Chapter 8

Bianchi Type III Cosmological Model with Hybrid Scale Factor in Presence of Van der Waal Fluid in Lyra Manifold

8.1 Introduction:

Recently cosmologists are started to believe that our universe is expanding with acceleration instead of deceleration as predicted by the Big Bang theory (Silk 1989). The idea of accelerated expansion of the universe was first proposed by (Riess et al. 1998). Cosmological observations like Type SNeIa supernovae (Schmidt et al. 1998; Perlmutter et al.1999; Riess et al. 2004; Astier et al. 2006; Amanullah et al. 2010; Suzuki et al. 2011), the large scale galaxies structures of universe (Daniel et al. 2008), Sachs-Wolfe effects (Nishizawa 2014), CMB (Cosmic Microwave Background) anisotropies (Bennett et al. 2003), and SDSS (Seljak et al. 2005) supports the above statement that the universe is undergoing an accelerated expansion. Again the cosmological results and data sets like Atacama Cosmology Telescope (ACT) (Sievers et al. 2013), Planck 2015 results- XIII (Ade et al. 2016), are also supporting this fact and allows the researchers to determine cosmological parameters such as the Hubble constant H and the deceleration parameter q. Studying the constraints given by the data from CMBR (Cosmic Microwave Background Radiation) investigation (Bernardis et al. 2000), (Spergel et al. 2003), WMAP (Wilkinson Microwave Anisotropy Probe) (Hinshaw et al. 2013), observations of clusters of galaxies at low red shift (Chaboyer et al. 1998; Salaris and Weiss 1998) etc. can be deduced that the universe is dominated by some mysterious components. This mysterious component of the energy is called dark energy which has negative pressure and positive energy density (giving negative EoS parameter). In the energy budget of the universe it has been estimated that about 73% of our universe is Dark energy, about 23% is occupied by Dark matter and the usual baryonic matter occupy about 4%. Different people giving different opinions about dark energy but there is no dearth of candidates for dark energy proposed still today. Therefore the further study of the nature of dark energy is required in the present research so it has become one of the most important topics in the field of fundamental physics. Due to this reason some authors like (Sahni et al. 2000; Padmanabhan 2003; Li et al. 2011; Bamba et al. 2012; Bahrehbakhsh et al. 2013; Wang et al. 2016; El-Nabulsi 2016a) made their contributions in this field.

In recent years many cosmologists and Physicists (of high energy physics) tried to explain the origin and behavior of dark energy and have suggested many cosmological models in different context. Some of the important cosmological models to explain the nature of dark energy are: Cosmological constant (Ostriker and Steinhardt 1995; Carroll 2001; Martin 2012), Dark Energy model with variable Cosmological Constant Λ (Sola and Stefancic 2005; Shapiro and Sola 2009), ΛCDM [Cosmological Constant Λ and Cold Dark Matter] (Pietro and Claeskens 2003; Riess et al 2004; Peebles and Ratra 2003), CPL [Chevalliear-Polarski-Linder parametrization] (Chevallier and Polarski 2001; Linder 2003), HDE [holographic dark energy- nature of DE according to some basic quantum gravitational principle] (Li 2004; Zhang and Wu 2005; Sadeghi et al. 2014; Nojiri and Odintsov 2006a; Saridakis 2008; Setare and Saridakis 2008; Cohen et al. 1999; Zhang and Wu 2007; Li et al. 2009; Zhang et al. 2015 ; Feng and Zhang 2015), Quintessence (Khurshudyan et al. 2014a,c; Zlatev et al. 1999; Saridakis and Sushkov 2010; Zhang 2005) which is a spatially homogeneous, slowly rolling scalar field that can also provide a negative pressure, driving thecosmic acceleration, which provides a possible mechanism for dynamical dark energy, Phantom (Dabrowski 2008; Dutta and Scherrer 2009) a scalar field that can provide a negative pressure driving the cosmic acceleration or DE from scalar field with a negative sign of the kinetic term, Quintom (Capozziello et al. 2006b; Cai et al. 2010), Tachyons (Padmanabhan and Choudhury 2002; Sen and Devi 2008), k-essence (Brax and Martin 1999; Picon et al. 2001) etc. can be mentioned from the literatures.

