Chapter 7

On Bianchi Type III Cosmological Model with Quadratic EoS in Lyra Geometry

7.1 Introduction:

According to Binney and Tremaine (2008), the evolution of the universe may be divided into four main periods, which are vacuum energy era (or, Planck era), radiation era and matter dominated era and the dark energy era (or, de Sitter era). The universe undergoes a phase of early inflation in the vacuum energy era (Planck era), that brings the universe from the Planck size $l_p = 1.62 \times 10^{-35} m$ to a 'macroscopic' size $a \sim 10^{-6} m$ in a lowest possible fraction of a second [Albrecht et al. (1982); Linde (1990)]. After that the universe enters in the radiation era and then into the matter dominated era, whenever the temperature cools down below 103 K approximately (Weinberg 1972). Finally, in the dark energy era, the universe undergoes a phase transition of late inflation (Copeland et al. 2006). The singularity problems are solved in the early inflation (Albrecht et al. 1982) where as late inflation is necessary to account for the observed accelerating expansion of the universe [Riess et al. (1998) ; Perlmutter, et al. (1999) ; Bennett et al., (2003) ; Seljak, et al. (2005) ; Astier, et al. (2006) ; Daniel, et al. (2008) ; Amanullah et al., (2010) ; Suzuki, et al. (2012) ; Nishizawa (2014)]. The cosmological results and data sets like Atacama Cosmology Telescope (ACT) (Sievers et al. 2013), Planck 2015 results- XIII (Ade et al. 2015), are also suggested about late time inflation of the universe and allows the researchers to determine cosmological parameters such as the Hubble constant H and the deceleration parameter q. Studying the constraints given by the data from CMBR (Cosmic Microwave Background Radiation) investigation [Bernardis et al. (2000) ; Spergel et al. (2003)], WMAP (Wilkinson Microwave Anisotropy Probe) (Hinshaw et al. 2013), observations of clusters of galaxies at low red shift [Chaboyer et al. (1998) ;

Salaris and Weiss (1998)] etc. it can be deduced that the universe is dominated by some mysterious components. This mysterious component of the energy is called dark energy which has negative pressure and positive energy density (giving negative EoS parameter). In the energy budget of the universe it has been estimated that about 73% of our universe is Dark energy, about 23% is occupied by Dark matter and the usual baryonic matter occupy about 4% . Therefore the study of the nature of dark energy has become one of the most important topics in the field of fundamental physics [Sahni et al. (2000) ; Padmanabhan (2003) ; Li et al. (2011) ; Bamba et al. (2012) ; Bahrehbakhsh and Farhoudi (2013) ; Wang et al. (2016) ; El-Nabulsi (2016a)]. Einstein's cosmological constant Λ is the simplest candidate for dark energy and physically it is equivalent to the quantum vacuum energy. The cosmological model with Λ and cold dark matter (CDM) is usually called the ΛCDM model.

In the year 1919 Einstein geometrizes gravitation in his theory of general relativity which is treated as a basis for model of the universe. After that many cosmologists and astrophysicists attempted to study gravitation in different contexts. Weyl (1918) attempted to describe both gravitation and electromagnetism geometrically by formulating a new kind of gauge theory that involved metric tensor (with an intrinsic geometrical significance). But due to the non-integrability condition it does not get importance in the cosmological society. With the concept of Einstein's general theory of relativity, Lyra (1951) recommended a modification by introducing a gauge function into the structure less manifold that removes the non-integrability condition of the length of a vector under parallel transport. These theories are recognized as the alternate theories of the gravitation or modified theories of the gravitation. Some important modified theories of gravitation are Brans-Dicke theory (Brans and Dicke 1961) ; Scalar-tensor theories [Barker (1978) ; Nordvedt (1970) ; Saez & Ballester (1985)] ; Vector-tensor theory (Bekenstein 2004), Weyl's theory (Weyl 1918a), F(R) gravity (Nojiri et al. 2011), mimetic gravity (Chamseddine et al. 2013), mimetic F(R) gravity (Nojiri et al. 2014b), Lyra geometry (Scheibe 1952) etc. These modified theories of gravitation may be used to study the accelerating expansion of the universe. Out of these modified theories of gravitation, in this paper we will discuss about the Lyra geometry. Soleng (1987) showed that the gauge vector ϕ_i in Lyra's geometry will either play the role of cosmological constant or creation field equal to Hoyle's creation field (Hoyle and Narlikar 1963). Sen (1957), Sen and Dunn (1971), Rosen (1983) are some Well-known researchers who have studied scalar-tensor theory of gravitation based on Lyra geometry. Halford (1970; 1972) pointed out that the constant displacement vector field ϕ_i in Lyra's geometry in the normal general relativistic treatment plays the role of cosmological constant. He also shows that the scalar-tensor treatment based on Lyra's geometry predicts the same effect within observational limits as in the Einstein's theory of relativity. Many researchers [Bhamra (1974), Kalyanshetti & Waghmode (1982), Rahaman (2003), Pradhan & Aotemshi (2003), Rahaman et al. (2009b), Casana et al. (2006), Mohanty et al. (2007; 2009b,c), Yadav et al. (2011), Mahanta et al. (2012), Darabi et al. (2015)], Mollah et al. (2015), Mollah & Singh (2016)] studied Einstein's field equations in the framework of Lyra's geometry and successfully obtained their solutions under different circumstances.

