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2. Singh K. M., Singh K. P., Dewri M. (2016), Interaction of Gravitational field and Brans-Dicke field in R/W universe containing Dark Energy like fluid
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Robertson–Walker model universe interacting with electromagnetic field and Brans–Dicke field in presence of hybrid scale factor

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ABSTRACT

We discuss the Robertson–Walker model universe with a hybrid scale factor for two cases: Flat and Open model interacting with Brans–Dicke field and electromagnetic field. Some exact solutions are obtained and the different characteristics and phenomena of the dark energy contained in it are discussed.

1. Introduction

Accelerated expansion of the present day universe is described and supported by recent cosmological observations [1–12]. The Brans–Dicke (B-D) theory [13] is one of the simplest scalar-tensor theory used by different authors to study about accelerated expansion of the universe. Using the B-D theory Banerjee and Beesham [14] discussed exponential and power-law solutions for the flat Robertson–Walker cosmological model. Ahmadi-Azar and Riazi [15], Etoh et al. [16], Singh and Beesham [17], Banerjee and Pavon [18], Chakraborty et al. [19] discussed different cosmological models in the context of B-D theory in different scenario. Singh et al. [20–22] studied different problems of interaction of gravitational field and Brans–Dicke field in R/W universe. Singh and Usham, Reddy and Rao, Bohra and Mehra [23–25] are some authors who discussed a field of charged distribution in Brans–Dicke field. Also, the presence of electromagnetic field was discussed in many papers by various authors [26–33]. El-Nabulsi discussed about relation concerning the dependence of the Hubble parameter with the scalar field in his number of papers [34–43]. These works have played a crucial role in this paper for taking power law relation between scale factor and scalar field. There have been lots of work done by authors [44–50] on hybrid scale factor in cosmological models. The work of Bohra and Mehra [25] in charged B-D field are the motivation behind this paper. In this paper, we studied isotropic cosmological model with hybrid scale factor interacting with the electromagnetic field and Brans–Dicke field considering Robertson–Walker metric.

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2. Metric and field equations

The spherically symmetric Robertson–Walker metric is

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right], \tag{1}$$

where k is the curvature index which can take values $-1, 0, 1$.

The Brans–Dicke (B-D) theory of gravity is described by the action (in units $h = c = 8\pi G = 1$)

$$S = \int d^4x\sqrt{|g|}\left[\frac{1}{16\pi}\left(\phi R - \frac{\omega}{\phi}g^{sl}\phi_{,l}\phi_{,s}\right) + L_m\right], \tag{2}$$

where R represents the curvature scalar associated with the 4D metric g_{ij}; g is the determinant of g_{ij}; ϕ is a scalar field; ω is a dimensionless coupling constant; L_m is the Lagrangian of the ordinary matter component. In the absence of the potential V (or the case of a nearly massless field with the potential V) the BD parameter is constrained to be greater than 4.0×10^4 from solar system experiments [54].

The Einstein field equations in the most general form are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = -\frac{\kappa}{\phi}T_{ij} - \frac{\omega}{\phi^2}\left[\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi^{,s}\phi_{,s}\right] - \frac{1}{\phi}(\phi_{,ij} - g_{ij}\phi^{;s}_{;s}), \tag{3}$$

where

$$(3+2\omega)\phi^{;s}_{;s} = \kappa T, \tag{4}$$

where $\kappa = 8\pi$, T is the trace of T_{ij}, Λ is the cosmological constant, R_{ij} is Ricci-tensor, g_{ij} is metric tensor, $\Box\phi = \phi^{;s}_{;s}$, $\Box$ is the Laplace-Beltrami operator and $\phi_{,i}$ is the partial differentiation with respect to x^i coordinate.

The energy-momentum tensor is

$$T_{ij} = M_{ij} + E_{ij}, \tag{5}$$

where

$$M_{ij} = (p+\rho)u_i u_j - pg_{ij}, \tag{6}$$

and

$$E_{ij} = -F_{il}F^l_j + \frac{1}{4}g_{ij}F_{lm}F^{lm}, \tag{7}$$

with $u_1 = u_2 = u_3 = 0$, $u_4 = 1$, u_i is four velocity vector satisfying $g^{ij}u_iu_j = 1$, p is the pressure and ρ is the energy density. A comma (,) or semicolon (;) followed by a subscript denotes partial differentiation or a covariant differentiation respectively. T_{ij} is the energy-momentum tensor for matter and E_{ij} is the electromagnetic energy-momentum tensor. The velocity of light is taken to be unity.

Then, the non-vanishing components of the electromagnetic energy-momentum tensor E^i_j are

$$E^1_1 = -E^2_2 = -E^3_3 = E^4_4 = -\frac{1}{2}g^{11}g^{44}F^2_{14} = \frac{1}{2}\frac{1-kr^2}{R^2}F^2_{14}. \tag{8}$$

Shear scalar is defined as

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3}H_i^2 - 3H^2\right). \tag{9}$$

The average anisotropy parameter Δ is defined as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2, \tag{10}$$

where $H_i, i = 1, 2, 3$ represent the directional Hubble parameters in x,y,z directions respectively and $\Delta = 0$ corresponds to isotropic expansion.

Gravitational variable [51] is defined as

$$G = \frac{1}{\phi}\left(\frac{4+2\omega}{3+2\omega}\right). \tag{11}$$

The deceleration parameter q is defined as

$$q = -\frac{R\ddot{R}}{\dot{R}^2}, \tag{12}$$

where q is the measure of the cosmic acceleration of the universe in cosmology.

3. Solutions of field equations

Now, considering Brans–Dicke scalar field ϕ is a function of t only i.e. $\phi' = 0$, for the metric (1), (3)–(5) gives

$$\frac{k}{R^2} + \frac{\dot{R}^2}{R^2} + \frac{2\ddot{R}}{R} - \Lambda = -\frac{8\pi p}{\phi} - \frac{4\pi}{\phi}\frac{1-kr^2}{R^2}F_{14}^2 - \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - 2\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} - \frac{\ddot{\phi}}{\phi}, \tag{13}$$

$$3\left(\frac{k}{R^2} + \frac{\dot{R}^2}{R^2}\right) - \Lambda = \frac{8\pi\rho}{\phi} + \frac{4\pi}{\phi}\frac{1-kr^2}{R^2}F_{14}^2 + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi}, \tag{14}$$

$$(3+2\omega)\left[\frac{3\dot{R}\dot{\phi}}{R} + \ddot{\phi}\right] = 8\pi(\rho - 3p). \tag{15}$$

From Eqs. (13), (14) and (15) we get

$$6\left(\frac{k}{R^2} + \frac{\dot{R}^2}{R^2} + \frac{\ddot{R}}{R}\right) - 4\Lambda = -\frac{8\pi}{\phi}\frac{1-kr^2}{R^2}F_{14}^2 + \omega\left[6\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} + 2\frac{\ddot{\phi}}{\phi} - \left(\frac{\dot{\phi}}{\phi}\right)^2\right]. \tag{16}$$

The power law relation between scale factor R and scalar field ϕ in the context of Robertson–Walker Brans–Dicke model [52] is given by

$$\phi = MR^n, \tag{17}$$

where $-6 < n < -2$ and M is a constant.

Using (17), (16) becomes

$$F_{14}^2 = \frac{\phi}{8\pi}\frac{R^2}{1-kr^2}\left[\left\{(n^2+4n)\omega - 6\right\}\left(\frac{\dot{R}}{R}\right)^2 + 2(n\omega - 3)\frac{\ddot{R}}{R} - \frac{6k}{R^2} + 4\Lambda\right]. \tag{18}$$

To find the solutions we consider the Hybrid Scale Factor [26] as

$$R = t^b e^{at}, \tag{19}$$

where a and b are positive constants.

3.1. Case I: Flat model $k = 0,\ \Lambda = 0$

From (18) and (19) we get

$$F_{14}^2 = \frac{M[t^b e^{at}]^{n+2}}{8\pi}\left[\left\{(n^2+6n)\omega - 12\right\}\left(\frac{b}{t} + a\right)^2 - 2(n\omega - 3)\frac{b}{t^2}\right]. \tag{20}$$

Using (19), (20), (13) and (14) gives

$$\rho = \frac{M(t^b e^{at})^n}{8\pi}\left[\left\{15 + 3n - 6n\omega - \frac{3}{2}n^2\omega\right\}\left(\frac{b}{t} + a\right)^2 + 2(n\omega - 3)\frac{b}{t^2}\right], \tag{21}$$

$$p = -\frac{M(t^b e^{at})^n}{8\pi}\left[\left\{n^2\left(\frac{2+3\omega}{2}\right) + (2+6\omega)n - 9\right\}\left(\frac{b}{t} + a\right)^2 + (8 + n - 2n\omega)\frac{b}{t^2}\right], \tag{22}$$

where $-6 < n < -2$

3.2. Case II: Open model $k = -1$ and $\Lambda = 0$

From (18) and (19) we get

$$F_{14}^2 = \frac{M[t^b e^{at}]^{n+2}}{8\pi(1+r^2)}\left[\left\{(n^2+6n)\omega - 12\right\}\left(\frac{b}{t} + a\right)^2 - 2(n\omega - 3)\frac{b}{t^2} + 6(t^b e^{at})^{-2}\right]. \tag{23}$$