But when we study these literatures we came to know that instead of general equation of state, enlarged and exotic equations of state were needed to model the acceleration of the cosmic expansion (Caldwell et al. 1998; Steinhardt et al. 1999; Peebles et al. 2003). In order to describe the standard matter and dark energy as a single fluid, many researchers like (Fabris et al. 2002; Kremer 2003b; Guo et al. 2007; Fabris et al. 2011; Avelino et al. 2014; Kahya et al. 2015; Singh et al. 2016a; Rao et al. 2016; Heydarzade et al. 2016) uses the Chaplygin gas-type equation of state that describe a transition from a matter dominated period to a cosmological constant dominated epoch. So in the present we can consider the Van der Walls equation of state which was introduced by the cosmologists Capozziello and his co-workers (Capozziello et al. 2002; Capozziello et al. 2003). Also we came to know that without introducing scalar fields, the transition from a scalar field dominated period to a matter dominated era can be described by the van der Waals equation of state. After Capozziaello, Kremer (2003a) also show that the van der Waals equation of state may be used to describe the transition from an inflationary period to a matter field dominated era. Malaver (2013) generated new exact solutions to the Einstein-Maxwell system, considering Van der Waals modified equation of state with polytropic exponent.

After Einstein, many Physicists and cosmologists have modified Einstein's theory of relativity in different context. Some of the important modified theories of gravitation are Weyl's theory (Weyl 1918), Brans-Dicke theory (Brans-Dicke 1961), Scalar-tensor theory (Barker 1978), F(R) Gravity (Buchdahl et al. 1970; Nojiri et al 2003), F(R, t) Gravity (Harko et al. 2011; Alves et al. 2016; Xu et al. 2016), Mimetic Gravity (Nojiri et al 2014b), Lyra geometry (Scheibe 1952) etc. Accelerated expansion of the universe may be discussed with the help of these modified theories of gravity so in this paper we discuss about the cosmological model by considering Lyra geometry. Many researchers have studied different cosmological models in Lyra geometry in different contexts. Similar to the Einstein's static model, Sen (1957) have obtained a static model with finite density by formulating a new scalar-tensor theory of gravitation based on Lyra geometry. But due to the non-integrability of length transfer so that the model has some unsatisfactory features did not gain general acceptance. Later, in the normal general relativistic treatment, (Halford 1970; Halford 1972) shown that the constant displacement vector field ϕ_k in Lyra's geometry plays the role of cosmological constant. As far as the classical solar system tests are concerned, the scalar-tensor treatment based on Lyra's geometry predicts the same effect, within observational limits as in the Einstein's theory of relativity. Some of the authors like (Beesham 1988; Singh and Singh 1993; Rahaman et al. 2003 and 2005; Casana et al. 2006; Mohanty et al. 2009c; Ram et al. 2010; Schingolev 2012; Kumari et al. 2013; Hova 2014; Gad 2015; Mollah et al. 2015; Ragab 2015; Ragab and Mazrooei 2016; Gad et al. 2016; Mollah and Singh 2016; Singh and Mollah 2016) etc. have studied Einstein's field equations in the framework of Lyra's geometry under different contexts and obtain their solutions successfully. Recently many well known researchers like (Hova 2013; Khurshudyan et al. 2014a,b; Ziaie et al. 2015; Darabi et al. 2015; Saadat 2015) suggested about accelerated expansion of the universe by investigating Einstein's field equations in the framework of Lyra's geometry under different contexts. Motivated from the study of the above literatures, here we studied a Bianchi type-III model universe interacting with Van der Wall fluid in Lyra manifold and find out the realistic solutions supporting the present observational facts. Physical and geometrical properties of the model are also discussed in details with the graphical presentations.

The paper is organized as follows. Section 8.2 consists of the field equations and their solutions, here we formulate the problem and obtained the exact solutions by considering Hybrid Scale factor and Van der Waal equation of state and plotted the graphs of four parameters. Section 8.3 is devoted to the detailed discussion of the results obtained. Finally in section 8.4, conclusions and further perspectives of our approach are mentioned.

8.2 Field Equations and their Solutions:

Cosmological model universe is considered with Bianchi type - III the line element as

$$ds^2 = A^2dx^2 + B^2e^{-2\alpha x}dy^2 + C^2dz^2 - dt^2 \tag{8.1}$$

where the metric potentials A, B, and C are the functions of time t alone and α is a non-zero constant.

