Abstract

Our investigation in the thesis entitled **"SOME PROBLEMS OF MATTER DISTRIBUTIONS WITH B-D SCALAR FIELD AND OTHER FIELDS IN RELATIVISTIC COSMOLOGY** comprises of nine chapters.

Chapter I is introductory in nature and deals with the basis and development of the relativistic cosmological problems. The motivation and physical relevance of the investigations presented in the later chapters are described in the light of related work of other authors.

Chapter II deals with the interaction of Gravitational field and Brans-Dicke field in R/W universe containing Dark Energy like fluid. In this chapter the universes we have investigated are found to behave in different ways and to show different manifestations under different conditions. Some of them show signs containing cosmological constant form and quintessence form of dark energy, whereas some others seem to contain fluids behaving like phantom and k-essence forms of dark energy, which can explain the present accelerated expansion of the universe. Thus the model universes we obtain in these cases may be taken as realistic models of our universe, and many more unknown properties of the universe and of dark energy may be realized and known from further studies of these models, which we will perform and report elsewhere afterwards. Further, one model of ours seems to undergo a gravitational collapse leading to a black hole; whereas another model surprisingly seems to face the fate of a Big Rip. Another new finding in some of our models is that they contain simultaneously two forms of dark energy, one due to cosmological constant and another due to Brans-Dicke scalar field. And, interestingly enough, one of our models seems to behave like a universe obeying the newly proposed cyclic theory of the universe.

Chapter III deals with isotropic Robertson-Walker model universe with dynamical cosmological parameter Λ in Brans-Dicke Theory of Gravitation. In this chapter, from Case-I we can see that the scale factor (R) and spatial volume (V) increases as time increses which shows expanding universe. Also, Brans-Dicke scalar $\phi \to 0$, gravitational variable $G \to \infty$, scalar expansion $\Theta \to 0$, cosmological constant $\Lambda \to 0$ as $t \to \infty$. For initial period of time i.e. $t = 0$, $R, \phi, G, H, \Theta, V, \Lambda$ become finite. Also, for $\alpha < 0, \rho > 0$ and $p < 0$ which gives positive energy density and negative pressure contributing to the dark energy model with accelerating universe. Here, for $2 < \beta < 3, q \leq 0$, the deceleration parameter is in the range $-1 \leq q \leq 0$ which is in agreement with the observations made by Riess et al. (1998) and Perlmutter et al. (1999) i.e. the expansion of the universe is accelerating. In Case-II also $R \to \infty, \phi \to 0, G \to \infty, V \to \infty, \Lambda \to 0$ as $t \to \infty$ and H, Θ remain finite for $t \to \infty$. Again, $R, \phi, G, H, \Theta, V, \Lambda$ remain finite for initial period of time $(t = 0)$. Also, for $\alpha < 0, \rho > 0$ and $p < 0$ which gives positive energy density and negative pressure contributing to the dark energy model with accelerating universe. Here, as $t \to \infty$, the deceleration parameter is in the range $-1 \leq q \leq 0$ which gives accelerated expansion of the universe. In Case-III $R \to \infty, \phi \to 0, G \to \infty, V \to \infty$ as $t \to \infty$ and H, Θ, Λ become finite for $t \to \infty$. For $t = 0$ we obtain $R, \phi, G, H, \Theta, V, \Lambda$ as finite. Also, for $\alpha < 0, \rho > 0$ and $p < 0$ which gives positive energy density and negative pressure contributing to the dark energy model with accelerating universe. Here, as $t \to \infty$. the deceleration parameter is in the range $-1 \leq q \leq 0$ which supports the observations made by Riess et al. (1998) and Perlmutter et al. (1999) for accelerating universe.

For all the cases, we can observe that our model is isotropic and shear free. The value of the cosmological constant for the model is found to be decreasing with the incease in time and tends to a small and positive value as time tends to infinity, which is supported by the observational data (Garnavich et al. 1998 and Schmidt et al. 1998).

