Chapter 6

Viscous Robertson-Walker model with Barotropic equation of state in Brans-Dicke Theory of Gravitation interacting with Electromagnetic field

6.1 Introduction

The present belief about our universe is that the current universe is in accelerating phase which is supported by recent cosmological observations (Riess et al.1998, Perlmutter et al.1998, Perlmutter et al. 1999, Garnavich et al.1998, Seljak et al. 2005, Astier et al. 2006, Bennett et al. 2003, Spergel et al. 2003, Komatsu et al. 2009, Ade et al. 2014, Tegmark et al. 2004, Abazajian et al., 2004, Adelman-McCarthy et al., 2008, Allen et al. 2004). Many relativist (Banerjee and Beesham 1997, Azar and Riazi 1995, Etoh et al. 1997, Singh and Beesham 1999, Banerjee and Pavon 2001, Chakraborty et al., 2003) has used the Brans-Dicke theory to investigate cosmological models. Also, various cosmological models filled with electromagnetic field were discussed in number of papers (Bohra and Mehra 1978, Reddy and Rao 1981, Singh and Usham 1989, Jimanez et al. 2009, Pandolfi 2014, Tripathy et al., 2015). Scale factor and the scalar field relationship has been used in different cosmological models (El-Nabulsi 2010,2011,2013, 2015). Pasqua and Chattopadhyay (2013) have investigated cosmological model by using logamediate form of scale factor. Different authors like (Al-Rawaft and Taha 1996, Al-Rawaft 1998, Overduin and Cooperstock 1998, Arbab 2003, Khadekar et al. 2006) like Al-Rawaft and Taha, Al-Rawaft, Overduin and Cooperstock, Arbab, Khadekar et al. studied about cosmological models with the variable cosmological constant of the form $\Lambda \propto \frac{\ddot{R}}{R}$ and some other form. In this paper, we studied Robertson-Walker model with Barotropic equation of state in Brans-Dicke Theory of Gravitation interacting with Electromagnetic field

6.2 Metric and Field equations

The spherically symmetric Robertson-Walker metric is

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\varphi^2)\right], \tag{6.1}$$

where k is the curvature index which can take values $-1, 0, 1$.

The B-D theory of gravity is described by the action (in units $h = c = 8\pi G = 1$)

$$S = \int d^4x\sqrt{|g|}\left[\frac{1}{16\pi}\left(\phi R - \frac{\omega}{\phi}g^{sl}\phi_{,l}\phi_{,s}\right) + L_m\right], \tag{6.2}$$

where R represents the curvature scalar associated with the 4D metric g_{ij}; g is the determinant of g_{ij}; ϕ is a scalar field; ω is a dimensionless coupling constant; L_m is the Lagrangian of the ordinary matter component.

The Einstein field equations in the most general form are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = -\frac{\kappa}{\phi}T_{ij} - \frac{\omega}{\phi^2}[\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi^{,s}\phi_{,s}] - \frac{1}{\phi}(\phi_{,ij} - g_{ij}\phi^{;s}_{;s}), \tag{6.3}$$

$$(3+2\omega)\phi^{;s}_{;s} = \kappa T, \tag{6.4}$$

where $\kappa = 8\pi$, T is the trace of T_{ij}, Λ is the cosmological constant, R_{ij} is Ricci-tensor, g_{ij} is metric tensor, $\Box\phi = \phi_{;s}^{;s}$, $\Box$ is the Laplace-Beltrami operator and $\phi_{,i}$ is the partial differentiation with respect to x^i coordinate.

The energy-momentum tensor is

$$T_{ij} = M_{ij} + E_{ij}, \tag{6.5}$$

where

$$M_{ij} = (\bar{p} + \rho)u_i u_j - \bar{p} g_{ij}, \tag{6.6}$$

$$\bar{p} = p - \eta u^i_{;i} \tag{6.7}$$

and

$$E_{ij} = -F_{il}F^l_j + \frac{1}{4} g_{ij} F_{lm} F^{lm}, \tag{6.8}$$

with $u_1 = u_2 = u_3 = 0$, $u_4 = 1$, u_i is four velocity vector satisfying $g^{ij}u_i u_j = 1$, p is the proper pressure, ρ is the energy density and η is the coefficient of bulk viscosity. Here a comma $(,)$or semicolon $(;)$followed by a subscript denotes partial differentiation or a covariant differentiation respectively. Also the velocity of light is assumed as unity.

