Chapter 3

Five Dimensional String Universes In Lyra Manifold

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3.1 Introduction

Many remarkable knowledge of cosmology are made by various experimental and theoretical results have made still today. But still now it is difficult to explain exactly the physical situation of the formation of our universe at the very early stage. To describe the events at the early stages of the universe we required to developed and study the concept of string theory. It is believed that universe might many phase transitions after big-bang.

Einstein formulation of General Relativity is the foundation of other geometric theories in order to explain the actual gravitational phenomena. A more general theory in which both gravitation and electromagnetism are described geometrically was proposed by Weyl (1918). Later Lyra (1951) suggested a modification of Riemannian geometry by introducing a Gauge function which removes the non-integrability condition of the length of a vector under parallel transport, which is known as Lyra's Geometry. In Lyra's geometry the connection is metric preserving as Riemannian geometry, and length transfers as integrable in contrast to Weyl's geometry. He also introduced a gauge function into the structure-less manifold, as a result of which a displacement field arises naturally. This alternating theory is of interest since it produces effects similar to Einstein's theory.

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[2]The work presented in this chapter has been published in "*International Journal of Astronomy and Astrophysics*" (IJAA), 2015, 5, 90-94; doi: 10.4236/ijaa.2015.52012

Many authors have investigated cosmology in Lyra's geometry with both a constant displacement field and time dependent one. Also cosmological models in the frame work of Lyra's geometry in different context are investigated by Pradhan and Kumar (2009a); Pradhan and Mathur (2009b); Pradhan and Yadav (2009c); Pradhan (2009); Pradhan et al. (2011c); Pradhan and Singh (2011b); Yadav (2012); Agarwal et al. (2011); Singh and Singh (2012). Cosmological models based on Lyra's manifold with constant displacement field vector was also studied by Bhamra (1974); Kalyanshetti and Waghmode (1982); Soleng (1987); Sen and Vanstone (1972); Karade and Borikar (1978); Reddy and Innaiah (1986); Reddy and Venkateswarlu (1987). But with this condition it is found as one of convenience and there is no priori reason for it. Recently, several authors like Asgar and Ansary (2014a); Kumari et al (2013); Asgar and Ansary (2014b); Zia and Singh (2012); Asgar and Ansary (2014c); Panigrahi and Nayak (2014) have studied cosmological models in the frame work of Lyra's geometry in various context.

Since we know that the constant vector displacement field in Lyra's geometry plays the role of cosmological constant in the normal general relativistic study as suggested by Halford (1970). Also, Halford (1972) shown that the scalar-tensor treatment based on Lyra's geometry predicts the same effects, within observational limits, as the Einstein theory.

As the necessity of study of higher-dimensional space-time in this field aiming to unify gravity with other interactions, the concept of extra dimension is relevant in cosmology, particularly for the early stage of universe and theoretically the present four dimensional stage of the universe might have been preceded by a multi-dimensional stage. So in this chapter we discussed about the five dimensional cosmological models in Lyra geometry by considering plane symmetric metric with some conditions to find out some solutions which are realistic with the observational facts.

3.2 Field Equations and their Solutions:

Here we consider the five dimensional plane symmetric metric in the form

$$ds^2 = A^2(dx^2 - dt^2) + B^2(dy^2 + dz^2) + C^2 dm^2 \tag{3.1}$$

where A, B and C are functions of time t only.

Einstein's field equations based on Lyra's geometry is-

$$R_{ij}-\frac{1}{2}g_{ij}R+\frac{3}{2}\phi_i\phi_j-\frac{3}{4}g_{ij}\phi^k\phi_k=-T_{ij} \tag{3.2}$$

where we use the units in which $\frac{8\pi G}{c^4}=1$ (Wesson 1992 ; Baysal et. al. 2001 ; Bali and Dave 2002) , and ϕ_i is the displacement vector defined by-

$$\phi_i=(\beta(t),0,0,0,0) \tag{3.3}$$

The energy momentum tensor of cosmic strings is

$$T_{ij}=\rho u_i u_j-\lambda x_i x_j \tag{3.4}$$

where, $\rho=\rho_p+\lambda$, is the energy density of the cloud of string, ρ_p being the rest energy density of particles attached to the strings and λ is the string tension density. $u^i=(0,0,0,0,A^{-1})$ is the five velocity vector for the cloud of particles and $x^i=(A^{-1},0,0,0,0)$ is the direction of strings. moreover the direction of strings satisfies

$$u^i u_i=-x^i x_i=-1\ ,\ and\ \ u^i x_i=0 \tag{3.5}$$

Using the comoving coordinate system and equations (3.3), (3.4) and (3.5) , the field equations (3.2) for the metric (3.1) yield-