Using (19), (20), (13) and (14) gives

$$\rho = \frac{M(t^b e^{at})^n}{8\pi}\left[\left\{15 + 3n - 6n\omega - \frac{3}{2}n^2\omega\right\}\left(\frac{b}{t} + a\right)^2 + 2(n\omega - 3)\frac{b}{t^2} - 3(t^b e^{at})^{-2}\right], \tag{24}$$

$$p = -\frac{M(t^b e^{at})^n}{8\pi}\left[\left\{n^2\left(\frac{2+3\omega}{2}\right) + (2+6\omega)n - 9\right\}\left(\frac{b}{t} + a\right)^2 + (8 + n - 2n\omega)\frac{b}{t^2} - (t^b e^{at})^{-2}\right], \tag{25}$$

where $-6 < n < -2$

3.3. For both the models i.e. Flat and Open

Brans–Dicke scalar field is

$$\phi = M[t^b e^{at}]^n. \tag{26}$$

Hubble's parameter and Scalar expansion are given by

$$H = \frac{b}{t} + a, \tag{27}$$

$$\Theta = 3\left(\frac{b}{t} + a\right). \tag{28}$$

The Gravitational variable is

$$G = \left(\frac{4+2\omega}{3+2\omega}\right) M^{-1}[t^b e^{at}]^{-n}. \tag{29}$$

Deceleration parameter is given by

$$q = -1 + \frac{b}{(at+b)^2}. \tag{30}$$

Spatial volume is given by

$$V = t^{3b} e^{3at}. \tag{31}$$

The directional Hubble's parameter on the x, y, z axes are

$$H_x = H_y = H_z = \frac{b}{t} + a. \tag{32}$$

The anisotropy parameter of the expansion is

$$\Delta = 0. \tag{33}$$

Shear scalar is

$$\sigma^2 = 0. \tag{34}$$

Red-shift is

$$z = t^{-b} e^{-at} - 1. \tag{35}$$

4. Discussion

For the Case I model, the metric comes out to be

$$ds^2 = dt^2 - t^{2b} e^{2at}\left[dr^2 + r^2(d\theta^2 + sin^2\theta d\varphi^2)\right]. \tag{36}$$

For the Flat model, the electric field component F_{14} is a decreasing function of t alone and it tends to zero as $t \to \infty$. F_{14} is physically realistic as $\omega > 40,000$ [53–55] and $-6 < n < -2$. The pressure and fluid density are functions of time t alone. For $-6 < n < -2$ fluid density is positive and pressure is negative and tends to zero as time tends to infinity.

For Open model, the electric field component F_{14} is a decreasing function of r as well as t and it tends to zero as $r \to \infty$ or $t \to \infty$. For $-6 < n < -2$ the solutions of F_{14} is physically realistic as the coupling constant $\omega > 40,000$ [53–55]. In this case, also pressure is negative and fluid density is positive and function of t and r.

For both the cases, the scale factor and spatial volume increases exponentially as $t \to \infty$, so the model universes are accelerating. Hubble's parameter H and scalar expansion Θ both tends to constants a and $3a$ as $t \to \infty$. From Eq. (26), we find that the scalar field ϕ is a decreasing function of t only. The solution for ϕ remains physically realistic even when $t \to \infty$. The gravitational variable G is increasing function of t and as $t \to \infty$, G becomes infinitely large. Again, the deceleration parameter is in the range $-1 \le q \le 0$ as $t \to \infty$ which is in agreement with the observations made by Riess et al. [1] and Perlmutter et al. [2] i.e. the expansion of the universe is accelerating. Incidentally, for both the universes the red-shift is seen to decrease with time. Also, anisotropy parameter, as well as the shear scalar, is zero which indicates isotropic and shear-free model. For both the models, the presence of dark energy confirms accelerated expansion of the universes.

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Research in Astronomy and Astrophysics

Interaction of Gravitational field and Brans-Dicke field in R/W universe containing Dark Energy like fluid

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Abstract On studying some new models of Robertson-Walker universes with a Brans-Dicke scalar field, it is found that most of these universes contain a dark energy like fluid which confirms the present scenario of the expansion of the universe. In one of the cases, the exact solution of the field equations gives a universe with a false vacuum, while in another it reduces to that of dust distribution in the Brans-Dicke cosmology when the cosmological constant is not in the picture. In one particular model it is found that the universe may undergo a Big Rip in the future, and thus it will be very interesting to investigate such models further.

Key words: Brans-Dicke scalar field — cosmological constant — dark energy — quintessence — k-essence — big rip

1 INTRODUCTION

Brans and Dicke (B-D) formulated a theory of gravitation (Brans & Dicke 1961), in which besides a gravitational part, a dynamical scalar field is introduced to incorporate a variable gravitational constant and Mach's principle in Einstein's theory. It can be considered as a natural extension of Einstein's general theory of relativity. The simplest case of the scalar tensor theory (Brans 1997) is defined by a constant coupling parameter ω and a scalar field ϕ. In B-D theory, the gravitational constant becomes time-dependent, varying as the inverse of a time-dependent scalar field which couples to gravity with a coupling parameter ω. One important property of this theory is that it gives expanding solutions (Mathiazhagan & Johri 1984; La et al. 1989) for scalar field $\phi(t)$ and scale factor $R(t)$ which are compatible with solar system observations (Perlmutter et al. 1999; Riess et al. 1998; Garnavich et al. 1998). The solar system observations (Bertotti et al. 2003) also impose lower bounds on ω. General relativity is recovered when ω goes to infinity (Weinberg 1972) and from timing experiments using the Viking space probe (Reasenberg et al. 1979), ω must exceed 500. This constraint ruled out many of the extended inflation theories (Weinberg 1989a; La & Steinhardt 1989) and provides a succession of improved models on extended inflation (Holman et al. 1990, 1991; Barrow & Maeda 1990; Steinhardt & Accetta 1990). Furthermore, all important features of the evolution of the universe such as: inflation (Mathiazhagan & Johri 1984), early and late time behavior of the universe (Shogin & Hervik 2014), cosmic acceleration and structure formation (Banerjee & Pavón 2001), quintessence and the coincidence problem (Sen & Seshadri 2003), self-interacting potential and cosmic acceleration (Errahmani & Ouali 2006), and a high energy description of dark energy in an approximate B-D context (Weinberg 1989b) could be explained successfully in the B-D formalism. For a large value of the $\omega-$ parameter, B-D theory gives the correct amount of inflation and early and late time behaviors, while small and negative values explain cosmic acceleration, structure formation and the coincidence problem. Dark energy, identified as being responsible for cosmic acceleration, determines the features related to future evolution of the universe. The nature of this kind of energy may lead to an improvement in our picture of particle physics and gravitation. Investigations into the nature of dark energy have lead to various candidates. Among them, the most popular ones are the cosmological constant Λ (Padmanabhan 2003; Mak et al. 2002; Caldwell et al. 2003a), a dynamical scalar field like quintessence (Bertolami & Martins 2000; Caldwell & Linder 2005; Caldwell 2002; Tsujikawa & Sami 2004; Caldwell et al. 2003b) or a similar phantom field (Cline et al. 2004; Nesseris & Perivolaropoulos 2007; Huang et al. 2007; Bento et al. 1991).

Astronomical observations indicate that the observable universe is undergoing a phase of accelerated expansion. The present day accelerated expansion of the universe is naturally addressed within the B-D theory with evolution described by the inverse of the Hubble scale and power law temporal behavior of a scale factor. The B-D scalar-tensor theory of gravitation is quite important in view of the fact that scalar fields play a vital role in inflationary cosmology. Many cosmological problems (Kolitch & Eardley 1995; Barrow et al. 1993; Bento et al. 1992; Sahoo & Singh 2002, 2003; Lukács 1976) can be successfully explained by using the B-D theory and its extended versions. The solutions of B-D field equations with the Robertson-Walker line element have been obtained by Luke & Szamosi (1972) using a self consistent numerical method. They derived a lower bound on $\frac{\dot{G}}{G}$ by taking $P = 0$ in the field equations of B-D (Dicke 1964) and concluded the presently available data cannot discriminate between different theories. Morganstern (1971) obtained a similar conclusion on the basis of the observed values of matter density, Hubble's constant, the deceleration parameter and the ages of different objects in the universe.

Since many forms of dark energy are always accompanied and interrelated with a scalar field, we are motivated to see whether the B-D scalar field can manifest some form of dark energy and what roles it can play in causing the accelerated expansion of the universe. We are also motivated to investigate different interesting forms of model universes containing a B-D field interacting with a gravitational field, and especially their interrelation with dark energy in the evolution of our universe. From our study, we find evidence for the existence of dark energy, in one form or another, in almost all model universes obtained by us under different conditions, during the periods of their evolution, which verifies the present accelerated expansion of the universe. One peculiarity of some of the models we obtain is the existence of two forms of dark energy simultaneously in such models, one from the cosmological constant and the other due to the B-D scalar field. In one case there is the possibility of our model universe collapsing and becoming a black hole. Interestingly enough, in yet another case, one of our models is facing the fate of a Big Rip, and one of the model universes we obtain seems to behave like a cyclic model of the universe.