The non-vanishing components of the electromagnetic energy-momentum tensor E^i_j are obtained as

$$E^1_1 = -E^2_2 = -E^3_3 = E^4_4 = -\frac{1}{2} g^{11} g^{44} F^2_{14} = \frac{1}{2} \frac{1 - kr^2}{R^2} F^2_{14}, \tag{6.9}$$

The shear scalar (σ) and the average anisotropy parameter (Δ) are defined as follows

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right), \tag{6.10}$$

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2, \tag{6.11}$$

where $H_i, i = 1, 2, 3$ represents the directional Hubble parameters in x, y, z directions respectively.

Gravitational variable is defined as

$$G = \frac{1}{\phi}\left(\frac{4+2\omega}{3+2\omega}\right), \tag{6.12}$$

The deceleration parameter is defined as

$$q = -\frac{R\ddot{R}}{\dot{R}^2}, \tag{6.13}$$

6.3 Solutions of field equations

Assuming $\phi^{'} = 0$, the metric (6.1) along with field equations (6.3)-(6.5) gives

$$\frac{k}{R^2} + \frac{\dot{R}^2}{R^2} + \frac{2\ddot{R}}{R} - \Lambda = -\frac{8\pi\bar{p}}{\phi} - \frac{4\pi}{\phi}\frac{1-kr^2}{R^2}F_{14}^2 - \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - 2\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} - \frac{\ddot{\phi}}{\phi}, \tag{6.14}$$

$$3\left(\frac{k}{R^2} + \frac{\dot{R}^2}{R^2}\right) - \Lambda = \frac{8\pi\rho}{\phi} + \frac{4\pi}{\phi}\frac{1-kr^2}{R^2}F_{14}^2 + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi}, \tag{6.15}$$

$$(3+2\omega)\left[\frac{3\dot{R}\dot{\phi}}{R} + \ddot{\phi}\right] = 8\pi(\rho - 3\bar{p}), \tag{6.16}$$

From eqs. (6.13), (6.14) and (6.15), we get

$$6\left(\frac{k}{R^2} + \frac{\dot{R}^2}{R^2} + \frac{\ddot{R}}{R}\right) - 4\Lambda = -\frac{8\pi}{\phi}\frac{1-kr^2}{R^2}F_{14}^2 + \omega\left[6\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} + 2\frac{\ddot{\phi}}{\phi} - \left(\frac{\dot{\phi}}{\phi}\right)^2\right], \tag{6.17}$$

Here, we consider relation between scale factor R and scalar field ϕ as

$$\phi = \phi_0 R^{\frac{1}{\omega}}, \tag{6.18}$$

ϕ_0 is a constant.

Using eq. (6.18), (6.17) becomes

$$F_{14}^2 = \frac{\phi_0 R^{\frac{1+2\omega}{\omega}}}{8\pi(1-kr^2)}\left[\left(\frac{1-2\omega}{\omega}\right)\left(\frac{\dot{R}}{R}\right)^2 - 4\frac{\ddot{R}}{R} - \frac{6k}{R^2} + 4\Lambda\right], \tag{6.19}$$

The logamediate form of Scale factor (Pasqua and Chattopadhyay 2013, Barrow and Nunes 2007) is given by

$$R = e^{A(logt)^{\alpha}}, \tag{6.20}$$

where A and α are two constant parameters which satisfy the condition $A\alpha > 0$ and $\alpha > 1$. Barrow and Nunes (2007) found that the observational ranges of the parameters A and α are $1.5 \times 10^{-92} \leq A \leq 2.1 \times 10^{-2}$ and $2 \leq \alpha \leq 50$ with their model.

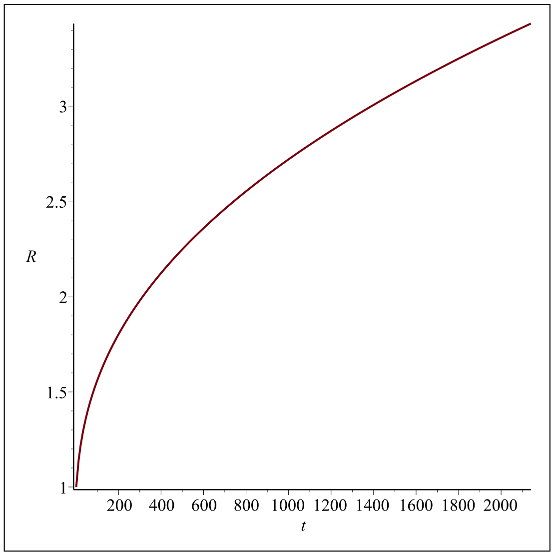

Figure 6.1: Graph of R $vs.$ t according to (6.20)