The equation of state in relativity and cosmology, which is nothing but the relationship among combined matter, temperature, pressure, energy and energy density for any region of space, plays a vital role. Many researchers like Ivanov (2002), Sharma & Maharaj (2007), Thirukkanesh & Maharaj (2008), Feroze & Siddiqui (2011), Varela et al. (2010) etc. studied cosmological models with linear and non linear equation of state. For the study of dark energy and general relativistic dynamics in different cosmological models, Quadratic equation of state plays a crucial role. Dark energy universe with different equations of state were already discussed by various authors like, Nojiri & Odintsov (2004), Nojiri & Odintsov (2005a), Nojiri et al. (2005b), Nojiri & Odintsov (2011), Capozziello et al. (2006a) and Bamba et al (2012) describing dark energy or unified dark matter. The general form of quadratic equation of state

$$p = p_0 + \alpha\rho + \beta\rho^2 ,$$

where p_0, α, β are the parameters, is nothing but the first term of the Taylor's expansion of any equation of state of the form $p = p(\rho)$ about $\rho = 0$.

Ananda and Bruni (2006) studied the general relativistic dynamics of Robertson-Walker models with a non-linear equation of state (EoS), focusing on the quadratic equation of state $p = p_0 + \alpha\rho + \beta\rho^2$. They have shown that the behavior of the anisotropy at the singularity found in the brane scenario can be recreated in the general relativistic context. Also by considering quadratic equation of state of the form

$$p = \alpha\rho + \frac{\rho^2}{\rho_c} ,$$

they have discussed the anisotropic homogeneous and inhomogeneous cosmological models in general relativity and tried to isotropize the universe at early times when the initial singularity is approached. In our present study, we have considered the quadratic equation of state of the form

$$p = \alpha\rho^2 - \rho \ ,$$

where $\alpha \neq 0$ is a constant quantity but we can take $p_0 = 0$ to avoid complexities in our calculations. This will not affect the quadratic nature of the equation of state.

Again Chavanis (2013) studied a four dimensional Friedmann-Lemaitre-Roberston-Walker (FLRW) cosmological model based on a quadratic equation of state in the form $\frac{p}{c^2} = -\frac{4\rho^2}{3\rho_p} + \frac{\rho}{3} - \frac{4\rho_\Lambda}{3}$ unifying vacuum energy, radiation and dark energy. Also in the year 2015, a cosmological model based on a quadratic equation of state describing the early inflation, the intermediate decelerating expansion, and the late accelerating expansion of the universe have been studied by the same author Chavanis (2015).

Many authors like Maharaj et al. (2012); Rahaman et al. (2009a); Feroze and Siddiqui (2011) have studied cosmological models based on quadratic equation of state under different circumstances. Recently Reddy et al. (2015), Adhav et al. (2015), Rao et al. (2015) studied Kaluza-Klein Space-time cosmological models with quadratic equation of state in general and modified theories of relativity.

Motivated from the above studies, here we have investigated a homogeneous and anisotropic bianchi type III cosmological model universe with quadratic equation of state in the framework of Lyra Geometry and find out the realistic solutions.

This chapter has been organized as follows: In Sec. 7.2, we have formulated the problem where the physical and kinematical parameters are defined. In this section, exact solutions of the field equations are also obtained and the graphs of some of the parameters are shown as well. Sec. 7.3 consists of the physical and geometrical properties of the derived model. Concluding remarks are given in sec. 7.4. In sec. 7.5, an acknowledgment to the funding authority finds place.