2 SOLUTIONS OF FIELD EQUATIONS

Here, we consider the spherically symmetric Robertson-Walker metric

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right], \tag{1}$$

where k is the curvature index which can take values $-1, 0, 1$.

The B-D theory of gravity is described by the action

$$S = \int d^4x\sqrt{|g|}\left[\frac{1}{16\pi}\left(\phi R - \frac{\omega}{\phi}g^{sl}\phi_{,l}\phi_{,s}\right) + L_m\right], \tag{2}$$

where R represents the curvature scalar associated with the 4D metric g_{ij}; g is the determinant of g_{ij}; ϕ is a scalar field; ω is a dimensionless coupling constant; L_m is the Lagrangian of the ordinary matter component.

The Einstein field equations in their most general form are given by

$$G_{ij} \equiv R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = -\frac{\kappa}{\phi}T_{ij} - \frac{\omega}{\phi^2}\left[\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi^{,s}\phi_{,s}\right] - \frac{1}{\phi}(\phi_{,ij} - g_{ij}\phi^{;s}_{;s}), \tag{3}$$

$$(3+2\omega)\phi^{;s}_{;s} = \kappa T, \tag{4}$$

where $\kappa = 8\pi$, Λ is the cosmological constant, R_{ij} is the Ricci-tensor, g_{ij} is a metric tensor, $\Box\phi = \phi^{;s}_{;s}$, $\Box$ is the Laplace-Beltrami operator and $\phi_{,i}$ is partial differentiation with respect to the x^i coordinate.

The energy-momentum tensor for the perfect fluid distribution is

$$T_{ij} = (P+\rho)U_iU_j - Pg_{ij}, \tag{5}$$

with U_i being a four velocity vector, P the proper pressure and ρ the proper rest mass density. Considering a comoving system, we get $U_1 = U_2 = U_3 = 0; U_4 = 1$ and $g^{ij}U_iU_j = 1$.

A comma $(,)$ or semicolon $(;)$ followed by a subscript denotes partial differentiation or a covariant differentiation respectively. The velocity of light is taken to be unity.

Now using the metric (1), the surviving field equations are

$$\begin{aligned} G_{11} &\equiv \frac{k}{R^2} + \frac{\dot{R}^2}{R^2} + \frac{2\ddot{R}}{R} - \Lambda \\ &= -\frac{\kappa P}{\phi} - \frac{\omega}{2\phi^2}\left[\frac{(1-kr^2)}{R^2}\phi'^2 + \dot{\phi}^2\right] - \frac{1}{\phi}\left[-\frac{2(1-kr^2)}{R^2 r}\phi' + \frac{2\dot{R}\dot{\phi}}{R} + \ddot{\phi}\right], \end{aligned} \tag{6}$$

$$G_{22} \equiv \frac{k}{R^2} + \frac{\dot{R}^2}{R^2} + \frac{2\ddot{R}}{R} - \Lambda$$
$$= -\frac{\kappa P}{\phi} - \frac{\omega}{2\phi^2}\left[-\frac{(1-kr^2)}{R^2}\phi'^2 + \dot{\phi}^2\right] - \frac{1}{\phi}\left[-\frac{(1-kr^2)}{R^2}\phi'' + \frac{(2kr^2-1)}{R^2 r}\phi' + \frac{2\dot{R}\dot{\phi}}{R} + \ddot{\phi}\right], \tag{7}$$

$$G_{33} = G_{22}, \tag{8}$$

$$G_{44} \equiv 3\left(\frac{k}{R^2} + \frac{\dot{R}^2}{R^2}\right) - \Lambda$$
$$= \frac{\kappa\rho}{\phi} + \frac{\omega}{2\phi^2}\left[\dot{\phi}^2 + \frac{(1-kr^2)}{R^2}\phi'^2\right] + \frac{1}{\phi}\left[\frac{(1-kr^2)}{R^2}\phi'' - \frac{(3kr^2-2)}{R^2 r}\phi' - \frac{3\dot{R}\dot{\phi}}{R}\right], \tag{9}$$

$$G_{14} \equiv \frac{\omega}{\phi^2}\phi'\dot{\phi} + \frac{\dot{\phi}'}{\phi} - \frac{\dot{R}\phi'}{R\phi} = 0. \tag{10}$$

From Equation (4), we get

$$(3+2\omega)\left[-\frac{(1-kr^2)}{R^2}\phi'' + \frac{(3kr^2-2)}{R^2 r}\phi' + \frac{3\dot{R}\dot{\phi}}{R} + \ddot{\phi}\right] = \kappa(\rho - 3P), \tag{11}$$

where a dot and dash denote differentiation with respect to time t and r respectively.

Subtracting Equation (6) from Equation (7), we get

$$0 = \frac{\phi'}{\phi}\left[\frac{1}{r} + \frac{kr}{1-kr^2} - \frac{\phi''}{\phi'} - \omega\frac{\phi'}{\phi}\right]. \tag{12}$$

From Equation (12), we get

$$\frac{\phi''}{\phi'} + \omega\frac{\phi'}{\phi} = \frac{1}{r} + \frac{kr}{1-kr^2}. \tag{13}$$

Integrating Equation (13), we get

$$\frac{1}{\omega+1}\phi^{\omega+1} = -\frac{A\sqrt{1-kr^2}}{k} + B, \tag{14}$$

where A and B are functions of time.

Integrating Equation (10), we get

$$\frac{1}{\omega+1}\phi^{\omega+1} = R(t)g(r) + Q(t). \tag{15}$$

From Equation (12), we get

$$\frac{\phi'}{\phi}\frac{d}{dr}\left[I_n\phi'\phi^{\omega}r^{-1}(1-kr^2)^{\frac{1}{2}}\right] = 0. \tag{16}$$

Using Equation (15) in Equation (16), we get

$$\frac{\phi'}{\phi}\frac{d}{dr}\left[I_n r^{-1}(1-kr^2)^{\frac{1}{2}} + I_n g'(r)\right] = 0, \tag{17}$$

from which it is obvious that ϕ is a function of r only, i.e. $Q(t) = 0$ in Equation (15) gives

$$\frac{1}{\omega+1}\phi^{\omega+1} = R(t)g(r). \tag{18}$$

Comparing Equations (14) and (15), we get $Q(t) = B = 0$. From Equation (14), we get

$$\frac{1}{\omega+1}\phi^{\omega+1} = -\frac{A\sqrt{1-kr^2}}{k}. \tag{19}$$

From Equations (17) and (18), we get

$$\frac{\dot{R}}{R} = \frac{\dot{A}}{A}. \tag{20}$$

Integrating, we get

$$R = NA, \tag{21}$$

where N is a constant of integration.

Using Equations (19) and (20) in Equations (6), (7), (9) and (11), we get

$$\frac{\kappa P}{\phi} = -\frac{k}{R^2} - \frac{2k}{R^2(\omega+1)} - \frac{\omega+3}{\omega+1}\frac{\dot{R}^2}{R^2} - \frac{2\omega+3}{\omega+1}\frac{\ddot{R}}{R} + \Lambda - \frac{\omega}{2}\left[\frac{k^2r^2}{R^2(\omega+1)^2(1-kr^2)} - \frac{\dot{R}^2}{(\omega+1)^2R^2}\right], \tag{22}$$

$$\frac{\kappa\rho}{\phi} = \frac{3k}{R^2} + \frac{3k}{R^2(\omega+1)} + \frac{3\omega+6}{\omega+1}\frac{\dot{R}^2}{R^2} - \Lambda - \frac{\omega}{2}\left[\frac{\dot{R}^2}{(\omega+1)^2R^2} - \frac{k^2r^2}{R^2(\omega+1)^2(1-kr^2)^2}\right], \tag{23}$$

and

$$\frac{\kappa}{\phi}(\rho-3P) = (3+2\omega)\left[\frac{3k}{R^2(\omega+1)} + \frac{\omega k^2r^2}{R^2(\omega+1)^2(1-kr^2)} + \frac{3\dot{R}^2}{(\omega+1)R^2} + \frac{\ddot{R}}{(\omega+1)R} - \frac{\omega}{(\omega+1)^2}\frac{\dot{R}^2}{R^2}\right]. \tag{24}$$

From Equations (22) and (23), we get

$$\begin{aligned}\frac{\kappa}{\phi}(\rho-3P) = {} & \frac{6k}{R^2} + \frac{\rho k}{R^2(\omega+1)} + \frac{6\omega+15}{\omega+1}\frac{\dot{R}^2}{R^2} + \frac{6\omega+9}{\omega+1}\frac{\ddot{R}}{R} \\ & -4\Lambda - \frac{\omega}{2}\left[\frac{4\dot{R}^2}{(\omega+1)^2R^2} - \frac{4k^2r^2}{R^2(\omega+1)^2(1-kr^2)}\right].\end{aligned} \tag{25}$$