From eq. (6.20), we get

$$q = -1 + \frac{1}{A\alpha(logt)^{\alpha-1}} + \frac{1-\alpha}{A\alpha(logt)^{\alpha}}, \tag{6.21}$$

Brans-Dicke scalar field is obtained as

$$\phi = \phi_0 e^{\frac{A(logt)^{\alpha}}{\omega}}, \tag{6.22}$$

The Gravitaional variable is

$$G = \left(\frac{4+2\omega}{3+2\omega}\right)\phi_0^{-1}e^{-\frac{A(logt)^{\alpha}}{\omega}} \tag{6.23}$$

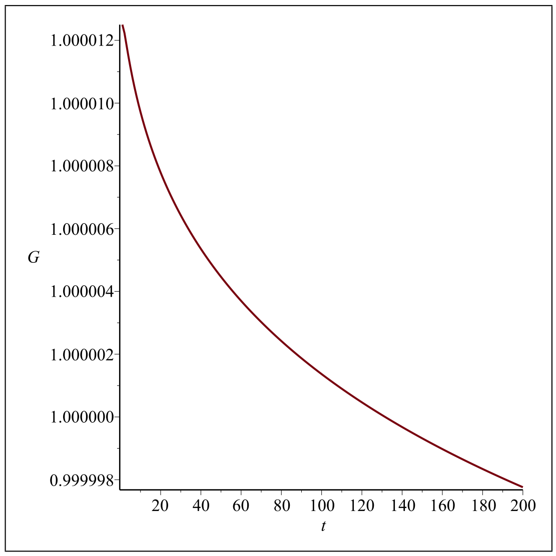

Figure 6.2: Graph of G *vs*. t according to (6.23)

Spatial volume, Hubble's parameter and Scalar expansion are given by

$$V = e^{3A(logt)^{\alpha}}, \tag{6.24}$$

$$H = \frac{A\alpha}{t}(logt)^{\alpha-1}, \tag{6.25}$$

$$\Theta = \frac{3A\alpha}{t}(logt)^{\alpha-1}, \tag{6.26}$$

The directional Hubble's parameter on the x, y, z axes are

$$H_x = H_y = H_z = \frac{A\alpha}{t}(logt)^{\alpha-1}, \tag{6.27}$$

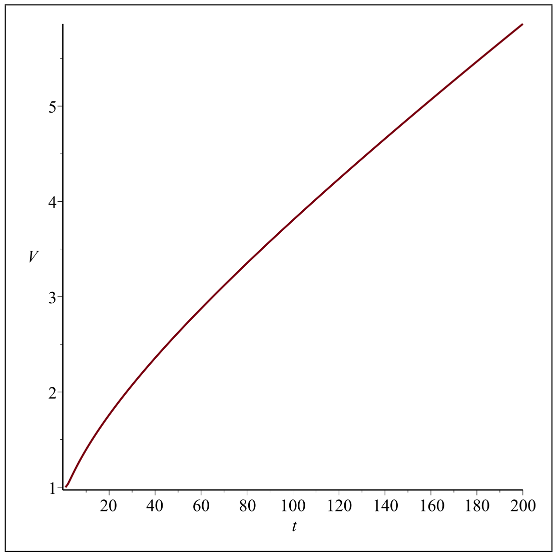

Figure 6.3: Graph of V $vs.$ t according to (6.24)

The anisotropy parameter, Shear scalar and Redshift of the expansion are obtained as

$$\Delta = 0, \tag{6.28}$$

$$\sigma^2 = 0, \tag{6.29}$$

$$z = e^{-A(logt)^{\alpha}} - 1, \tag{6.30}$$

6.3.1 Case A: Flat model $k = 0$, $\Lambda = a\left(\frac{\dot{R}}{R}\right)^2 + b\frac{\ddot{R}}{R}$

Using eq. (6.20) , eq. (6.19) becomes

$$\begin{aligned} F_{14}^2 = \frac{\phi_0 e^{\frac{(1+2\omega)A(logt)^{\alpha}}{\omega}}}{8\pi}[B_1\left\{\frac{A\alpha}{t}(logt)^{\alpha-1}\right\}^2 \\ +B_2\left\{\frac{A\alpha}{t^2}(logt)^{\alpha-1} + \frac{A\alpha(1-\alpha)}{t^2}(logt)^{\alpha-2}\right\}], \end{aligned} \tag{6.31}$$