7.2 Field Equations and their Solutions:

Let us consider the homogeneous and anisotropic space-time metric described by the Bianchi type III line element in the form

$$ds^2 = A^2dx^2 + B^2e^{-2\alpha x}dy^2 + C^2dz^2 - dt^2 \tag{7.1}$$

where A, B, and C are the scale factors, which are functions of cosmic time t only and $\alpha \neq 0$ is a constant.

The Einstein field equations in Lyra Geometry [as taken by Sen (1957); Sen and Dunn (1972)] in geometric units ($c = G = 1$) is given by

$$R_{ij} - \frac{1}{2}Rg_{ij} + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi^k\phi_k = -8\pi T_{ij} \tag{7.2}$$

and the energy momentum tensor T_{ij} is taken as

$$T^i_j = (\rho + p)u^i u_j + pg^i_j \tag{7.3}$$

where, $u^i = (0,0,0,1)$ is the four velocity vector, $\phi_i = (0,0,0,\beta(t))$ is the displacement vector; R_{ij} is the Ricci tensor; R is the Ricci scalar, ρ is the energy density and p is the pressure so that

$$g_{ij}u^i u^j = 1 \tag{7.4}$$

Therefore we have

$$T^1_1 = T^2_2 = T^3_3 = p \;\; ; \;\; T^4_4 = -\rho \quad and \quad T^i_j = 0 \quad for\,all \;\; i \neq 0 \tag{7.5}$$

Also, the general quadratic form of the Equation of state (EoS) for the matter of distribution is given by

$$p = p(\rho) = a\rho^2 + b\rho + c$$

where $a \neq 0$, b and c are constants.

From the field equations (7.2), the continuity equation is given by

$$\dot{\rho} + \frac{3}{2}\beta\dot{\beta} + \left[(\rho + p) + \frac{3}{2}\beta^2\right]\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0 \tag{7.6}$$

Again by the use of the energy conservation equation $T^i_{j;i}$, the continuity equation for matter can be written as

$$\left(R^i_j-\frac{1}{2}g^i_jR\right)_{;i}+\frac{3}{2}\left(\phi^i\phi_j\right)_{;i}-\frac{3}{4}\left(g^i_j\phi^k\phi_k\right)_{;i}=0 \tag{7.7}$$

Simplifying equation (7.7) we have

$$\frac{3}{2}\phi_j\left[\frac{\partial\phi^i}{\partial x^i}+\phi^l\Gamma^i_{li}\right]+\frac{3}{2}\phi^i\left[\frac{\partial\phi_j}{\partial x^i}-\phi_l\Gamma^l_{ij}\right]-\frac{3}{4}g^i_j\phi_k\left[\frac{\partial\phi^k}{\partial x^i}+\phi^l\Gamma^k_{lj}\right]-\frac{3}{4}g^i_j\phi^k\left[\frac{\partial\phi_k}{\partial x^i}-\phi_l\Gamma^l_{kj}\right]=0 \tag{7.8}$$

This equation (7.8) identically satisfied for $j=1,2,3$.

But for $j=4$, this equation reduces to

$$\frac{3}{2}\beta\dot{\beta}+\frac{3}{2}\beta^2\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \tag{7.9}$$

Therefore the continuity equation (7.6) reduces to

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \tag{7.10}$$

Therefore in a comoving coordinate system, the Bianchi type III space-time metric (7.1) for the energy momentum tensor (7.3), the Einstein's field equations (7.2) reduces to

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}+\frac{3}{4}\beta^2=-8\pi p \tag{7.11}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}+\frac{3}{4}\beta^2=-8\pi p \tag{7.12}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\alpha^2}{A^2}+\frac{3}{4}\beta^2=-8\pi p \tag{7.13}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-\frac{\alpha^2}{A^2}-\frac{3}{4}\beta^2=-8\pi\rho \tag{7.14}$$

$$\alpha\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=0 \tag{7.15}$$

where the overhead dot denote derivatives with respect to the cosmic time t.