From Equations (24) and (25), we get

$$\frac{\rho k}{R^2(\omega+1)} + \frac{6\dot{R}^2}{R^2(\omega+1)} + \frac{4\omega+6}{\omega+1}\frac{\ddot{R}}{R} + \frac{(2\omega+1)\omega}{(\omega+1)^2}\frac{\dot{R}^2}{R^2} - \frac{(2\omega+1)k^2r^2\omega}{R^2(\omega+1)^2(1-kr^2)} - 4\Lambda = 0. \tag{26}$$

2.1 Case I: When $\omega = 0$

In this case, Equations (22), (23) and (26) reduce to

$$\frac{\kappa P}{\phi} = -\frac{3k}{R^2} - \frac{3\dot{R}^2}{R^2} - \frac{3\ddot{R}}{R} + \Lambda, \tag{27}$$

$$\frac{\kappa\rho}{\phi} = \frac{6k}{R^2} + \frac{6\dot{R}^2}{R^2} - \Lambda, \tag{28}$$

$$\frac{6k}{R^2} + \frac{6\dot{R}^2}{R^2} + \frac{6\ddot{R}}{R} - 4\Lambda = 0. \tag{29}$$

Integrating Equation (29), we get

$$R = \sqrt{\frac{3}{\Lambda}}\cosh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}, \text{ when } \quad k=1, \tag{30}$$

$$R = \sqrt{\frac{3}{\Lambda}}\sinh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}, \text{ when } \quad k=-1, \tag{31}$$

$$R = e^{\sqrt{\frac{\Lambda}{3}}(t+D)}, \text{ when } \quad k=0, \tag{32}$$

where D is an arbitrary constant of integration.

Case I(a): When $k = 1$, we get

$$R = \sqrt{\frac{3}{\Lambda}}\cosh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}. \tag{33}$$

From Equation (21), we get

$$A = \frac{1}{N}\sqrt{\frac{3}{\Lambda}}\cosh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}. \tag{34}$$

From Equation (19), we get

$$\phi = -\frac{\sqrt{1-r^2}}{N}\sqrt{\frac{3}{\Lambda}}\cosh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}. \tag{35}$$

The gravitational variable is given by

$$G = -\sqrt{\frac{\Lambda}{3}}\frac{4N}{3\sqrt{1-r^2}}\frac{1}{\cosh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}}. \tag{36}$$

From Equations (27) and (28), we get

$$P = -\frac{\sqrt{3\Lambda(1-r^2)}\cosh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}}{\kappa N}, \tag{37}$$

and

$$\rho = -\frac{\sqrt{3\Lambda(1-r^2)}\cosh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}}{\kappa N}. \tag{38}$$

Hubble's parameter is given by

$$H = \sqrt{\frac{\Lambda}{3}}\tanh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}. \tag{39}$$

Scalar expansion is given by

$$\Theta = \sqrt{3\Lambda}\tanh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}. \tag{40}$$

In this model universe, it is seen that the gravitational variable G has a tendency to increase the pressure and decrease the density of the fluid whereas the B-D scalar field has a tendency to decrease the pressure and increase the density of this universe. This model has a singularity at $r = 1$.

Case I(b): When $k = -1$, we get

$$R = \sqrt{\frac{3}{\Lambda}}\sinh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}. \tag{41}$$

From Equation (21), we get

$$A = \frac{1}{N}\sqrt{\frac{3}{\Lambda}}\sinh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}. \tag{42}$$

From Equation (19), we get

$$\phi = \frac{\sqrt{1+r^2}}{N}\sqrt{\frac{3}{\Lambda}}\sinh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}, \tag{43}$$

which is a function of both r and t. When $t \to \infty$, both R and A tend to ∞, and when $r \to \infty$ and $t \to \infty$, the B-D scalar ϕ tends to ∞. Therefore, we conclude that for $k = -1$ the B-D scalar ϕ is an increasing function of both r and t.

The gravitational variable is given by

$$G = \sqrt{\frac{\Lambda}{3}}\frac{4N}{3\sqrt{1+r^2}}\frac{1}{\sinh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}}, \tag{44}$$

which shows that gravitational variable G decreases as r and t increase and tends to zero when either $r \to \infty$ or $t \to \infty$.

From Equations (27) and (28), we get

$$P = -\frac{\sqrt{3A(1+r^2)}}{\kappa N}\sinh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}, \tag{45}$$

and

$$\rho = \frac{\sqrt{3\Lambda(1+r^2)}}{\kappa N}\sinh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}. \tag{46}$$

Hubble's parameter is given by

$$H=\sqrt{\frac{\Lambda}{3}}\coth\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}. \tag{47}$$

Scalar expansion is given by

$$\Theta=\sqrt{3\Lambda}\coth\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}. \tag{48}$$

For this model universe, the scalar field helps in the expansion of the universe. Also, the expansion factor R increases with time thus accurately describing the expansion of the universe. Here in this type of model universe it is seen that pressure is negative and the equation of state $\omega_1=\frac{P}{\rho}=-1$. Thus this universe seems to be a universe containing dark energy due to cosmological constant Λ. Again, here the scalar field ϕ also contributes to the expansion of this universe. Thus some part of the dark energy contained may be interpreted as quintessence in the form of dark energy which is in agreement with present observations, using equation of state $\omega_1\simeq-1$.

Case I(c): When $k=0$, we get

$$R=e^{\sqrt{\frac{\Lambda}{3}}(t+D)}, \tag{49}$$

and

$$A=\frac{1}{N}e^{\sqrt{\frac{\Lambda}{3}}(t+D)}. \tag{50}$$

From Equation (13), we get

$$\phi=\frac{1}{2N}r^2e^{\sqrt{\frac{\Lambda}{3}}(t+D)}, \tag{51}$$

which is a function of both r and t. When $t\to\infty$, $R\to\infty$, and either $r\to\infty$ or $t\to\infty$, the B-D scalar ϕ tends to infinity.

The gravitational variable is given by

$$G=\frac{8N}{3r^2e^{\sqrt{\frac{\Lambda}{3}}(t+D)}}, \tag{52}$$

which shows that gravitational variable G decreases as r and t increase and tends to zero as $r\to\infty$ or $t\to\infty$. From Equations (27) and (28), we get

$$P=-\frac{\Lambda r^2}{2\kappa N}e^{\sqrt{\frac{\Lambda}{3}}(t+D)}, \tag{53}$$

and

$$\rho=\frac{\Lambda r^2}{2\kappa N}e^{\sqrt{\frac{\Lambda}{3}}(t+D)}. \tag{54}$$

Hubble's parameter is given by

$$H=\sqrt{\frac{\Lambda}{3}}. \tag{55}$$

Scalar expansion is given by

$$\Theta=\sqrt{3\Lambda}. \tag{56}$$

Again for the solution in this case, it is found that the B-D scalar field ϕ is singular at the origin. However, on the other hand, at the origin, the gravitational force is very strong. As time t increases, the pressure decreases whereas the density increases. Thus there is the possibility that the model universe in this case contracts gradually and at some stage the density will be very high, thereby making it possible for the universe to become a black hole in the course of time. Or, in a different situation, the equation of state is $\omega_1=\frac{P}{\rho}=-1$ whereas the pressure is negative. This implies that our model universe is expanding and contains dark energy due to the cosmological constant which is in agreement with present observational data, namely, $\frac{p}{\rho}\simeq-1$.

2.2 Case II: When $\omega=0$ and $\Lambda=0$

From Equation (29), we get

$$\frac{6k}{R^2}+\frac{6\dot{R}^2}{R^2}+\frac{6\ddot{R}}{R}=0. \tag{57}$$

Integrating, we get

$$R=\sqrt{-kt^2+2at+2b}, \tag{58}$$

where a and b are constants of integration. From Equation (21), we get

$$A=\frac{1}{N}\sqrt{-kt^2+2at+2b}. \tag{59}$$

Case II(a): When $k=1$.

From Equations (58) and (59), we get

$$R=\sqrt{-t^2+2at+2b}, \tag{60}$$

and

$$A=\frac{1}{N}\sqrt{-t^2+2at+2b}. \tag{61}$$

From Equation (19), we get

$$\phi=-\frac{1}{N}\sqrt{-t^2+2at+2b}\sqrt{1-r^2}, \tag{62}$$

which is a function of both r and t. The gravitational variable is given by

$$G=-\frac{4N}{3\sqrt{-t^2+2at+2b}\sqrt{1-r^2}}, \tag{63}$$

when $N<0$. From Equations (60), (61), (62) and (63), we see that the reality condition for R, A, ϕ and k is $(a^2+2b)>(t-a)^2$ and $r^2<1$.

From Equations (27) and (28), we get

$$P=0, \tag{64}$$

and

$$\rho=-\frac{6(a^2+2b)\sqrt{(1-r^2)}}{\kappa N(-t^2+2at+2b)^{\frac{3}{2}}}, \tag{65}$$

which is a function of both r and t. The reality condition is the same as above. Hubble's parameter is given by

$$H = \frac{t-a}{t^2-2at-2b}. \tag{66}$$

Scalar expansion is given by

$$\Theta = \frac{3(t-a)}{t^2-2at-2b}. \tag{67}$$

For this model universe, it is seen that at time t given by $t^2-2at-2b=0$ there may be a gravitational collapse. Since, in this case, the energy density is negative, there is the possibility that this universe contains a phantom form of dark energy. But there is doubt in this case as here the pressure is zero and this universe is closed, since dark energy is assumed to help in the accelerated expansion of the universe. Thus, when $k=1$, $\omega=0$ and $\Lambda=0$, the problem reduces to the case of dust distribution.