Here the Einstein field equations based on Lyra's geometry is taken as (Sen 1957 ; Sen and Dunn 1971)

$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi^k\phi_k = -8\pi T_{ij} \tag{8.2}$$

with $\frac{8\pi G}{c^2} = 1$ where, R_{ij} is the Ricci tensor; R is the Ricci scalar, ϕ_i is the displacement vector given by

$$\phi_i = (0,0,0,\beta(t)) \tag{8.3}$$

and T_{ij} is the-energy momentum tensor for cosmic fluid which is to be taken for ideal fluid in the form

$$T_{ij} = (\rho + p)u_iu_j + pg_{ij} \tag{8.4}$$

where, ρ is the energy density , p is the pressure of the cosmic fluid and $u^i = (0,0,0,1)$ is the four velocity vector satisfying

$$g_{ij}u^i u^j = u^i u_i = -1 \tag{8.5}$$

Therefore we have

$$T_1^1 = T_2^2 = T_3^3 = p \;\; ; \;\; T_4^4 = -\rho \quad and \quad T_j^i = 0 \;\; for \;\; i \neq 0 \tag{8.6}$$

Let us consider that the fluid satisfy the energy conservation equation $T^i_{i;j} = 0$ for perfect fluid, so that we have

$$\dot{\rho} + \frac{\dot{V}}{V}(\rho + p) = 0 \tag{8.7}$$

In comoving coordinate system, the field equations (8.2) for the line element (8.1) with the help energy momentum tensor given by equation (8.4) reduces to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{3}{4}\beta^2 = -p \tag{8.8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} + \frac{3}{4}\beta^2 = -p \tag{8.9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} + \frac{3}{4}\beta^2 = -p \tag{8.10}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} - \frac{3}{4}\beta^2 = \rho \tag{8.11}$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0 \tag{8.12}$$

where the overhead dot (.) denotes differentiation with respect to time t.

The physical quantities such as Volume V, average Scale factor R , Expansion Scalar θ , Hubble's expansion factor H , Shear Scalar σ , Anisotropy Parameter Δ and Deceleration parameter q which are playing significant role in describing the properties of the model universe are given by

$$V = R^3(t) = ABCe^{-\alpha x} \tag{8.13}$$

$$\theta = 3H = u^i_{;i} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \tag{8.14}$$

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{8.15}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right) = \frac{1}{3}\left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{B}\dot{C}}{BC}\right) \tag{8.16}$$

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{1}{3H^2}\left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - 3H^2\right) \tag{8.17}$$

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = 3\frac{d}{dt}\left(\frac{1}{\theta}\right) - 1 \tag{8.18}$$

where H_i ; $i = 1, 2, 3$ represent the directional Hubble's parameters in x, y, z directions respectively.

From equation (8.12) we have

$$A = k_0 B$$

where k_0 is an integrating constant. Let us choose $k_0 = 0$, which will not affect the generality of the solution so that we have

$$A = B \tag{8.19}$$

Therefore the equations (8.8)-(8.11) reduces to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{3}{4}\beta^2 = -p \tag{8.20}$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{\alpha^2}{B^2} + \frac{3}{4}\beta^2 = -p \tag{8.21}$$

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{B^2} - \frac{3}{4}\beta^2 = \rho \tag{8.22}$$

Subtraction of equation (8.20) from the equation (8.21) will give

$$\frac{d}{dt}\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)+\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)\left(2\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right)=\frac{\alpha^2}{B^2} \tag{8.23}$$

Integrating Equation (8.23), we get

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=\frac{k_1}{R^3}e^{\int\frac{\frac{\alpha^2}{B^2}}{\frac{\dot{B}}{B}-\frac{\dot{C}}{C}}dt} \tag{8.24}$$

where $k_1 > 0$ is a constant of integration.

Here we have three independent field equations (8.20)-(8.22) containing five unknown parameters viz- B, C, β, ρ and p, so in order to solve these equations it is required two more physical relations involving them. These two relations are taken as

(i) Following (Adhav 2011), first we take the relation between the metric potentials B and C as

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=\frac{\alpha^2}{B^2} \tag{8.25}$$

and

(ii) Hybrid scale factor (Jimanez, et al. 2009) is taken as

$$R=t^b e^{at} \tag{8.26}$$

where a and b are positive constants.