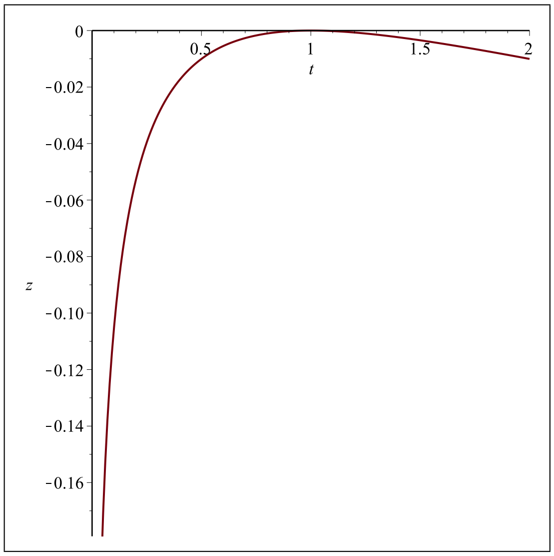

Figure 6.4: Graph of z $vs.$ t according to (6.30)

where $B_1 = \frac{\{4(a+b)-6\}\omega+1}{\omega}, B_2 = 4(1-b)$

Using eqs. (6.31) and (6.20), eqs. (6.14) and (6.15) gives

$$\bar{p} = -\frac{\phi_0 e^{\frac{A(logt)^{\alpha}}{\omega}}}{8\pi}\left[L_1\left\{\frac{A\alpha}{t}(logt)^{\alpha-1}\right\}^2 - L_2\left\{\frac{A\alpha}{t^2}(logt)^{\alpha-1} + \frac{A\alpha(1-\alpha)}{t^2}(logt)^{\alpha-2}\right\}\right], \tag{6.32}$$

$$\rho = \frac{\phi_0 e^{\frac{A(logt)^{\alpha}}{\omega}}}{8\pi}\left[M_1\left\{\frac{A\alpha}{t}(logt)^{\alpha-1}\right\}^2 + M_2\left\{\frac{A\alpha}{t^2}(logt)^{\alpha-1} + \frac{A\alpha(1-\alpha)}{t^2}(logt)^{\alpha-2}\right\}\right]. \tag{6.33}$$

Now, limiting the distribution by considering Barotropic equation of state as

$$p = (\gamma-1)\rho, 0 \leq \gamma \leq 2, \tag{6.34}$$

and using eq. (6.7), we obtain the explicit form of physical quantities p and η as

$$p = \frac{(\gamma-1)\phi_0 e^{\frac{A(logt)^{\alpha}}{\omega}}}{8\pi}[M_1\left\{\frac{A\alpha}{t}(logt)^{\alpha-1}\right\}^2 + M_2\left\{\frac{A\alpha}{t^2}(logt)^{\alpha-1} + \frac{A\alpha(1-\alpha)}{t^2}(logt)^{\alpha-2}\right\}], \tag{6.35}$$

$$\eta = \frac{\phi_0 e^{\frac{A(logt)^{\alpha}}{\omega}}}{24\pi}\left[N_1\left\{\frac{A\alpha}{t}(logt)^{\alpha-1}\right\} + N_2\left\{1 + \frac{(1-\alpha)}{t(logt)}\right\}\right], \tag{6.36}$$

where $N_1 = \frac{\{6(\gamma-1)+(4-3\gamma)(a+b)\}\omega^2+5\omega+1}{\omega^2}$, $N_2 = \frac{\{(3\gamma-4)b+2(1-\gamma)\}\omega-1}{\omega}$

6.3.2 Case B: Open model $k = -1$ and $\Lambda = a\left(\frac{\dot{R}}{R}\right)^2 + b\frac{\ddot{R}}{R}$

Using eq. (6.20) , eq. (6.19) becomes

$$F_{14}^2 = \frac{\phi_0 e^{\frac{(1+2\omega)A(logt)^{\alpha}}{\omega}}}{8\pi(1+r^2)}[B_1\left\{\frac{A\alpha}{t}(logt)^{\alpha-1}\right\}^2 + B_2\left\{\frac{A\alpha}{t^2}(logt)^{\alpha-1} + \frac{A\alpha(1-\alpha)}{t^2}(logt)^{\alpha-2}\right\} + 6e^{-2A(logt)^{\alpha}}], \tag{6.37}$$