Case II(b): When $k=-1$.

From Equations (58) and (59), we get

$$R = \sqrt{t^2+2at+2b}, \tag{68}$$

and

$$A = \frac{1}{N}\sqrt{t^2+2at+2b}. \tag{69}$$

From Equation (19), we get

$$\phi = \frac{1}{N}\sqrt{t^2+2at+2b}\sqrt{1-r^2}, \tag{70}$$

which is a function of both r and t. When $t\to\infty$, the radius of the universe R tends to infinity, and the B-D scalar ϕ tends to infinity either when $r\to\infty$ or $t\to\infty$. The gravitational variable is given by

$$G = \frac{4N}{3\sqrt{t^2+2at+2b}\sqrt{1-r^2}}. \tag{71}$$

From Equation (71), we see that the gravitational variable G decreases when t and r increase and tends to zero when $r\to\infty$ or $t\to\infty$. From Equations (27) and (28), we get

$$P = 0, \tag{72}$$

and

$$\rho = \frac{6(a^2-2b)\sqrt{(1+r^2)}}{\kappa N(t^2+2at+2b)^{\frac{3}{2}}}, \tag{73}$$

which is real where $a^2-2b>0$. Hubble's parameter is given by

$$H = \frac{t+a}{t^2+2at+2b}. \tag{74}$$

Scalar expansion is given by

$$\Theta = \frac{3(t+a)}{t^2+2at+2b}. \tag{75}$$

In the solution for this case, it is obtained that as time t increases, the radius of our (model) universe increases, that is our universe is expanding which is the sign of being a realistic model. But here it is seen that this universe expands initially at a high rate and gradually the expansion slows down until it stops at infinitely large time when preparing for contraction. In this model universe, the B-D field influences the area given by $r=1$, and is inversely proportional to the gravitational potential due to G. Thus, when $k=-1$, $\omega=0$ and $\Lambda=0$, the problem reduces to the case of dust distribution.

Case II(c): When $k=0$.

From Equations (58) and (59), we get

$$R = \sqrt{2at+2b}, \tag{76}$$

and

$$A = \frac{1}{N}\sqrt{2at+2b}. \tag{77}$$

From Equation (76), we know that radius of the universe R tends to infinity when t tends to infinity. From Equation (13), we get

$$\phi = \frac{r^2\sqrt{2at+2b}}{2N}, \tag{78}$$

which is a function of both r and t. When either $r\to\infty$ or $t\to\infty$, the B-D scalar ϕ tends to infinity. The gravitational variable is given by

$$G = \frac{8N}{3r^2\sqrt{2at+2b}}, \tag{79}$$

which shows that the gravitational variable G decreases when r and t increase and tends to zero when either $r\to\infty$ or $t\to\infty$.

From Equations (27) and (28), we get

$$P = 0, \tag{80}$$

and

$$\rho = \frac{3a^2r^2}{\kappa N(2at+2b)^{\frac{3}{2}}}. \tag{81}$$

Hubble's parameter is given by

$$H = \frac{t}{2(at+b)}. \tag{82}$$

Scalar expansion is given by

$$\Theta = \frac{3t}{2(at+b)}. \tag{83}$$

From Equation (81), we see that ρ decreases when r is fixed and t increases and ρ increases when r increases and t decreases.

Regarding our model universe in this case, we have seen, from the expressions of R and ϕ, that the scalar field has a tendency to increase the radius of the universe, thereby helping in the expansion of the universe. The density of this universe is also seen to decrease with time which is a sign of a realistic universe. The expansion factor here is found to increase with time, thereby implying that our universe is expanding, which accurately describes the present universe. Thus, when $k = 0$, $\omega = 0$ and $\Lambda = 0$, the problem reduces to the case of dust distribution.

2.3 Case III: When $\omega \neq 0$ and $\Lambda = 0$

Since R is a function of t, we only consider the case $k = 0$. Then, Equation (26) reduces to

$$\frac{6\dot{R}^2}{R^2(\omega+1)} + \frac{4\omega+6}{\omega+1}\frac{\ddot{R}}{R} + \frac{(2\omega+1)\omega}{(\omega+1)^2}\frac{\dot{R}^2}{R^2} = 0. \quad (84)$$

Integrating, we get

$$R = \left[\frac{(4+3\omega)(at+b)}{(2+2\omega)}\right]^{\frac{2+2\omega}{4+3\omega}}, \quad (85)$$

where a and b are arbitrary constants of integration. From Equation (21), we get

$$A = \frac{1}{N}\left[\frac{(4+3\omega)(at+b)}{(2+2\omega)}\right]^{\frac{2+2\omega}{4+3\omega}}. \quad (86)$$

If $\omega > 0$, the radius of the universe increases as t increases and tends to infinity as t tends to infinity. From Equation (13), we get

$$\phi = \left\{\frac{(\omega+1)r^2}{2N}\right\}^{\frac{1}{\omega+1}}\left\{\frac{(4+3\omega)(at+b)}{2+2\omega}\right\}^{\frac{2}{4+3\omega}}, \quad (87)$$

which is a function of both r and t. If $\omega > 0$, the B-D scalar ϕ tends to infinity either when $r \to \infty$ or $t \to \infty$. The gravitational variable is given by

$$G = \frac{4+2\omega}{3+2\omega}\left\{\frac{2N}{(\omega+1)r^2}\right\}^{\frac{1}{\omega+1}}\left\{\frac{2+2\omega}{(4+3\omega)(at+b)}\right\}^{\frac{2}{4+2\omega}}. \quad (88)$$

If $\omega > 0$, G decreases as r and t increase and tends to zero when either $r \to \infty$ or $t \to \infty$. From Equations (22) and (23), we get

$$P = -\frac{4a^2(2\omega+3)^2}{\kappa(4+3\omega)^2(at+b)^2}\left\{\frac{(\omega+1)r^2}{2N}\right\}^{\frac{1}{\omega+1}}\left\{\frac{(4+3\omega)(at+b)}{2+2\omega}\right\}^{\frac{2}{4+3\omega}}, \quad (89)$$

and

$$\rho = \frac{2a^2(2\omega+3)}{\kappa(3\omega+4)(at+b)^2}\left\{\frac{(\omega+1)r^2}{2N}\right\}^{\frac{1}{\omega+1}}\left\{\frac{(4+3\omega)(at+b)}{2+2\omega}\right\}^{\frac{2}{4+3\omega}}. \quad (90)$$

Hubble's parameter is given by

$$H = \frac{2a(\omega+1)}{(4+3\omega)(at+b)}. \quad (91)$$

Scalar expansion is given by

$$\Theta = \frac{6a(\omega+1)}{(4+3\omega)(at+b)}. \quad (92)$$

Considering the solution obtained in this case, the gravitational variable G is found to vary inversely with the scalar field ϕ. Thus in this case, the B-D scalar field has a tendency to decrease the gravitational potential. For this universe it is seen that the equation of state $\omega_1 < -1$, namely, $\omega_1 = \frac{P}{\rho} = -\frac{2(2\omega+3)}{4+3\omega} = -1 - \frac{\omega+2}{4+3\omega} < -1$. Thus the dark energy contained in this universe may be taken as the k-essence form of energy. Here we see that for the k-essence energy, with $\omega_1 < -1$, the scalar field grows in the future. Since the k-essence fields are similarly uniform on a small scale, the abundance of k-essence energy within a bound object actually grows with time, thereby increasing its influence on the internal dynamics. Ultimately, there is the possibility that the repulsive k-essence energy will overcome the forces holding this model together and pulls this universe apart in a Big Rip. Thus, when $k = 0$, $\omega \neq 0$ and $\Lambda = 0$, the problem reduces to the case of dust distribution.

2.4 Case IV: When $\omega \neq 0$ and $\Lambda \neq 0$

Since R is only a function of t, we just consider the case $k = 0$.

Then, Equation (26) reduces to

$$\frac{6\dot{R}^2}{R^2(\omega+1)} + \frac{4\omega+6}{\omega+1}\frac{\ddot{R}}{R} + \frac{(2\omega+1)\omega}{(\omega+1)^2}\frac{\dot{R}^2}{R^2} - 4\Lambda = 0. \quad (93)$$

Integrating, we get

$$R = e^{\frac{2(\omega+1)\sqrt{\Lambda}}{\sqrt{(2\omega+3)(3\omega+4)}}t}, \tag{94}$$

and

$$A = \frac{1}{N} e^{\frac{2(\omega+1)\sqrt{\Lambda}}{\sqrt{(2\omega+3)(3\omega+4)}}t}. \tag{95}$$

If $\omega > 0$, the radius of the universe R tends to infinity as t tends to infinity.

