}{c_1 e^{b_0 t} + c_2 e^{-(2+a)b_0 t}} - \frac{2(a+3)b_0 c_1^2 e^{b_0 t}}{[c_1 e^{b_0 t} + c_2 e^{-(2+a)b_0 t}]^2}]^{\frac{1}{2}} \tag{2.32}$$

In this case,

$$\chi\rho - \chi\lambda = (4+2a)b_0^2 + \frac{(5+2a)b_0^2 c_1 e^{b_0 t} - (a^2+4a+4)b_0^2 c_2 e^{-(2+a)b_0 t}}{c_1 e^{b_0 t} + c_2 e^{-(2+a)b_0 t}} \qquad (2.33)$$

The variations of some parameters with respect to time of the **Case-II** are shown in Figures 5-7, , by considering particular values of the integrating constants as $a = b_0 = b_1 = c_1 = c_2 = 1$.
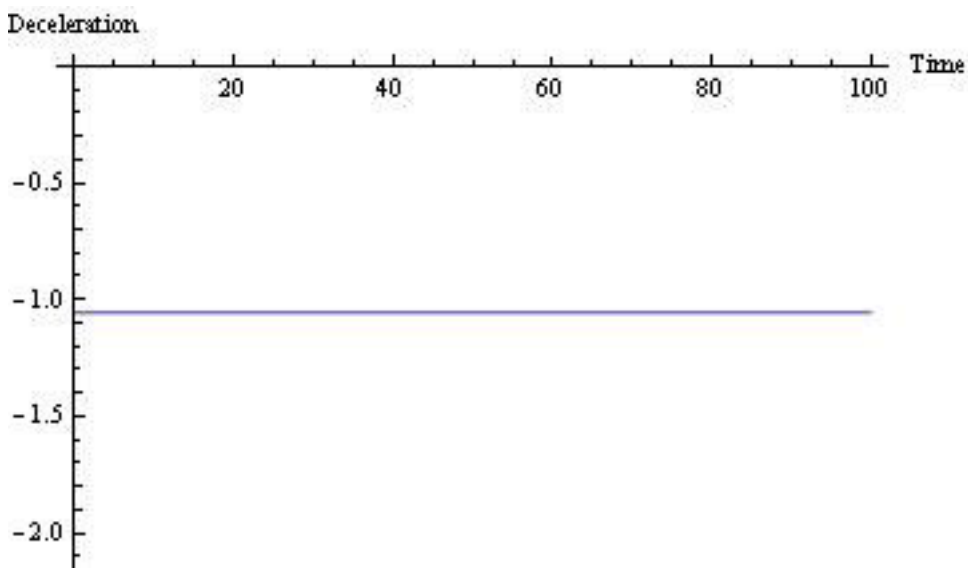

Figure-2.5 : The Variation of Deceleration Parameter q vs. Time t. Here $a = b_0 = b_1 = c_1 = c_2 = 1$.
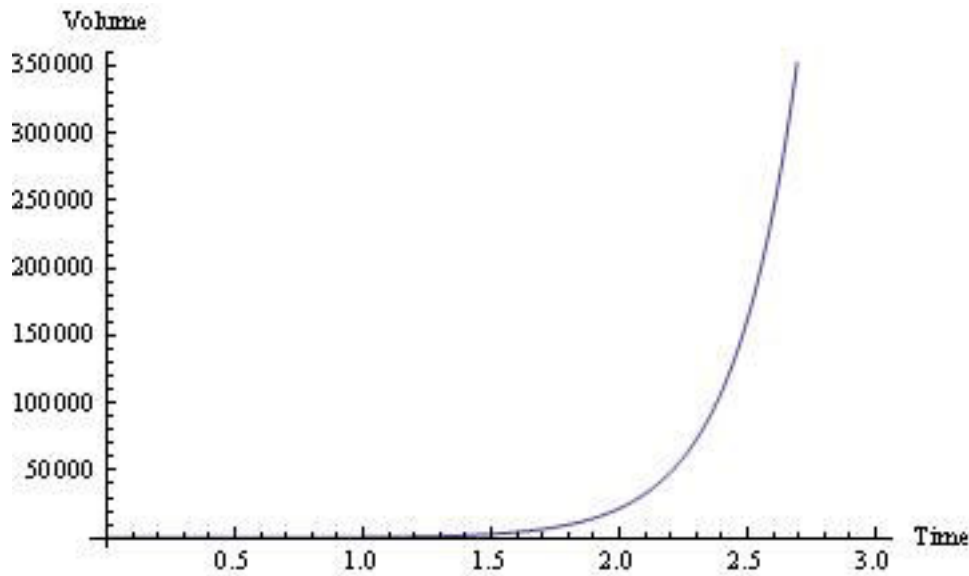
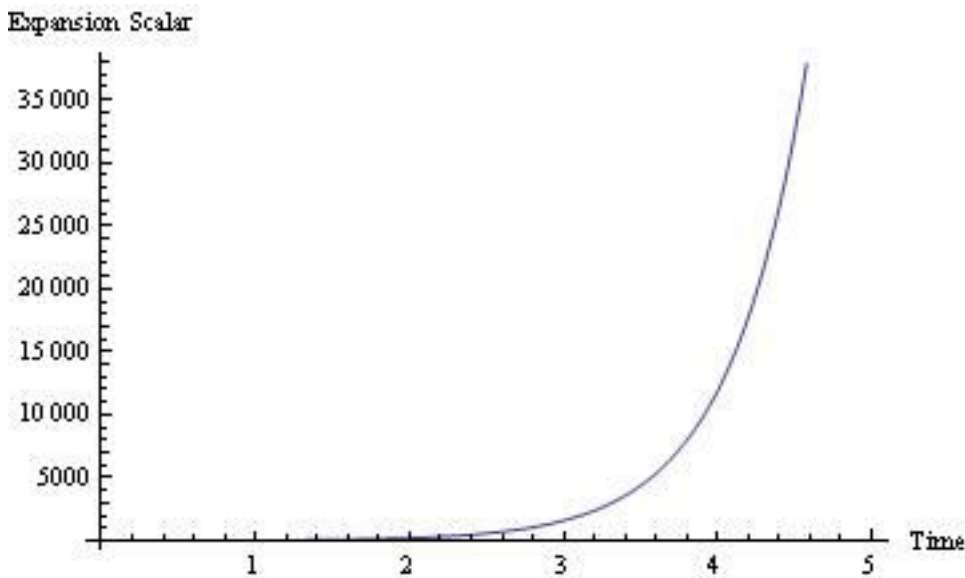**Figure-2.6 :** The Variation of Volume V vs. Time t. Here $a = b_0 = b_1 = c_1 = c_2 = 1$.