$$2\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}^2}{B^2}+2\frac{\dot{B}\dot{C}}{BC}-2\frac{\dot{A}\dot{B}}{AB}-\frac{\dot{A}\dot{C}}{AC}+\frac{3}{4}\beta^2=\lambda A^2 \tag{3.6}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}-\frac{\dot{A}^2}{A^2}+\frac{\dot{B}\dot{C}}{BC}-\frac{3}{4}\beta^2=0 \tag{3.7}$$

$$\frac{\ddot{A}}{A}+2\frac{\ddot{B}}{B}-\frac{\dot{A}^2}{A^2}+\frac{\dot{B}^2}{B^2}-\frac{3}{4}\beta^2=0 \tag{3.8}$$

$$\frac{\dot{B}^2}{B^2}+2\frac{\dot{A}\dot{B}}{AB}+2\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{3}{4}\beta^2=\rho A^2 \tag{3.9}$$

Now (3.7) and (3.8) gives

$$\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}-\frac{\ddot{C}}{C}-\frac{\dot{B}\dot{C}}{BC}=0 \tag{3.10}$$

A solution of (3.10) is

$$B = e^{b_0 t + b_1} \tag{3.11}$$

$$C = e^{b_0 t - c_1} \tag{3.12}$$

Thus (3.11) and (3.12) together with (3.7) and (3.8) gives

$$A = (a_0 t + a_1)^{\frac{1}{2}} \tag{3.13}$$

and

$$\beta^2 = \frac{4}{3}[3b_0^2 - \frac{a_0{}^2}{2}(a_0 t + a_1)^{-2}] \tag{3.14}$$

Now from (3.9) we have

$$\rho = \frac{3}{2}a_0 b_0 (a_0 t + a_1)^{-2} + \frac{a_0^2}{2}(a_0 t + a_1)^{-3} \tag{3.15}$$

and from (3.6) we have

$$\lambda = 9b_0^2 (a_0 t + a_1)^{-1} - \frac{3}{2}a_0 b_0 (a_0 t + a_1)^{-2} - \frac{a_0^2}{2}(a_0 t + a_1)^{-3} \tag{3.16}$$

Therefore from the relation $\rho_p = \rho - \lambda$ we have

$$\rho_p = a_0^2 (a_0 t + a_1)^{-3} + 3a_0 b_0 (a_0 t + a_1)^{-2} - 9b_0^2 (a_0 t + a_1)^{-1} \tag{3.17}$$

For the metric (3.1), the expansion factor θ and shear scalar σ are obtained as

$$\theta = a_0 (a_0 t + a_1)^{-1} + 3b_0 \tag{3.18}$$

and

$$\sigma = \frac{1}{\sqrt{6}}[2b_0 - a_0 (a_0 t + a_1)^{-1}] \tag{3.19}$$

Therefore from equations (3.18) and (3.19) we have

$$\frac{\sigma}{\theta} = \frac{1}{\sqrt{6}} \cdot \frac{2b_0(a_0t + a_1) - a_0}{3b_0(a_0t + a_1) + a_0} \tag{3.20}$$

Here the deceleration parameter q is given by

$$q = \frac{4}{3} \cdot \frac{[a_0^2 - 2b_0^2(a_0t + a_1)^2][a_0^2 - 12b_0^2(a_0t + a_1)^2]}{[a_0^2 - 4b_0^2(a_0t + a_1)^2]^2} \tag{3.21}$$

3.3 Physical Interpretations of the Solutions:

In the universe we obtain here it is seen that the energy density has a finite value at the beginning and then it gradually decreases until it shrinks almost to zero at infinite time. The string tension density is also found to be a decreasing function of time until it almost tends to zero as time tends to infinity. Here, with the advent of time, the density of the string decreases more rapidly than density of the particles attached to them. Thus our universe ultimately becomes a universe dominated by particles, where strings are becoming invisible in course of time. Here, for our universe, we see that the special dimensions expand isotropically implying the expansion of our universe which bears testimony to our universe being a realistic one.

Moreover, from the expressions of the expansion factor and deceleration parameter obtained here it can be inferred that our universe is expanding, but the rate of expansion is decreasing slowly and slowly until at infinite time it is expanding at a constant rate. Here the gauge function β^2 is found to be constant at the initial epoch of time and gradually increases with time until it becomes a finite constant $4b_0^2$ at infinite time.

Interacting with the pressure-less matter here the displacement vector can play the same role as a cosmological constant (term). Thus it will be nice to study more to be whether the displacement vector plays a role in disturbing the rate of expansion of the universe.

Here also, $\frac{\sigma}{\theta} \neq 0$, first our universe seems to be anisotropic one, but it will become gradually an isotropic one until it becomes perfectly isotropic at time given by $t = -a_1 \frac{1}{a_0} \left(\frac{a_0}{b_0} \right)$. It can be seen that even though an anisotropic parameter is produced in this universe, its anisotropy does not promote anisotropy in the expansion, thus in course of time our universe becomes an isotropic one.