From Equation (13), we get

$$\phi = \left\{ \frac{(\omega+1)r^2}{2N} \right\}^{\frac{1}{\omega+1}} e^{\frac{2\sqrt{\Lambda}}{\sqrt{(2\omega+3)(3\omega+4)}}t}, \tag{96}$$

which is a function of both r and t. When either $r \to \infty$ or $t \to \infty$, the B-D scalar ϕ tends to infinity. The gravitational variable is given by

$$G = \frac{4+2\omega}{3+2\omega} \left\{ \frac{2N}{(\omega+1)r^2} \right\}^{\frac{1}{\omega+1}} e^{-\frac{2\sqrt{\Lambda}}{\sqrt{(2\omega+3)(3\omega+4)}}t}, \tag{97}$$

which is a function of both r and t. From Equation (97), we see that the gravitational variable G decreases when r and t increase and tends to zero when either $r \to \infty$ or $t \to \infty$. From Equations (22) and (23), we get

$$P = \frac{\Lambda}{\kappa(4+3\omega)} \tag{98}$$

and

$$\rho = \frac{\Lambda}{\kappa} \left\{ \frac{(\omega+1)r^2}{2N} \right\}^{\frac{1}{\omega+1}} e^{\frac{2\sqrt{\Lambda}}{\sqrt{(2\omega+3)(3\omega+4)}}t}. \tag{99}$$

Hubble's parameter is given by

$$H = \frac{2(\omega+1)\sqrt{\Lambda}}{\sqrt{(2\omega+3)(3\omega+4)}}. \tag{100}$$

Scalar expansion is given by

$$\Theta = \frac{6(\omega+1)\sqrt{\Lambda}}{\sqrt{(2\omega+3)(3\omega+4)}}. \tag{101}$$

In this model universe, the scalar field is seen to have a tendency to increase the expansion of the universe, thereby flattening the universe. Here, also the B-D field has a tendency to decrease the gravitational potential, and the gravitational variable G tends to decrease the pressure and density of the universe. Since here, as $t \to \infty$, it is found that $R \to \infty$ as well as $\rho \to \infty$, there is the possibility of a bounce at some point in time, thereby indicating that this universe shows cyclic behavior. If $8\sqrt{\Lambda} > \frac{\omega\sqrt{(2\omega+3)(3\omega+4)}}{(\omega+1)^2}$, then this model universe will have an accelerated expansion instigated by the negative pressure. Also, in this model the vacuum energy due to the cosmological constant may be taken as the dark energy part causing the accelerated expansion of the universe.

3 CONCLUSIONS

The universes we have investigated are found to behave in different ways and to show different manifestations under different conditions. Some of them show signs of containing a cosmological constant form and quintessence form of dark energy, whereas others seem to contain fluids behaving like phantom and k-essence forms of dark energy, which can explain the present accelerated expansion of the universe. Thus the model universes we obtain in these cases may be taken as realistic models of our universe, and many more unknown properties of the universe and of dark energy may be realized and known from further studies of these models, which we will perform and report elsewhere afterwards. Furthermore, one model of ours seems to undergo a gravitational collapse leading to a black hole; whereas another model surprisingly seems to face the fate of a Big Rip. Another new finding in some of our models is that they simultaneously contain two forms of dark energy, one due to a cosmological constant and another due to a B-D scalar field. Also, interestingly enough, one of our models seems to behave like a universe obeying the newly proposed cyclic theory of the universe.

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ISOTROPIC ROBERTSON-WALKER MODEL UNIVERSE WITH DYNAMICAL COSMOLOGICAL PARAMETER Λ IN BRANS-DICKE THEORY OF GRAVITATION

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Abstract. *This paper discusses about Robertson-Walker space-time with quadratic equation of state and dynamical cosmological parameter Λ. Some exact solutions of Einstein's field equations for three cases have been obtained. Physical behaviors of the models are discussed in detail.*

Keywords: Brans-Dicke theory, dark energy, quadratic equation of state.

I. INTRODUCTION

The Brans-Dicke (B-D) theory [1] of gravitation is one of the simplest and best understood scalar-tensor theories. As a result B-D theory has attained significant attention in recent years. Cosmological models in Brans-Dicke theory is discussed by many authors [2–10]. The cosmological and astronomical data obtained from the Supernovae Ia(SNeIa), the cosmic microwave background (CMB) radiation anisotropies, the Large Scale Structure (LSS) and X-ray experiments support the discovery of accelerated expansion of the present day universe [11–19]. The accelerated expansion of universe is due to the presence of dark energy which has positive energy density and adequate negative pressure [20, 21]. Chen and Wu [22] considered Λ varying as R^{-2}, Carvalho and Lima [23] generalized it. Beesham [24], Tiwari [25], Harpreet and Tiwari [26], Kotambkar *et al.* [27] are some of the authors who studied cosmological model with variable G and Λ. Nojiri and Odintsov [28], Capozziello [29], Chavanis [30], Sharma and Rantnapal [31], Takisa *et al.* [32], Feroze and Siddiqui [33] are some of the authors who have discussed about cosmological models with equation of state in quadratic nature. Reddy *et al.* [34] have obtained Bianchi type-I model with a quadratic equation of state. Ngudelanga [35] has studied about a star with quadratic equation of state. Recently, Adhav *et al.* [36, 37] have obtained some cosmological models with the help of quadratic equation of state.

In this paper, we discussed an isotropic cosmological model with perfect fluid in Brans-Dicke theory of gravitation by considering equation of state in quadratic form. In Sec. II we give the field equations and their solution. The discussion is given in Sec. III.

II. FIELD EQUATIONS AND SOLUTIONS

Here, we consider the spherically symmetric Robertson Walker metric

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right], \tag{1}$$

where k is the curvature index which can take values $-1, 0, 1$.

The Brans-Dicke (B-D) theory of gravity is described by the action

$$S = \int d^4x\sqrt{|g|}\left[\frac{1}{16\pi}\left(\phi R - \frac{\omega}{\phi}g^{sl}\phi_{,l}\phi_{,s}\right) + L_m\right], \tag{2}$$

where R represents the curvature scalar associated with the 4D metric g_{ij}; g is the determinant of g_{ij}; ϕ is a scalar field; ω is a dimensionless coupling constant; L_m is the Lagrangian of the ordinary matter component.

The Einstein field equations in the most general form are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = \frac{-\kappa}{\phi}T_{ij} - \frac{\omega}{\phi^2}\left[\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi^{,s}\phi_{,s}\right] - \frac{1}{\phi}\left[\phi_{,ij} - g_{ij}\phi^{,s}{}_{;s}\right], \tag{3}$$

$$(3+2\omega)\,\phi^{,s}{}_{;s} = \kappa T, \tag{4}$$

where $\kappa = 8\pi$, Λ is the cosmological constant, R_{ij} is Ricci-tensor, g_{ij} is metric tensor, $\Box\phi = \phi^{,s}{}_{;s}$, and ϕ_i is the partial differentiation with respect to x^i coordinate.

The energy-momentum tensor for the perfect fluid distribution is

$$T_{ij} = (p+\rho)\,u_i u_j - p g_{ij} \tag{5}$$

with u_i = four velocity vector, p = proper density and ρ = proper rest mass density.

Considering a co-moving system, we get

$$u_1 = u_2 = u_3 = 0, u_4 = 1 \text{ and } g^{ij}u_i u_j = 1.$$

A comma $(,)$ or semicolon $(;)$ followed by a subscript denotes partial differentiation or a covariant differentiation, respectively. The velocity of light is taken to be unity.

Now for the metric (1) surviving field equations are

$$\frac{k}{R^2} + \frac{\dot{R}^2}{R^2} + 2\frac{\ddot{R}}{R} - \Lambda = \frac{-\kappa p}{\phi} - \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - 2\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} - \frac{\ddot{\phi}}{\phi}, \tag{6}$$

$$3\left(\frac{k}{R^2} + \frac{\dot{R}^2}{R^2}\right) - \Lambda = \frac{\kappa\rho}{\phi} + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi}. \tag{7}$$

From Eq. (4), we get

$$(3+2\omega)\left[3\frac{\dot{R}\dot{\phi}}{R} + \ddot{\phi}\right] = \kappa(\rho - 3p) \tag{8}$$

The energy momentum equation $T^{ij}{}_{;j} = 0$ leads to the form

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{9}$$

We consider [38] ansatz

$$\Lambda = \beta H^2 \tag{10}$$

and equation of state in quadratic form as

$$p = \alpha\rho^2 - \rho, \tag{11}$$

where $\alpha \neq 0$.

From equations (9) and (11), we get

$$\rho = R^{-3\alpha}, \tag{12}$$

$$p = \alpha\left[R^{-3\alpha}\right]^2 - R^{-3\alpha}. \tag{13}$$

From equations (6), (7),(8) and (10), we get

$$3\frac{k}{R^2} + (3-2\beta)\frac{\dot{R}^2}{R^2} + 3\frac{\ddot{R}}{R} = \omega[\frac{\ddot{\phi}}{\phi} - \frac{1}{2}\frac{\dot{\phi}^2}{\phi^2} + 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi}] \tag{14}$$

where a dot $(.)$ denotes differentiation with respect to time t.