Figure-2.7 : The Variation of Expansion Scalar θ vs. Time t. Here $a = b_0 = b_1 = c_1 = c_2 = 1$.

2.3   Physical Interpretation of the Solutions:

Here in a five-dimensional Lyra manifold, we have constructed a model universe containing string cloud with standard matter that interacts with the vector field ϕ_k. With the assumption of a constant displacement vector field, our model behaves as a dark energy model universe, particularly the phantom energy type. The displacement field, which is considered as a

component of the total energy, plays the role of dark energy. In one case our universe seems to be dominated by cosmological constant type of dark energy.

In **Case - I** we see that the energy density of our universe has a finite value at $t = 0$, that is, at the beginning of the evolution; and even when $t \to \infty$ it has some finite value and there is no singularity; thereby there is possibility that our model is that of the oscillating type. The string density has some definite value at the beginning of the epoch until it gradually decreases so that it is zero at time t given by

$$(b_0 c_0 t^2 + b_0 c_1 t + 2c_0 t + b_0 c_2 + c_1)^2 = 4b_0^4 c_0^2 t^4 + 8b_0^4 c_0 c_1 t^3.$$

From the expression of the deceleration parameter and its variation with time as shown in the above **Fig-2.1** we see that our universe is undergoing an accelerated expansion. Again, the values of the volume V and the expansion factor θ show that our universe is expanding without any singularity, though at $t = 0$ and $t \to \infty$ the expansion rate is constant.

Thus we see that, with the advent of time, the density of string decreases more rapidly than density of the particles attached to them indicating that our universe becomes a universe consisting only of particles, matter and energy, where strings become invisible.

In this model the gauge function β^2 is found to be constant at the beginning of the evolution and gradually decreases with time until it takes a finite value (another constant value) at infinite time. Here it is seen that the displacement vector, interacting with pressureless matter, plays the same role as the cosmological constant. From the above characteristics we may conclude that this model universe is one containing and dominated by dark energy.

In **Case - II** the energy density of our universe as well as the tension density of the string are found to be decreasing functions of time, and the string density becomes zero at the point $6b_0^2 = a^2 + 2a + 3$. And their difference takes a definite quantity as $t \to \infty$ which implies that as time passes the string vanish leaving the particles in this universe. The volume of this universe is found here to expand at a high rate. And more from the expressions of the expansion factor and the deceleration parameter with their variations with time as shown in the above graphical representations of this case it can be concluded that the universe is expanding at an accelerated rate. Since the energy density was negative in the past, increases towards a positive value then decreases toward zero with time. The expansion of the early universe is due to the negative energy density with negative pressure and the late-time accel-

erated expansion comes from the positive energy and negative pressure which behaves like a dark energy. It is amazing that the non-singular universe is expanding with time whereas a transition from negative energy density to positive energy density occurs its dynamical evolution. Similar scenario occurs in Kim and Yoon (2007) and negative energy density in accelerated universe was discussed recently by Sawicki and Vikman (2013). Thus here it is seen that our model behaves as a dark energy filled universe.

In a phantom dark energy filled universe, for the phantom scalar ϕ_{ph} we know that $(\dot{\phi}_{ph})^2 < 0$. Strikingly similar in our problem here it is found that the displacement vector β^2 is less than zero. Thus it is interesting here to think, without loss of generality, of the Lyra manifold defining itself as a source of dark energy, as our universe here behaves and have the characteristics of a universe filled with phantom form of dark energy. Thus it is very much stimulating to see that a universe in Lyra manifold can define itself as a dark energy model universe, perhaps the dark energy here being the phantom energy type in most of the cases. Thus the geometry itself of the Lyra manifold may be taken as a hidden source of dark energy. Large number of literatures exists to address the observational fact of the current expansion and evolution of universe from the higher dimensional point of view as we mentioned above. But regarding the source of dark energy and its connections to extra-dimensions, we are not aware of models similar to the one developed in this paper. Further details, consequences and numerical confrontations with observations are in progress.

2.4   Conclusion:

Different mines provide different opinions about our universe but as a concluding remarks, we can say that our model behaves as a dark energy model universe, particularly the phantom energy type. The displacement field, which is considered as a component of the total energy, plays the role of dark energy. And in one case our universe seems to be dominated by cosmological constant type of dark energy. However what we have presented is nothing else but a toy model which has to be worked out in detail, first of all by matching the observations. It will be the issue of our forthcoming paper.