Now for the metric (7.1) , important physical quantities like Volume V, average Scale factor R , Expansion Scalar θ , Hubble's parameter H , Shear Scalar σ , Anisotropy Parameter Δ and Deceleration parameter q are defined as

$$V = R^3 = ABCe^{-\alpha x} \tag{7.16}$$

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \tag{7.17}$$

$$H = \frac{1}{3}\theta = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{7.18}$$

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{B}\dot{C}}{BC}\right) \tag{7.19}$$

$$\Delta = \frac{1}{3H^2}\left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2}\right) - 1 \tag{7.20}$$

and

$$q = 3\frac{d}{dt}\left(\frac{1}{\theta}\right) - 1 \tag{7.21}$$

Since $\alpha \neq 0$, so from (7.15) we have

$$\frac{\dot{A}}{A} = \frac{\dot{B}}{B}$$

Integrating it we get

$$A = kB \tag{7.22}$$

where k is a constant. Without loss of generality we may choose $k = 0$ then we have

$$A = B \tag{7.23}$$

Therefore the equations (7.11)-(7.14) reduces to

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}+\frac{3}{4}\beta^2=-8\pi p \tag{7.24}$$

$$2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}-\frac{\alpha^2}{B^2}+\frac{3}{4}\beta^2=-8\pi p \tag{7.25}$$

$$\frac{\dot{B}^2}{B^2}+2\frac{\dot{B}\dot{C}}{BC}-\frac{\alpha^2}{B^2}-\frac{3}{4}\beta^2=8\pi\rho \tag{7.26}$$

The equations (7.24) - (7.26) represents a system of three independent simultaneous equations involving five unknown parameters viz. B, C, β, ρ and p , so in order to find exact solution of the above system it is required two more physical conditions involving these parameters. These two conditions are taken as follows-

i) choosing $b=-1$ and $c=0$ in general form of quadratic equation of state we have considered the equation of state in the form

$$p=a\rho^2-\rho \tag{7.27}$$

where a is a constant and

ii) we assume that the expansion scalar θ is proportional to the shear tensor σ_1^1 so that we get

$$BC=A^n \tag{7.28}$$

where n is a constant.

So, from equations (7.23) - (7.26) and (7.28) it may be obtained

$$n\frac{\ddot{B}}{B}+(n-1)^2\frac{\dot{B}^2}{B^2}+\frac{3}{4}\beta^2=-8\pi p \tag{7.29}$$

$$2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}-\frac{\alpha^2}{B^2}+\frac{3}{4}\beta^2=-8\pi p \tag{7.30}$$

$$(2n-1)\frac{\dot{B}^2}{B^2}-\frac{\alpha^2}{B^2}-\frac{3}{4}\beta^2=8\pi\rho \tag{7.31}$$

Subtracting (7.30) from (7.29) and adding (7.30) & (7.31) we have

$$(n-2)\frac{\ddot{B}}{B}+(n^2-2n)\frac{\dot{B}^2}{B^2}+\frac{\alpha^2}{B^2}=0 \tag{7.32}$$

and

$$\frac{\ddot{B}}{B}+n\frac{\dot{B}^2}{B^2}-\frac{\alpha^2}{B^2}=4\pi(\rho-p) \tag{7.33}$$

From equations (7.32) and (7.33), it can be obtained that

$$\frac{\ddot{B}}{B}+n\frac{\dot{B}^2}{B^2}=\frac{4\pi}{n-1}(\rho-p) \tag{7.34}$$

The equations (7.16) and (7.34) will give us

$$\dot{V}=\sqrt{m\rho V^2+k_1} \tag{7.35}$$

where k_1 and $m=\frac{8\pi(n+1)}{n-1}$ are integrating constants.

Integrating equation (7.35) we have

$$\int\frac{dV}{\sqrt{m\rho V^2+k_1}}=t+k_2 \tag{7.36}$$

where k_2 is an integrating constant that represent a shift of cosmic time t. Therefore it can be chosen as zero.

Equations (7.10) and (7.27) will give us

$$\rho=(alogV)^{-1} \tag{7.37}$$

Now, if we take $k_1=k_1=0$ then from equations (7.36), the volume V is obtained as

$$V=exp\left[\left(\frac{9m}{4a}t^2\right)^{\frac{1}{3}}\right] \tag{7.38}$$

So, the scale factors A, B, and C are obtained from equations (7.16), (7.23), (7.28) and (7.38) as

$$A = B = exp\left[\frac{1}{n+1}\left(\frac{9m}{4a}t^2\right)^{\frac{1}{3}}\right]e^{\frac{\alpha x}{n+1}} \tag{7.39}$$

and

$$C = exp\left[\frac{n-1}{n+1}\left(\frac{9m}{4a}t^2\right)^{\frac{1}{3}}\right]e^{\frac{(n-1)\alpha x}{n+1}} \tag{7.40}$$