From equations (8.24)and (8.25) we have

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=\frac{k_1}{R^3}e^t \tag{8.27}$$

Therefore from equations (8.19), (8.26) and (8.27), the values of of the scale factor A, B and C are obtained as

$$A=B=k_2 t^b e^{at} e^{\frac{k_1}{3}F(t)} \tag{8.28}$$

and

$$C = k_2^{-2} t^b e^{at} e^{-\frac{2k_1}{3}F(t)} \tag{8.29}$$

where $k_1 > 0$ and $k_2 > 0$ are integrating constants and $F(t) = \int t^{-3b} e^{(1-3a)t} dt$

Using these values of scale factors A, B and C in equation (8.1) we have

$$ds^2 = k_2^2 t^{2b} e^{2at} e^{\frac{2k_1}{3}F(t)} [dx^2 + e^{-2\alpha x} dy^2] + k_2^{-4} t^{2b} e^{2at} e^{-\frac{4k_1}{3}F(t)} dz^2 - dt^2 \tag{8.30}$$

The van der Waals equation of state [Kremer (2004)] is taken in the form

$$p = \frac{8\omega\rho}{3-\rho} - 3\rho^2 \tag{8.31}$$

Now when $\omega = 0$ then the equation (8.31) reduces to

$$p = -3\rho^2 \tag{8.32}$$

Therefore from equations (8.7) and (8.32) we have the energy density ρ and pressure p as

$$\rho = \frac{1}{3 + c_0 t^{3b} e^{3at}} \tag{8.33}$$

and

$$p = -\frac{3}{\left(3 + c_0 t^{3b} e^{3at}\right)^2} \tag{8.34}$$

where $c_0 > 0$ is an integrating constant.

Again the equation (8.20) will give us the value of displacement vector β as follows

$$\beta^2 = \frac{4}{\left(3 + c_0 t^{3b} e^{3at}\right)^2} + \frac{8}{3}\frac{b}{t^2} - 4\left(a + \frac{b}{t}\right)^2 + \frac{4k_1}{9} t^{-3b} e^{(1-3a)t} - \frac{4k_1^2}{9} t^{-6b} e^{2(1-3a)t} \tag{8.35}$$

Therefore from equations (8.13)-(8.18) the values of parameters Volume V, Expansion Scalar

θ , Hubble's expansion factor H , Shear Scalar σ , Anisotropy Parameter Δ and Deceleration parameter q for our model universe are obtained as

$$V = t^{3b} e^{3at} \tag{8.36}$$

$$\theta = 3\left(a + \frac{b}{t}\right) \tag{8.37}$$

$$H = \left(a + \frac{b}{t}\right) \tag{8.38}$$

$$\sigma^2 = \frac{1}{3} k_1^2 t^{-6b} e^{2(1-3a)t} \tag{8.39}$$

$$\Delta = \frac{2k_1^2}{9}\left(a + \frac{b}{t}\right)^{-2} t^{-6b} e^{2(1-3a)t} \tag{8.40}$$

and

$$q = \frac{b}{(at+b)^2} - 1 \tag{8.41}$$

The variations of the parameters like Volume V, Expansion Scalar θ, Deceleration parameter q, energy density ρ and pressure p with respect to cosmic time t for this model universe are presented in **Figures 1-5**.