For any cosmological model scale factor $R(t)$ should be known and equation (14) can be integrated by taking the separation constant as zero. So, from (14) we can consider

$$3\frac{k}{R^2} + (3-2\beta)\frac{\dot{R}^2}{R^2} + 3\frac{\ddot{R}}{R} = 0, \tag{15}$$

$$\frac{\ddot{\phi}}{\phi} - \frac{1}{2}\frac{\dot{\phi}^2}{\phi^2} + 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} = 0. \tag{16}$$

The gravitational variable [39] is given by

$$G = (\frac{4+2\omega}{3+2\omega})\frac{1}{\phi} \tag{17}$$

The anisotropy parameter is given by

$$\Delta = \frac{1}{3}\sum_{n=1}^{3}\left(\frac{H_i - H}{H}\right)^2.$$

Shear scalar is given by

$$\sigma^2 = \frac{1}{2}\left[\sum_{i=1}^{3} H_i - \frac{1}{3}\Theta^2\right].$$

II.1. Case I: $k = 0$ and $0 < \beta < 3$

From (15), we get

$$R = M_1\{c_1(3-\beta)(c_1 t + c_2)\}^{\frac{3}{2(3-\beta)}}, \tag{18}$$

where $M_1 = 2^{\frac{-3}{2(\beta-3)}} 3^{\frac{3}{2(\beta-3)}}$ and c_1, c_2 are constants.

From equation (16), we get

$$\phi = A^2 2^{\frac{9}{(\beta-3)}} 3^{\frac{-3}{(\beta-3)}} \{c_1(3-\beta)\}^{\frac{9}{(\beta-3)}} \left(\frac{\beta-3}{2\beta+3}\right)^2 (c_1 t + c_2)^{\frac{2\beta+3}{(\beta-3)}}, \tag{19}$$

where A is a constant.

The gravitational variable is given by

$$G=\left(\frac{4+2\omega}{3+2\omega}\right)\left[A^2 2^{\frac{9}{(\beta-3)}}3^{\frac{-3}{(\beta-3)}}\{c_1(3-\beta)\}^{\frac{9}{(\beta-3)}}\left(\frac{\beta-3}{2\beta+3}\right)^2(c_1t+c_2)^{\frac{2\beta+3}{(\beta-3)}}\right]^{-1}. \tag{20}$$

From equations (12) and (13), we get

$$\rho=\left[M_1\{c_1(3-\beta)(c_1t+c_2)\}^{\frac{3}{2(3-\beta)}}\right]^{-3\alpha} \tag{21}$$

$$p=\alpha\left[M_1\{c_1(3-\beta)(c_1t+c_2)\}^{\frac{3}{2(3-\beta)}}\right]^{-6\alpha}-\left[M_1\{c_1(3-\beta)(c_1t+c_2)\}^{\frac{3}{2(3-\beta)}}\right]^{-3\alpha}. \tag{22}$$

Spatial volume is given by

$$V=\left[M_1\{c_1(3-\beta)(c_1t+c_2)\}^{\frac{3}{2(3-\beta)}}\right]^3. \tag{23}$$

Hubble's parameter is given by

$$H=\frac{3}{2c_1(3-\beta)^2(c_1t+c_2)}. \tag{24}$$

Scalar expansion is given by

$$\Theta=\frac{9}{2c_1(3-\beta)^2(c_1t+c_2)}. \tag{25}$$

Deceleration parameter is given by

$$q=-\left(\frac{2\beta-5}{3}\right). \tag{26}$$

The anisotropy parameter is given by

$$\Delta=0. \tag{27}$$

Shear scalar is given by

$$\sigma=0. \tag{28}$$

Cosmological constant is given by

$$\Lambda=\beta\left[\frac{3}{2c_1(3-\beta)^2(c_1t+c_2)}\right]^2. \tag{29}$$

II.2. Case II: $k=-1$ **and** $\beta=3$.

From (15), we

$$R=M_2\left(e^{\frac{c_4}{c_3}}e^{\frac{t}{c_3}}+e^{\frac{-c_4}{c_3}}e^{\frac{-t}{c_3}}\right). \tag{30}$$

where $M_2=\frac{c_3}{2}$, c_3, c_4 are constants.

From equation (16), we get

$$\phi=c_5\left[\left\{\frac{2e^{2\frac{c_4}{c_3}}e^{2\frac{t}{c_3}}+1}{4\left(e^{2\frac{c_4}{c_3}}e^{2\frac{t}{c_3}}+1\right)^2}\right\}\right]^2, \tag{31}$$

where $c_5 = {M_2}^{-6}$ is a constant.

The gravitational variable is given by

$$G = {c_5}^{-1}\left(\frac{4+2\omega}{3+2\omega}\right)\left[\left\{\frac{2e^{2\frac{c_4}{c_3}}e^{2\frac{t}{c_3}}+1}{4\left(e^{2\frac{c_4}{c_3}}e^{2\frac{t}{c_3}}+1\right)^2}\right\}\right]^{-2}. \tag{32}$$

From equations (12) and (13), we get

$$\rho = \left[M_2\left(e^{\frac{c_4}{c_3}}e^{\frac{t}{c_3}}+e^{\frac{-c_4}{c_3}}e^{\frac{-t}{c_3}}\right)\right]^{-3\alpha}, \tag{33}$$

$$p = \alpha\left[M_2\left(e^{\frac{c_4}{c_3}}e^{\frac{t}{c_3}}+e^{\frac{-c_4}{c_3}}e^{\frac{-t}{c_3}}\right)\right]^{-6\alpha} - \left[M_2\left(e^{\frac{c_4}{c_3}}e^{\frac{t}{c_3}}+e^{\frac{-c_4}{c_3}}e^{\frac{-t}{c_3}}\right)\right]^{-3\alpha}. \tag{34}$$

Spatial volume is given by

$$V = \left[M_2\left(e^{\frac{c_4}{c_3}}e^{\frac{t}{c_3}}+e^{\frac{-c_4}{c_3}}e^{\frac{-t}{c_3}}\right)\right]^{3}. \tag{35}$$

Hubble's parameter is given by

$$H = \frac{1}{c_3}\left[\frac{1-e^{-2\frac{c_4}{c_3}}e^{-2\frac{t}{c_3}}}{1+e^{-2\frac{c_4}{c_3}}e^{-2\frac{t}{c_3}}}\right]. \tag{36}$$

Scalar expansion is given by

$$\Theta = \frac{3}{c_3}\left[\frac{1-e^{-2\frac{c_4}{c_3}}e^{-2\frac{t}{c_3}}}{1+e^{-2\frac{c_4}{c_3}}e^{-2\frac{t}{c_3}}}\right]. \tag{37}$$

Deceleration parameter is given by

$$q = \frac{-1}{{c_3}^4}\left[\frac{1+e^{-2\frac{c_4}{c_3}}e^{-2\frac{t}{c_3}}}{1-e^{-2\frac{c_4}{c_3}}e^{-2\frac{t}{c_3}}}\right]. \tag{38}$$

The anisotropy parameter is given by

$$\Delta = 0. \tag{39}$$

Shear scalar is given by

$$\sigma = 0. \tag{40}$$

Cosmological constant is given by

$$\Lambda = \frac{3}{{c_3}^2}\left[\frac{1-e^{-2\frac{c_4}{c_3}}e^{-2\frac{t}{c_3}}}{1+e^{-2\frac{c_4}{c_3}}e^{-2\frac{t}{c_3}}}\right]^2. \tag{41}$$

II.3. Case III: $k = 1$ **and** $\beta = 3$.

From (15), we get

$$R = M_3\left(e^{\frac{c_7}{c_6}}e^{\frac{t}{c_6}} - e^{\frac{-c_7}{c_6}}e^{\frac{-t}{c_6}}\right) \tag{42}$$

where $M_3 = \frac{c_6}{2}$, c_6, c_7 are constants.

From equation (16), we get

$$\phi = c_8\left[\left\{\frac{2e^{2\frac{c_7}{c_6}}e^{2\frac{t}{c_6}} - 1}{4\left(e^{2\frac{c_7}{c_6}}e^{2\frac{t}{c_6}} - 1\right)^2}\right\}\right]^2, \tag{43}$$

where $c_8 = {M_3}^{-6}$ is a constant.