Chapter IV deals with Robertson-Walker model with Van der Waals equation of state and special form of deceleration parameter in Brans-Dicke Theory of Gravitation. In this chapter, we have considered the special form of Deceleration Parameter $q = -1 + \frac{\zeta}{1+R^{\zeta}}$ which yields time-dependent scale factor $R = \left(\alpha_1 e^{\alpha_2 \zeta t} - 1\right)^{\frac{1}{\zeta}}$. Here, scale factor and spatial volume are the exponential functions of time, tends to infinity as $t \to \infty$, so the model universes are accelerating. Hubble's parameter and scalar expansion tends to α_2 and $3\alpha_2$ as time tends to infinity. The models found here are non-singular. At the initial phase of the universe, the value of deceleration parameter is positive while as $t \to \infty$, the value of q becomes -1. Hence, the universe had a decelerated expansion in the past. There is accelerated expansion at present. For $2 < \zeta \leq 4$ and $\omega > 40,000$ (Reasenberg et al. 1979, Faraoni 2004, Calcagni et al. 2012), the electric field component F_{14} increases as time increases in Flat and open model. For $2 < \zeta \leq 4$, $\alpha < 0$ and $\beta < 0$, fluid density is positive but pressure is negative which are again functions of time t alone.Here, the scalar field ϕ is also increasing function of t only. So, the gravitational variable G is decreasing as time increases. For both the universes, the red-shift is seen to decrease as time increases. Also, the model is isotropic and shear-free as anisotropy parameter, as well as the shear scalar, is zero. Here dynamical cosmological constant tends $\frac{\alpha_2^2}{4}$ as $t \to \infty$ which is positive and small. For large values of ω, the dark energy is present in both cases, which is in good harmony with recent cosmological observations (Riess et al.1998, Perlmutter et al.1997, 1998, 1999, Schmidt et al.1998, Garnavich et al. 1998).

Chapter V deals with some spherically symmetric R/W universe interacting with vacuum B-D scalar field. Since, we assume the validity of Hubble's principle for all the solutions of the universe corresponding to $k = \pm 1$. The B-D scalar ϕ does not exist in the flat model of the universe. Here, we have seen that the role played by the scalar ϕ relating to the expansion and contraction of the universe is that the B-D scalar ϕ which is a negative, decreasing function of time may be treated as something reflecting the expansion of the universe while the B-D scalar ϕ which is a positive increasing function of time can be treated as something reflecting the contraction of the universe. For $\omega < -\frac{4}{3}$ but not equal to zero, ϕ is found to be a negative decreasing function of time thereby causing the expansion of the universe till it becomes infinitely large. Corresponding to a case where $\omega > -\frac{4}{3}$, ϕ is found to be a positive increasing function of time thereby causing the universe to contract and the universe becomes concentrated to a point when ϕ becomes positively infinite.

Chapter VI deals with viscous Robertson-Walker model with Barotropic equation of state in Brans-Dicke Theory of Gravitation interacting with Electromagnetic field. Here, we have considered the logamediate form of Scale factor $R = e^{A(logt)^{\alpha}}$ which gives spatial volume as the exponenetial function of time. This gives exponential expantion of the universe, so the model universes are accelerating. Hubble's parameter and scalar expansion tends to zero as time tends to infinity for the range of A and α given by Barrow and Nunes (2007). The value of deceleration parameter also lies in the range of observational data as time increses. We see that for all the cases accelerated expansion can be achieved for a flat and open model of the Universe for large values of ω (Reasenberg et al. 1979, Faraoni 2004, Calcagni et al. 2012). For all the models, the electric field component F_{14} increases as time increases. Here, the fluid density is positive which is again functions of time t alone. Here, we find that the scalar field ϕ is also increasing function of t only. The gravitational variable G is decreasing function of t and as $t \to \infty$, G becomes very small. For all model universe, coefficient of viscosity exists and the red-shift is seen to decrease with time. Also, the models are isotropic and shear-free as anisotropy parameter, as well as the shear scalar, is zero. Here in conformity with the experimental evidences, the cosmological constant decreases with time from large value at an initial epoch to small positive value at late time of evolution.

Chapter VII deals with interaction of electromagnetic field and Brans-Dicke field in Robertson-Walker cosmological model with time dependent deceleration parameter. In this chapter We study a cosmological model with time-dependent deceleration parameter of the form $q = -\frac{R\ddot{R}}{\dot{R}^2} = -1 + m.sech^2(\alpha t)$ which gives scale factor as $R = [sinh(\alpha t)]^{\frac{1}{m}}$. We observe that scale factor and spatial volume increases as time increases. i.e. the universe starts evolving with zero volume at $t = 0$ and expands with cosmic time t. The parameters H, Θ tends to zero for $t \to \infty$. Here the anisotropy parameter and shear scalar are zero, hence the model is isotropic and shear free throughout the evolution of the universe. The model has point-type singularity. For the Flat model, the electric field component F_{14} is a increasing function of t alone and it tends to large value as $t \to \infty$. The pressure and fluid density are functions of time t alone. For $\omega > 40000$ (Reasenberg et al. 1979, Faraoni 2004, Calcagni et al. 2012), fluid density is positive and pressure is negative. Here we find that the scalar field ϕ is a increasing function of t only. The gravitational variable G is drecreasing function of t and as $t \rightarrow \infty$, G becomes negligible. Again, the deceleration parameter $q > 0$ for $t < \alpha^{-1} tanh^{-1}(1-\frac{1}{m})^{\frac{1}{2}}$ and $q < 0$ for $t > \alpha^{-1} tanh^{-1}(1-\frac{1}{m})^{\frac{1}{2}}$ (Maurya et al. 2016) which shows that the model is decelerating at early phase and accelerating in later phase of time. Hence, the dark energy model is consistent with the recent cosmological observations (Riess et al.1998, Perlmutter et al.1997, Perlmutter et al.1998, Perlmutter et al.1999, Schmidt et al.1998, Garnavich et al. 1998). The red-shift is seen to decrease as time passes.