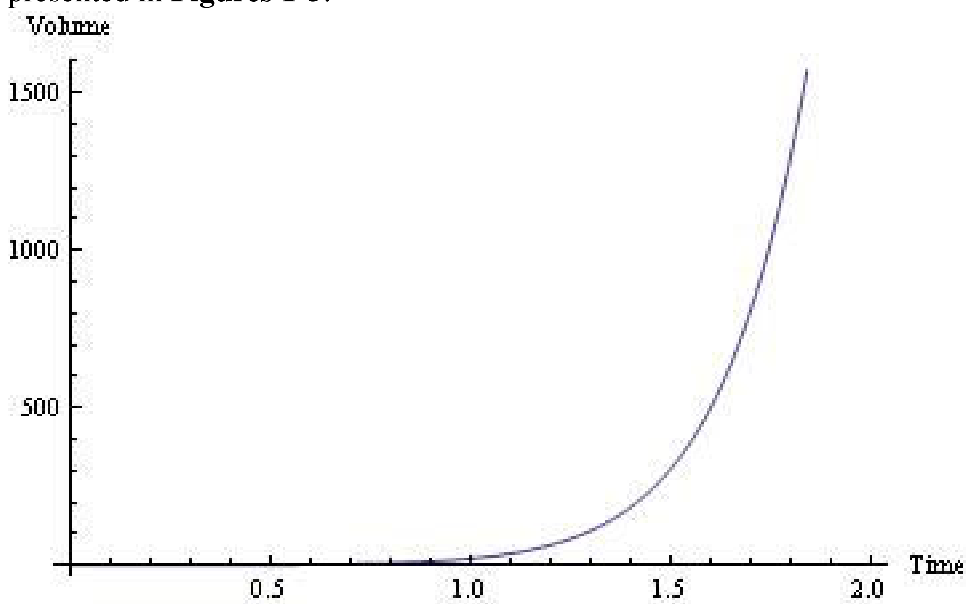

Figure-8.1 : The plot of Volume V vs. Time t , whenever $a = b = 1$.

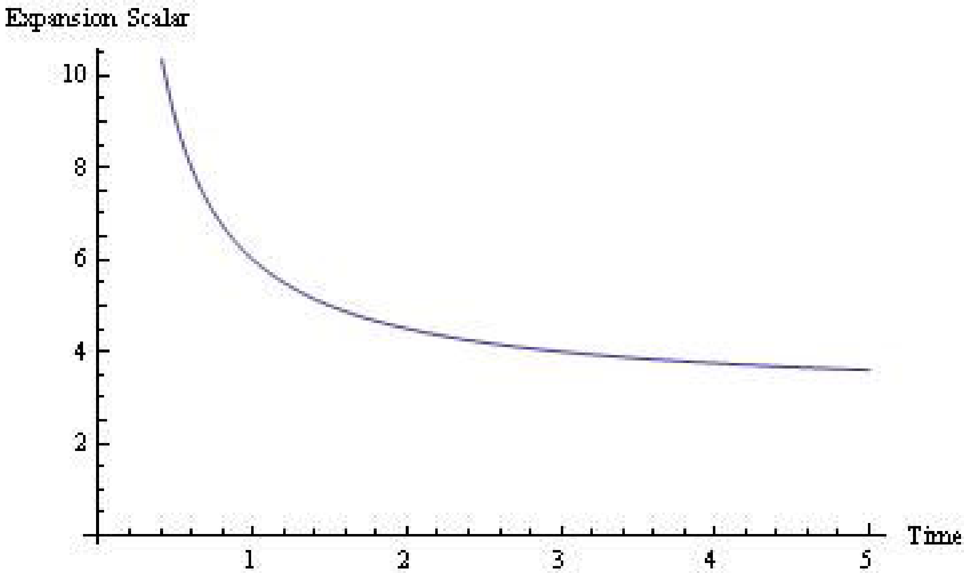

Figure-8.2 : The plot of Expansion scalar θ vs. Time t , whenever $a = b = 1$.

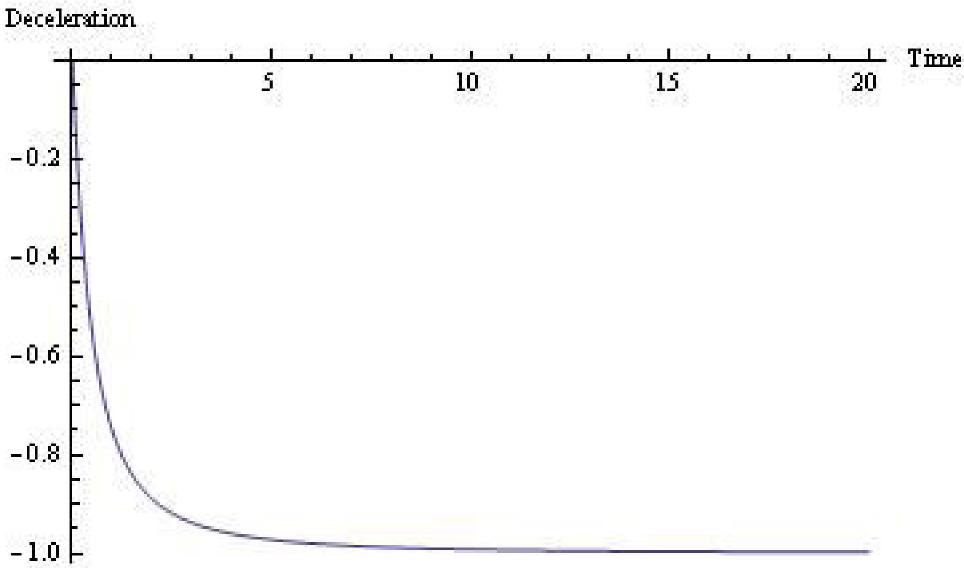

Figure-8.3 : The plot of Deceleration Parameter q vs. Time t , whenever $a = b = 1$.