where $B_1 = \frac{\{4(a+b)-6\}\omega+1}{\omega}, B_2 = 4(1-b)$

Using eqs. (6.37) and (6.20), eqs. (6.14) and (6.15) gives

$$\bar{p} = -\frac{\phi_0 e^{\frac{A(logt)^{\alpha}}{\omega}}}{8\pi}[L_1\left\{\frac{A\alpha}{t}(logt)^{\alpha-1}\right\}^2 - L_2\left\{\frac{A\alpha}{t^2}(logt)^{\alpha-1} + \frac{A\alpha(1-\alpha)}{t^2}(logt)^{\alpha-2}\right\} + 2e^{-2A(logt)^{\alpha}}], \tag{6.38}$$

$$\rho = \frac{\phi_0 e^{\frac{A(logt)^{\alpha}}{\omega}}}{8\pi}[M_1\left\{\frac{A\alpha}{t}(logt)^{\alpha-1}\right\}^2 + M_2\left\{\frac{A\alpha}{t^2}(logt)^{\alpha-1} + \frac{A\alpha(1-\alpha)}{t^2}(logt)^{\alpha-2}\right\} - 3e^{-2A(logt)^{\alpha}}], \tag{6.39}$$

Now, limiting the distribution by considering Barotropic equation of state as

$$p = (\gamma - 1)\rho, 0 \leq \gamma \leq 2, \tag{6.40}$$

and using eq. (6.7), we obtain the explicit form of physical quantities p and η as

$$p = \frac{(\gamma-1)\phi_0 e^{\frac{A(logt)^{\alpha}}{\omega}}}{8\pi}[M_1\left\{\frac{A\alpha}{t}(logt)^{\alpha-1}\right\}^2 + M_2\left\{\frac{A\alpha}{t^2}(logt)^{\alpha-1} + \frac{A\alpha(1-\alpha)}{t^2}(logt)^{\alpha-2}\right\} - 3e^{-2A(logt)^{\alpha}}] \tag{6.41}$$

$$\eta = \frac{\phi_0 e^{\frac{A(logt)^{\alpha}}{\omega}}}{24\pi}\left[N_1\left\{\frac{A\alpha}{t}(logt)^{\alpha-1}\right\} + N_2\left\{1 + \frac{(1-\alpha)}{t(logt)}\right\} + (5-3\gamma)e^{-2A(logt)^{\alpha}}\right] \tag{6.42}$$

where $L_1 = \frac{(a+b)\omega^2 - 3\omega + 1}{\omega^2}, L_2 = \frac{b\omega+1}{\omega}$, $M_1 = \frac{6\omega - 3(a+b)\omega + 2}{\omega}$, $M_2 = 3b - 2$,
$N_1 = \frac{\{6(\gamma-1)+(4-3\gamma)(a+b)\}\omega^2 + 5\omega + 1}{\omega^2}$, $N_2 = \frac{\{(3\gamma-4)b + 2(1-\gamma)\omega - 1\}}{\omega}$
Lastly we obtain the Λ parameter as

$$\Lambda = (a+b)\left\{\frac{A\alpha}{t}(logt)^{\alpha-1}\right\}^2 + b\left\{\frac{A\alpha}{t^2}(logt)^{\alpha-1} + \frac{A\alpha(1-\alpha)}{t^2}(logt)^{\alpha-2}\right\} \tag{6.43}$$

6.4 Some physical models

For both the Flat and Open model the line element comes out to be

$$ds^2 = dt^2 - e^{2A(logt)^{\alpha}}\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\varphi^2)\right], \tag{6.44}$$

where k is the curvature index which can take values $-1, 0, 1$.
Corresponding to the three extreme cases of equation of state $p = (\gamma - 1)\rho$, we discuss three physical models.

6.4.1 Case I: False vacuum model $\gamma = 0$

We have the false vacuum model when $\gamma = 0$. The cosmological model in this case is given by eq. (6.44) and the physical quantities in this case take the form

$$\begin{aligned}\rho = -p = -\frac{\phi_0 e^{\frac{A(logt)^{\alpha}}{\omega}}}{8\pi}[M_1\left\{\frac{A\alpha}{t}(logt)^{\alpha-1}\right\}^2 \\ + M_2\left\{\frac{A\alpha}{t^2}(logt)^{\alpha-1} + \frac{A\alpha(1-\alpha)}{t^2}(logt)^{\alpha-2}\right\} + 3ke^{-2A(logt)^{\alpha}}],\end{aligned} \tag{6.45}$$

$$\eta = \frac{\phi_0 e^{\frac{A(logt)^{\alpha}}{\omega}}}{24\pi}\left[N_1\left\{\frac{A\alpha}{t}(logt)^{\alpha-1}\right\} + N_2\left\{1 + \frac{(1-\alpha)}{t(logt)}\right\} - 5ke^{-2A(logt)^{\alpha}}\right], \tag{6.46}$$