Using equations (7.39) and (7.40) in equation (7.1), the geometry of the model universe is given by

$$\begin{aligned} ds^2 =& exp\left[\frac{2}{n+1}\left(\frac{9m}{4a}t^2\right)^{\frac{1}{3}}\right]e^{\frac{2\alpha x}{n+1}}\left[dx^2+e^{-2\alpha x}dy^2\right] \\ &+exp\left[\frac{2(n-1)}{n+1}\left(\frac{9m}{4a}t^2\right)^{\frac{1}{3}}\right]e^{\frac{2(n-1)\alpha x}{n+1}}dz^2-dt^2 \end{aligned} \tag{7.41}$$

The use of equation (7.38) in Equation (7.37), the energy density ρ is obtained as

$$\rho = \frac{1}{a}\left(\frac{9m}{4a}\right)^{-\frac{1}{3}}t^{-\frac{2}{3}} \tag{7.42}$$

Therefore from equation (7.27), the pressure p can be obtained as

$$p = \frac{1}{a}\left(\frac{9m}{4a}\right)^{-\frac{1}{3}}t^{-\frac{2}{3}}\left[\left(\frac{9m}{4a}\right)^{-\frac{1}{3}}t^{-\frac{2}{3}}-1\right] \tag{7.43}$$

From equation (7.31) the displacement vector β is given by

$$\frac{3}{4}\beta^2 = \frac{n-2}{n-1}\frac{4\pi}{a}\left(\frac{9m}{4a}\right)^{-\frac{1}{3}}t^{-\frac{2}{3}}\left[\frac{2}{n+1}-\left(\frac{9m}{4a}\right)^{-\frac{1}{3}}t^{-\frac{2}{3}}\right] \tag{7.44}$$

For the model universe (7.41), the other physical and geometrical properties like expansion scalar θ, Hubble's expansion factor H, shear scalar σ, anisotropy parameter Δ and deceleration parameter q can be easily obtained from equations (7.17)-(7.21).

$$\theta = \frac{2}{3}\left(\frac{9m}{4a}\right)^{\frac{1}{3}} t^{-\frac{2}{3}} \tag{7.45}$$

$$H = \frac{2}{9}\left(\frac{9m}{4a}\right)^{\frac{1}{3}} t^{-\frac{2}{3}} \tag{7.46}$$

$$\sigma^2 = \frac{4(n-2)^2}{27(n+1)^2}\left(\frac{9m}{4a}\right)^{\frac{2}{3}} t^{-\frac{2}{3}} \tag{7.47}$$

$$\Delta = \frac{2(n-2)^2}{(n+1)^2} = constant \neq 0 \;\; for \;\; n \neq 2 \;\; and \;\; n \neq -1 \tag{7.48}$$

and

$$q = \frac{3}{2}\left(\frac{9m}{4a}\right)^{-\frac{1}{3}} t^{-\frac{2}{3}} - 1 \tag{7.49}$$

It is well known that the different values of the parameters will give rise different graph, so the variations of some parameters are shown, by taking particular values of the integrating constants as $a = 10$ and $n = 10$ so that $m = 30.730159$, in **Figures 1-6**.

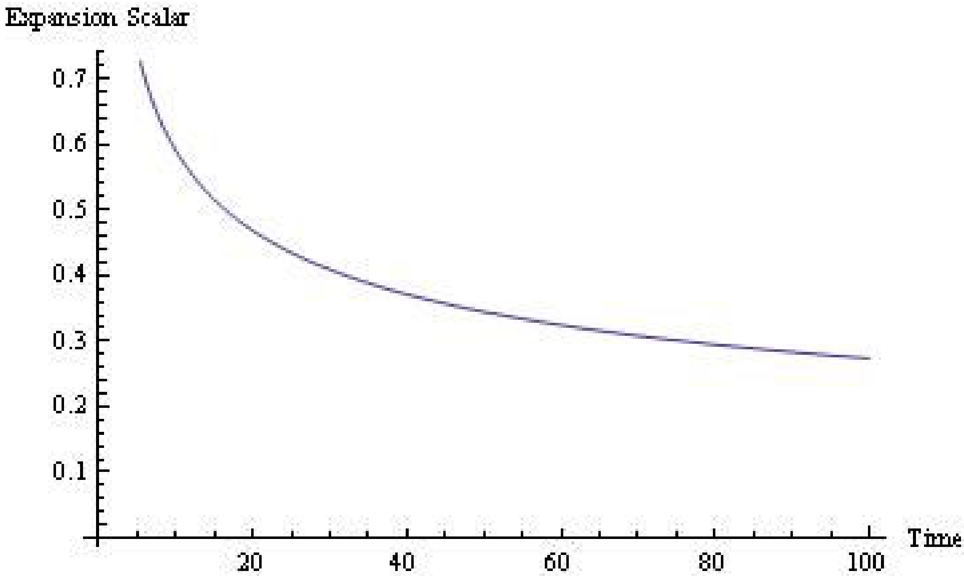

Figure-7.1 : The variation of Expansion Scalar θ vs. Time t, when $a = 10$ and $n = 10$ so that $m = 30.730159$.