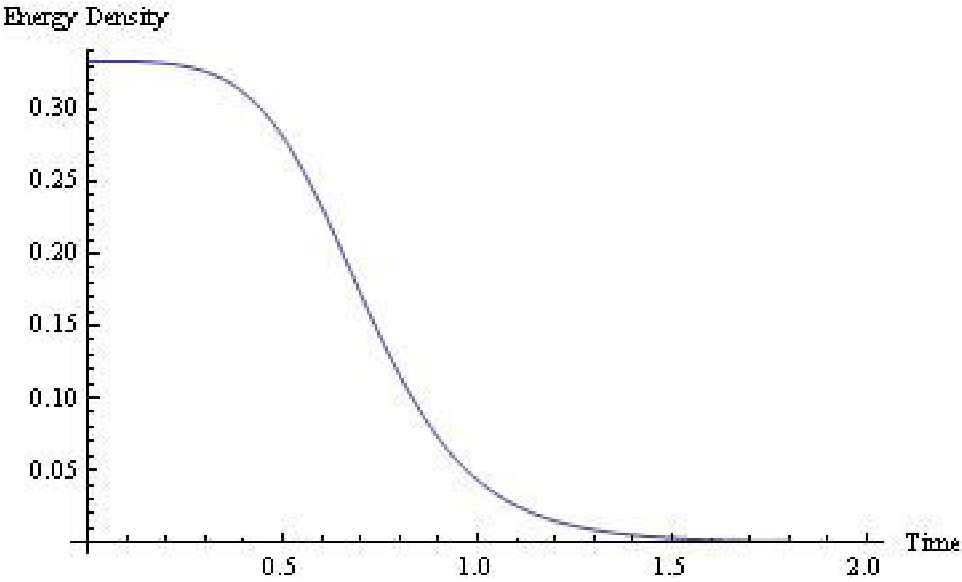

Figure-8.4 : The plot of Energy density ρ vs. Time t , $a = b = c_0 = 1$.

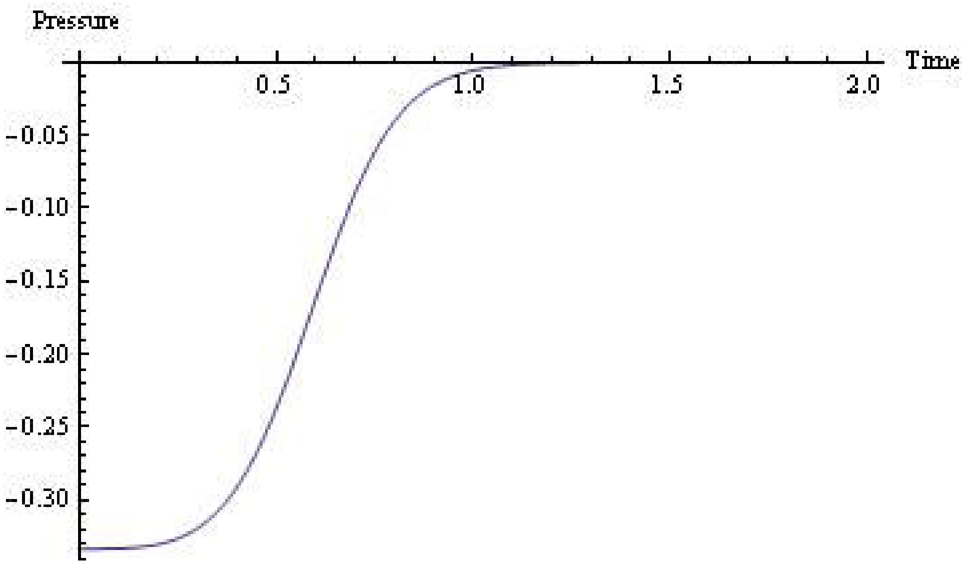

Figure-8.5 : The plot of Pressure p vs. Time t , $a = b = c_0 = 1$.

8.3 Physical Interpretations of the Solutions:

We discussed the physical and geometrical properties of proposed model with the following few points.