Chapter VIII deals with viscous Robertson-Walker model with Polytropic equation of state and charge in Brans-Dicke Theory of Gravitation. In this chapter, we have considered a scale factor $R = e^{Bt^{\beta}}$ which gives scale factor and spatial volume as the exponenetial functions of time, tends to infinity as $t \rightarrow \infty$, so the model universes are accelerating. Hubble's parameter and scalar expansion tends to zero as time tends to infinity. The value of deceleration parameter lies in the range of observational data as time tends to increses. In the end, we see that for all the cases accelerated expansion can be achieved for a flat and open model of the Universe for large values of ω (Reasenberg et al. 1979, Faraoni 2004, Calcagni et al. 2012). For the Flat model as well as an open model, the electric field component F_{14} increases as time increases. Here the fluid density is positive which is again functions of time t alone. Here, we find that the scalar field ϕ is also increasing function of t only. The gravitational variable G is decreasing function of t and as $t \rightarrow \infty$, G becomes very small. For both the universes, the red-shift is seen to decrease as time increases. Also, the model is isotropic and shear-free as anisotropy parameter, as well as the shear scalar, is zero. For dust-filled universe, there is no distinction between barotropic and polytropic equations of state because for $\alpha = 0$, we have $p = 0$, whatever may be the value of n in the equation of state. Also for the non-dust cases there exists quintessence ($-1 < \alpha < 0$) or vacuum fluid ($\alpha = -1$) or phantom energy $\alpha < -1$. Since the present models do not depend on any particular value of n in the polytropic equation of state which indicates the generality of the present investigation. Here, in conformity with the experimental evidences, the cosmological constant decreases with time from large value at an initial epoch to small positive value at late time of evolution.For all model universe, coefficient of viscosity exists giving us viscous isotropic and shear free models.

Chapter IX deals with Robertson-Walker model universe interacting with Electromagnetic field and Brans-Dicke field in presence of Hybrid scale factor. In this chapter, for the Case-I model the metric comes out to be $ds^2 = dt^2 - t^{2b}e^{2at}\left[dr^2 + r^2(d\theta^2 + sin^2\theta d\varphi^2)\right]$

For the Flat model, the electric field component F_{14} is a decreasing function of t alone and it tends to zero as $t \to \infty$. F_{14} is physically realistic as $\omega > 40000$ (Reasenberg et al. 1979, Faraoni 2004, Calcagni et al. 2012) and $-6 < n < -2$. The pressure and fluid density are functions of time t alone. For $-6 < n < -2$, fluid density is positive and pressure is negative and tends to zero as time tends to infinity. For the Case-II model the metric comes out to be $ds^2 = dt^2 - t^{2b}e^{2at}\left[\frac{dr^2}{1+r^2} + r^2(d\theta^2 + sin^2\theta d\varphi^2)\right]$

For Open model, the electric field component F_{14} is a decreasing function of r as well as t and it tends to zero as $r \to \infty$ or $t \to \infty$. For $-6 < n < -2$ the solutions of F_{14} is physically realistic as the coupling constant $\omega > 40000$ (Reasenberg et al. 1979, Faraoni 2004, Calcagni et al. 2012). In this case, also pressure is negative and fluid density is positive and function of t and r.

For both the cases, the scale factor and spatial volume increases exponentially as $t \to \infty$, so the model universes are accelerating. Hubble's parameter H and scalar expansion Θ both tends to constants a and $3a$ as $t \to \infty$. Here, we find that the scalar field ϕ is a decreasing function of t only. The solution for ϕ remains physically realistic even when $t \to \infty$. The gravitational variable G is increasing function of t and as $t \to \infty$, G becomes infinitely large. Again, the deceleration parameter is in the range $-1 \leq q \leq 0$ as $t \to \infty$ which is in agreement with the observations made by Riess et al. (1998) and Perlmutter et al. (1999) i.e. the expansion of the universe is accelerating. Incidentally, for both the universes the red-shift is seen to decrease with time. Also, anisotropy parameter, as well as the shear scalar, is zero which indicates isotropic and shear-free model. For both the models, the presence of dark energy confirms accelerated expansion of the universes.