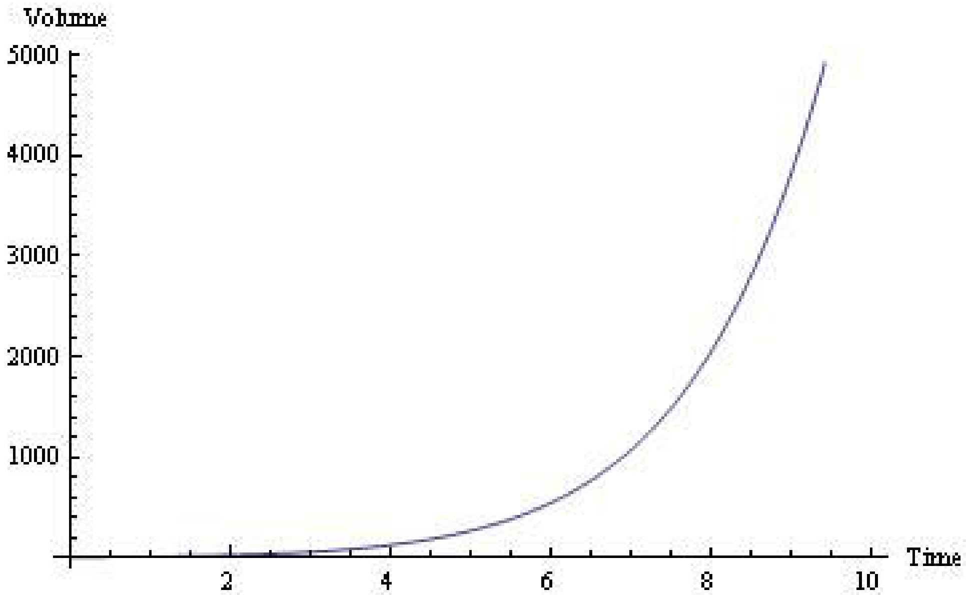

Figure-7.2 : The variation of Volume V vs. Time t, when $a = 10$ and $n = 10$ so that $m = 30.730159$.

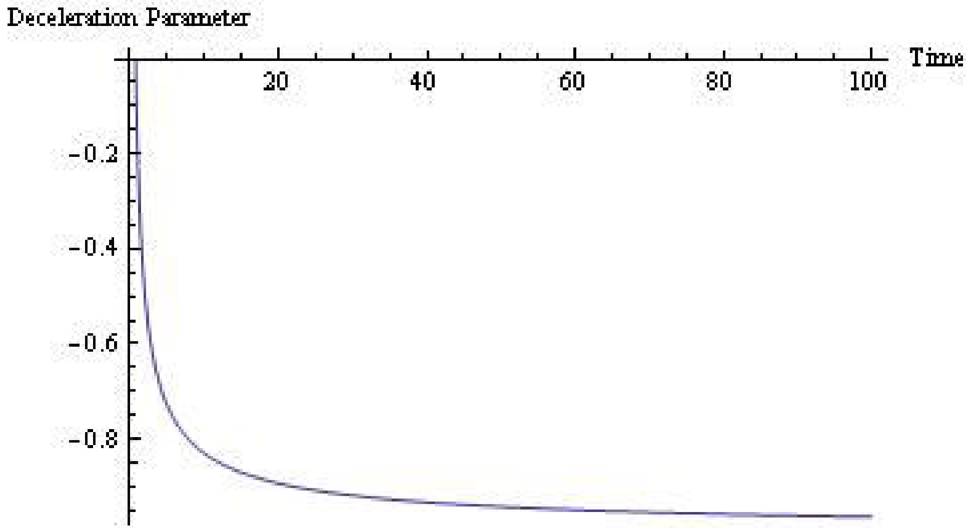

Figure-7.3 : The variation of Deceleration Parameter q vs. Time t, when $a = 10$ and $n = 10$ so that $m = 30.730159$.