(i) From equation (8.36) and the graph of Volume vs. time which shown in **figure-8.1** we see that the value of volume V is positive for all values of t and increases with the increase of time t. Also from equation (8.37) and the graph of expansion scalar vs. time as given in **figure-8.2**, it seems that the expansion scalar θ are positive, so our model given by equation (8.30) represents an expanding universe.

From the expression of the expansion scalar θ obtained in the equation (8.37), we also see that our model possesses a big-bang singularity at $t = 0$ and then expand until at $a = -\frac{b}{t}$ it stops expansion, which shows the period of initial expansion. And at this point of time our universe becomes isotropic. However at $t \to \infty$ the universe is expanding at a constant rate which indicates that our universe will be a flat one.

(ii) Also, **Figure-8.3** depicts the behavior of deceleration parameter against time. From equation (8.41) and **figure-8.1** it is seen that the deceleration parameter q is negative and is a decreasing function of time t which approaches to '-1' whenever $t \to \infty$. Therefore the deceleration parameter of our model universe agrees with the result of (Xu et al. 2009) that three different ranges for deceleration parameter: $0.715 \pm 0.045, -0.658^{+0.061}_{-0.057}, 0.461^{+0.031}_{-0.033}$. Thus with the evolution of time, our model represent an accelerating universe, since for all values of t, the value of deceleration parameter q is negative and the value of expansion factor is positive, so our model universe is expanding at an accelerated rate.

From the expression of the deceleration parameter we also see that the rate of accelerated expansion will increase with time until it attains its maximum value at $t \to \infty$.

(iii) Again since the anisotropy parameter $\Delta \neq 0$, therefore our model is found to be an anisotropic one.

(iv) On the other hand when we observed that the variation of energy density ρ and pressure p against time which are presented in **Figure-8.4** and **Figure-8.5** with the values of the respective parameters, it has been observed that, the energy density ρ decreases from a positive value with the increase of time i.e. when $t \to \infty$ then $\rho \to 0$, whereas the pressure p is a negative quantity but is an increasing function of time and increases from negative to zero with the evolution of time t. i.e. $p \to 0$ whenever $t \to \infty$, which shows that our model universe represents a dark energy model.

Also as the energy density of the universe tend to 0 (zero) as $t \to \infty$. However its density tends to a fixed quantity $\frac{1}{3}$ at $t = 0$. Thus our model seems to be an oscillatory universe with a bounce at time $t = 0$ only to begin the process of evolution with a big-bang.

(v) In another view here also we see that $\omega = \frac{p}{\rho} = 3\rho$. Thus here according to the value of the energy density the nature of dark energy may be determined. We see that during the period when the dark energy is of quintessence type the density of the fluid will range from $0 < \rho < \frac{1}{3}$ and correspondingly the pressure will range between $-\frac{1}{3} < p < 0$.

(vi) Here $\beta^2 \to \infty$ as $t \to 0$ and $\beta^2 \to -4a^2$ as $t \to \infty$. And we know that $(\dot{P}_{ph})^2 < 0$ for the phantom scalar ϕ_{ph} in a dark energy filled universe. Thus at the late stage of our universe a part of the dark energy may behaves as a phantom types this part being generated by Lyra manifold itself which may be taken as a source of dark energy.

(vii) Here,

$$\omega = \frac{p}{\rho} = \frac{-3}{3 + c_0 t^{3b} e^{3at}}$$

at $t = 0, \omega = -1$. Thus at just beginning of the epoch the dark energy content of our universe behaves like the cosmological constant type. Also from this relation we see that $(3 + c_0 t^{3b} e^{3at})$ increases with the time until at $t \to \infty$, the fluid becomes pressure less. Thus for one model the equation of state parameter ω ranges from 0 to -1 which shows the type of dark energy involved (found) in this universe is quintessence.

(viii) Also it is seen the energy density of the Van der Wall fluid does not tend to zero as the scale-factor tend to zero which shows that there is no singularity at the beginning of the evolution.

8.4 Conclusion:

In the attempt to explain some of the unknown phenomenon of nature, including dark energy and dark matter by considering a universe consisting of Van der Wall fluid we come to know that this universe behaves as one containing with dark energy. So there is possibility that the Van der Wall fluid is some source of dark energy. We are planning to study such model universe in the context of different Bianchi space-time in general relativity. Further study about such model universe will be beneficial to the young researchers to study the source of dark energy.