6.4.2 Case II: Stiff fluid model $\gamma = 2$

For $\gamma = 2$, the distribution reduces to the equation of state $\rho = p$ which is known as Zeldovich fluid or bulk viscous stiff fluid model. The cosmological model in this case is given by eq. (6.44) and the physical quantities in this case take the form

$$\begin{aligned}\rho = p = \frac{\phi_0 e^{\frac{A(logt)^{\alpha}}{\omega}}}{8\pi}[M_1\left\{\frac{A\alpha}{t}(logt)^{\alpha-1}\right\}^2 + M_2\left\{\frac{A\alpha}{t^2}(logt)^{\alpha-1} + \frac{A\alpha(1-\alpha)}{t^2}(logt)^{\alpha-2}\right\} \\ + 3ke^{-2A(logt)^{\alpha}}],\end{aligned} \tag{6.47}$$

$$\eta = \frac{\phi_0 e^{\frac{A(logt)^{\alpha}}{\omega}}}{24\pi}\left[N_1\left\{\frac{A\alpha}{t}(logt)^{\alpha-1}\right\} + N_2\left\{1 + \frac{(1-\alpha)}{t(logt)}\right\} + ke^{-2A(logt)^{\alpha}}\right], \tag{6.48}$$

6.4.3 Case III: Radiation model $\gamma = \frac{4}{3}$

For $\gamma = \frac{4}{3}$, the distribution reduces to the special case with equation of state $\rho = 3p$ which is known as Radiation dominated model. The cosmological model in this case is given by eq.

(6.44) and the physical quantities in this case take the form

$$\rho = \frac{\phi_0 e^{\frac{A(logt)^{\alpha}}{\omega}}}{8\pi}[M_1\left\{\frac{A\alpha}{t}(logt)^{\alpha-1}\right\}^2 + M_2\left\{\frac{A\alpha}{t^2}(logt)^{\alpha-1} + \frac{A\alpha(1-\alpha)}{t^2}(logt)^{\alpha-2}\right\} + 3ke^{-2A(logt)^{\alpha}}], \tag{6.49}$$

$$p = 3\rho = \frac{3\phi_0 e^{\frac{A(logt)^{\alpha}}{\omega}}}{8\pi}[M_1\left\{\frac{A\alpha}{t}(logt)^{\alpha-1}\right\}^2 + M_2\left\{\frac{A\alpha}{t^2}(logt)^{\alpha-1} + \frac{A\alpha(1-\alpha)}{t^2}(logt)^{\alpha-2}\right\} + 3ke^{-2A(logt)^{\alpha}}], \tag{6.50}$$

$$\eta = \frac{\phi_0 e^{\frac{A(logt)^{\alpha}}{\omega}}}{24\pi}\left[N_1\left\{\frac{A\alpha}{t}(logt)^{\alpha-1}\right\} + N_2\left\{1 + \frac{(1-\alpha)}{t(logt)}\right\} - ke^{-2A(logt)^{\alpha}}\right], \tag{6.51}$$

6.5 Conclusion

In this chapter, we have considered the logamediate form of scale factor $R = e^{A(logt)^{\alpha}}$. This scale factor gives spatial volume as the exponential functions of time. This provides the exponential expansion of the universe, so the model universes are accelerating. Hubble's parameter and scalar expansion tend to zero as time tends to infinity for the range of A and α given by Barrow and Nunes (2007). The value of deceleration parameter also lies within the range of observational data as time increases. For $\omega > 40000$ (Calcagni et al. 2012) found that accelerated expansion of the model universe can be achieved. For all the models, the electromagnetic field component F_{14} is an increasing function of time. Here the fluid density is positive which is again functions of time t alone. Here, we find that the scalar field ϕ is also increasing function of t only. The gravitational variable G is decreasing function of t and as $t \to \infty$, $G \to 0$. Also, the coefficient of bulk viscosity exists and the red-shift decreases with time. Also, an isotropic and shear-free model has been found. Here the cosmological constant decreases with time from large value at an initial epoch to small positive value at late time of evolution which in conformity with the experimental evidence.