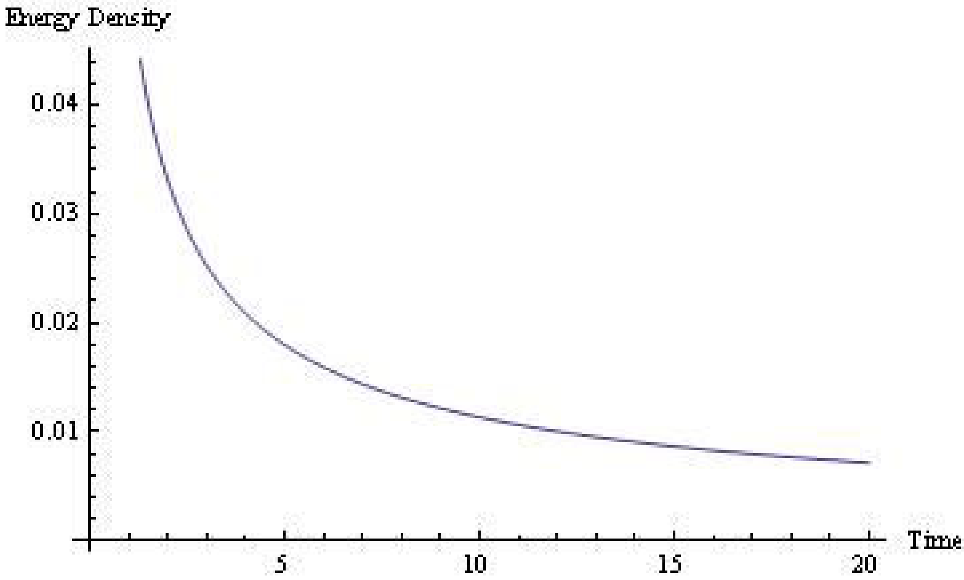

Figure-7.4 : The variation of Energy Density ρ vs. Time t, when when $a = 10$ and $n = 10$ so that $m = 30.730159$.

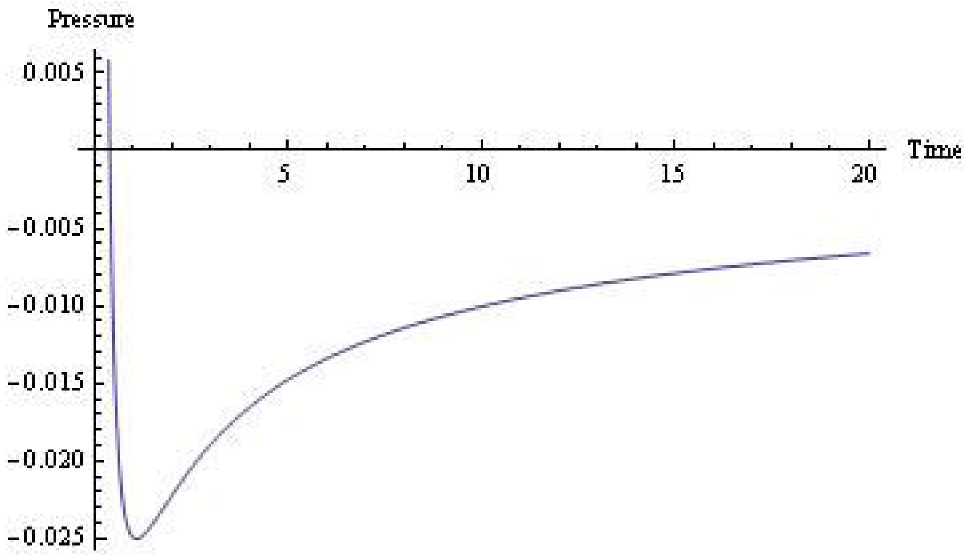

Figure-7.5 : The variation of Pressure p vs. Time t, when when $a = 10$ and $n = 10$ so that $m = 30.730159$.

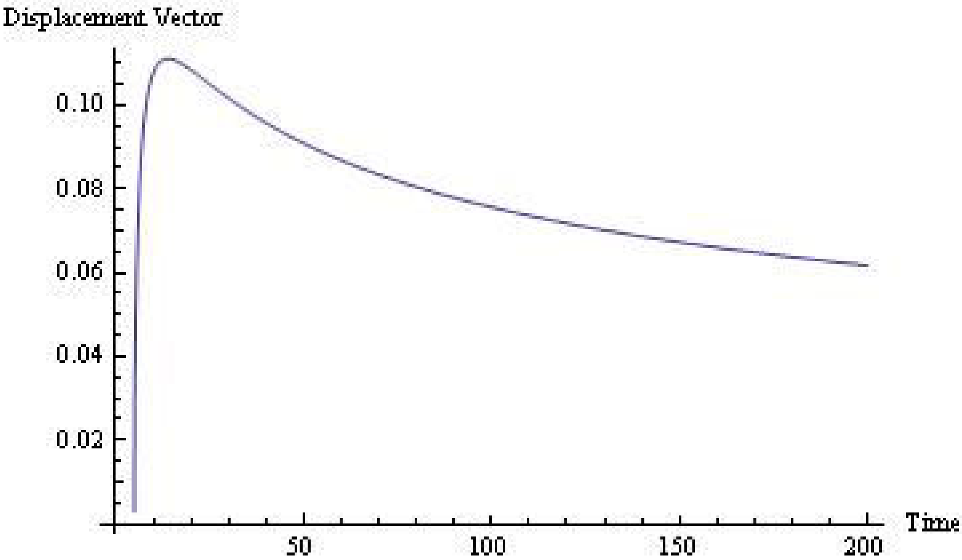

Figure-7.6 : The variation of Displacement Vector β vs. Time t, when $a = 10$ and $n = 10$ so that $m = 30.730159$.

7.3 Physical and Geometrical Properties of the Model Universe:

The evolution of expansion scalar θ has been shown in **Figure 7.1** corresponding to the equation (7.45) and it is observed that the expansion scalar θ starts with infinite value at initial epoch of cosmic time $t = 0$ but as time t progresses it decreases and becomes constant after some finite time that explains the Big-Bang scenario. From equation (7.38) and **Figure 7.2**, it is seen that the value of volume V of the model universe is an increasing function of cosmic time t and increases from zero at initial epoch of time to infinite volume whenever $t \to \infty$, so our model represents an expanding universe.

Now from equation (7.49) and **Figure 7.3**, it is clear that initially the value of deceleration parameter q is negative and tends to -1 at infinite time i.e. the values of q is in the range $-1 \leq q \leq 0$ implying that the values of q satisfies the present observational data like Riess et al. (1998) and Perlmutter et al. (1999). Also from the expression (7.46) for Hubble's expansion factor H it has been observed that dH/dt is negative, so our model universe expands with an accelerated rate.

Since the present day universe is isotropic, it is important to observe whether our models evolve into an isotropic or an anisotropic one. In order to investigate the isotropy of the model universe, here we have considered the simple anisotropy parameter Δ. From equation (7.48), it has been observed that the anisotropy parameter Δ is independent of cosmic time t and $\Delta = 0$ for $n = 2$ but whenever $n \neq 2$ and $n \neq 1$ then $\Delta \neq 0$, which shows that our model universe is isotropic throughout the evolution whenever $n = 2$ but for $n \neq 2$ and $n \neq 1$, the model remains anisotropic throughout the evolution.

Again it is known that the energy conditions for Bianchi type III model is energy density ρ is positive i.e. $\rho > 0$. From **Figure 7.4** of equation (7.42), it is seen that the energy ρ is always positive. Also the **Figure 7.5** showing the evolution of pressure p depicts that initially when $t \to 0$ then p is positive but as the time progresses p changes sign from positive to negative. Therefore our model universe trespasses through the transition from matter dominated period to inflationary period.

From equation (7.44), the displacement vector β is found to be positive which increases rapidly at initial epoch of time but with the increase of time it decreases and at infinite time the displacement vector β becomes a small positive constant. The variation of the parameter 'displacement vector β' is shown in **Figure 7.6**. Also from equation (7.47) it can be seen that the shear scalar σ is a decreasing function of cosmic time t and vanishes as $t \to \infty$. So our model represents a shear free dark energy cosmological model universe for large values of cosmic time t.

7.4 Conclusion:

Investigating a homogeneous and anisotropic space-time described by Bianchi type III metric in presence of perfect fluid in the context of Lyra geometry under the assumption of quadratic equation of state (EoS), exact solutions of the Einstein's field equations have been obtained. Here we have got a model universe which is expanding with acceleration that also passes through the transition from matter dominated period to inflationary period. The displacement vector β and shear scalar ρ becomes zero as $t \to \infty$. So our model represents a shear free dark energy cosmological model universe for large values of cosmic time t.