The gravitational variable is given by

$$G = {c_8}^{-1}\left(\frac{4+2\omega}{3+2\omega}\right)\left[\left\{\frac{2e^{2\frac{c_7}{c_6}}e^{2\frac{t}{c_6}} - 1}{4\left(e^{2\frac{c_7}{c_6}}e^{2\frac{t}{c_6}} - 1\right)^2}\right\}\right]^{-2}. \tag{44}$$

From equations (12) and (13), we get

$$\rho = \left[M_3\left(e^{\frac{c_7}{c_6}}e^{\frac{t}{c_6}} - e^{\frac{-c_7}{c_6}}e^{\frac{-t}{c_6}}\right)\right]^{-3\alpha}, \tag{45}$$

$$p = \alpha\left[M_3\left(e^{\frac{c_7}{c_6}}e^{\frac{t}{c_6}} - e^{\frac{-c_7}{c_6}}e^{\frac{-t}{c_6}}\right)\right]^{-6\alpha} - \left[M_3\left(e^{\frac{c_7}{c_6}}e^{\frac{t}{c_6}} - e^{\frac{-c_7}{c_6}}e^{\frac{-t}{c_6}}\right)\right]^{-3\alpha}. \tag{46}$$

Spatial volume is given by

$$V = \left[M_3\left(e^{\frac{c_7}{c_6}}e^{\frac{t}{c_6}} - e^{\frac{-c_7}{c_6}}e^{\frac{-t}{c_6}}\right)\right]^3. \tag{47}$$

Hubble's parameter is given by

$$H = \frac{1}{c_6}\left[\frac{1 + e^{-2\frac{c_7}{c_6}}e^{-2\frac{t}{c_6}}}{1 - e^{-2\frac{c_7}{c_6}}e^{-2\frac{t}{c_6}}}\right]. \tag{48}$$

Scalar expansion is given by

$$\Theta = \frac{3}{c_6}\left[\frac{1 + e^{-2\frac{c_7}{c_6}}e^{-2\frac{t}{c_6}}}{1 - e^{-2\frac{c_7}{c_6}}e^{-2\frac{t}{c_6}}}\right] \tag{49}$$

Deceleration parameter is given by

$$q = \frac{-1}{{c_6}^4}\left[\frac{1 - e^{-2\frac{c_7}{c_6}}e^{-2\frac{t}{c_6}}}{1 + e^{-2\frac{c_7}{c_6}}e^{-2\frac{t}{c_6}}}\right]. \tag{50}$$

The anisotropy parameter is given by

$$\Delta = 0. \tag{51}$$

Shear scalar is given by

$$\sigma = 0. \tag{52}$$

Cosmological constant is given by

$$\Lambda = \frac{3}{{c_6}^2}\left[\frac{1+e^{-2\frac{c_7}{c_6}}e^{-2\frac{t}{c_6}}}{1-e^{-2\frac{c_7}{c_6}}e^{-2\frac{t}{c_6}}}\right]^2. \tag{53}$$

III. DISCUSSION

Here, we have got the following results:
Case I: In this case $R \to \infty$, $\phi \to 0$, $G \to \infty$, $H \to 0$, $\Theta \to 0$, $V \to \infty$, $\Lambda \to 0$ as $t \to \infty$. For $t = 0$, R, ϕ, G, H, Θ, V, Λ become finite. Also, for $\alpha < 0$, $\rho > 0$ and $p < 0$ which gives positive energy density and negative pressure contributing to the dark energy model with accelerating universe. Here, for $2 < \beta < 3$, $q \leq 0$, the deceleration parameter is in the range $-1 \leq q \leq 0$ which is in agreement with the observations made by Riess *et al.* [12] and Perlmutter *et al.* [13] i.e. the expansion of the universe is accelerating. Also, $\Delta = 0$, $\sigma = 0$ this shows that our model is isotropic and shear free. The value of the cosmological constant for the model is found to be small and positive, which is supported by the observations Garnavich et al. [40, 41] and Schmidt *et al.* [42].
Case II: In this case $R \to \infty$, $\phi \to 0$, $G \to \infty$, $V \to \infty$ as $t \to \infty$ and H, Θ, Λ remains finite for $t \to \infty$. Again $R, \phi, G, H, \Theta, V, \Lambda$ become finite for $t = 0$. Also, for $\alpha < 0$, $\rho > 0$ and $p < 0$ which gives positive energy density and negative pressure contributing to the dark energy model with accelerating universe. Here, as $t = 0$ and $t \to \infty$, the deceleration parameter is in the range $-1 \leq q \leq 0$ which gives accelerated expansion of the universe. Here, $\Delta = 0$, $\sigma = 0$ this shows that our model is isotropic and shear free. The time dependent cosmological constant for the model is small and positive.
Case III: In this case $R \to \infty$, $\phi \to 0$, $G \to \infty$, $V \to \infty$, as $t \to \infty$ and H, Θ, Λ become finite for $t \to \infty$. For $t = 0$, $R, \phi, G, H, \Theta, V, \Lambda$ become finite. Also, for $\alpha < 0$, $\rho > 0$ and $p < 0$ which gives positive energy density and negative pressure contributing to the dark energy model with accelerating universe. Here, as t varies from 0 to ∞, the deceleration parameter is in the range $-1 \leq q \leq 0$ which supports the observations made by Riess *et al.* [12] and Perlmutter *et al.* [13] for accelerating universe. Also, $\Delta = 0$, $\sigma = 0$ this shows that our model is isotropic and shear free. The time dependent cosmological constant for this model also is small and positive.

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SOME SPHERICALLY SYMMETRIC R/W UNIVERSE INTERACTING WITH VACUUM B–D SCALAR FIELD

We study a spherically symmetric vacuum cosmological model of the Universe interacting with the Brans–Dicke (B–D) scalar field in the Robertson–Walker (R/W) metric. Exact time-dependent solutions of B–D vacuum field equations are obtained in two different cases. The physical and dynamical properties of the model are discussed in detail.

Keywords: Brans–Dicke theory, vacuum cosmological model, spherically symmetric scalar field.

1. Introduction

The Brans–Dicke (B–D) theory [1] describes most of the important features of the progress of the Universe during the late-time dynamical epoch. As a result, the B–D theory has attained a significant attention in recent years. The scalar-tensor theories are considered the simplest and best understood modification of gravity theory. The Brans–Dicke theory is, in fact, a modification of Einstein's General Relativity allowing the variable gravity with certain coupling parameter ω. It is somewhat classical in nature, for that reason it is expected to play a crucial role in the late-time evolution of the Universe. It is also realized that most of the inflationary models based on the B–D scalar theory overcharge many important elements about the evolution of the Universe [2, 3]. Hence, the B–D theory gives a connection between the accelerated expansion of the Universe and fundamental physics. Earlier, Brans and Dicke [1] obtained the vacuum solutions of B–D field equations followed by three more solutions for a spherically symmetric metric. Nariri [4] proposed a Hamiltonian approach to the dynamics of the expanding homogeneous Universe. Janis *et al.* [5] established a theorem to generate the B–D vacuum state solutions. Tabensky and Taub [6] obtained B–D vacuum static solutions with plane symmetric self-gravitating fluids. Rao *et al.* [7] discussed about cylindrically symmetric B–D fields. Various authors [8–13] discussed about vacuum solutions in the Brans–Dicke theory of gravitation for the metric tensors viz. plane symmetry, static cylindrical symmetry, zero-mass scalar field, conformal scalar field, for spatially homogeneous and anisotropic configuration, axisymmetric stationary and spherical symmetries, static fields, *etc.* Bhadra and Sarkar *et al.* [14] obtained that only two classes are independent among the four classes of static spherically symmetric solutions of the vacuum Brans–Dicke theory of gravity. Adhav *et al.* [15] obtained an exact solution of the vacuum Brans–Dicke field equations for the metric tensor of a spatially homogeneous and anisotropic model. Static, cylindrically symmetric vacuum solutions with and without a cosmological constant in the B–D theory were obtained by Baykal *et al.* [16]. Rai *et al.* [17] obtained an exact solution of the vacuum Brans–Dicke field equations for the metric tensor of a spatially homogeneous and anisotropic model. Here, we studied the problem of a B–D scalar field interacting with the spherically symmetric Robertson–Walker metric. The paper is organized as follows: in Section 2, we consider the metric and give solutions of the field equations in different cases; in Section 3, we give conclusion about the solutions.

2. Solutions of Field Equations

The vacuum Brans–Dicke field equations in the general form are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = \frac{\omega}{\phi^2}\left[\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}g^{sl}\phi_{,l}\phi_{,s}\right] - \frac{1}{\phi}[\phi_{,ij} - g_{ij}\phi^{;s}_{;s}], \quad (1)$$

$$(3+2\omega)\phi^{;s}_{;s} = 4\Lambda, \quad (2)$$

where ϕ is the scalar field, Λ is the cosmological constant, ω is the dimensionless Dicke coupling constant, R_{ij} is the Ricci tensor, R is the Riemann curvature scalar, g_{ij} is the metric tensor, $\Box\phi = \phi^{;s}_{;s}$, $\Box$ is the Laplace—Beltrami operator, and $\phi_{,i}$ is the partial differentiation with respect to the x^i coordinate.

Let us consider the R/W space time metric

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right], \quad (3)$$

where $R(t)$ is the scale factor, and k is the curvature index, which can take up the values $(-1, 0, +1)$ for open, flat, and closed models of the Universe, respectively. Corresponding to metric (3), the Brans–Dicke field equation (1) becomes

$$\frac{k}{R^2} + \frac{\dot{R}^2}{R^2} + \frac{2\ddot{R}}{R} - \Lambda = \frac{\omega}{2\phi^2}\left[\frac{(kr^2-1)}{R^2}\phi'^2 - \dot{\phi}^2\right] + \frac{1}{\phi}\left[\frac{2(1-kr^2)}{R^2 r}\phi' - \frac{2\dot{R}\dot{\phi}}{R} - \ddot{\phi}\right], \quad (4)$$

$$\frac{k}{R^2} + \frac{\dot{R}^2}{R^2} + \frac{2\ddot{R}}{R} - \Lambda = \frac{\omega}{2\phi^2}\left[\frac{(1-kr^2)}{R^2}\phi'^2 - \dot{\phi}^2\right] + \frac{1}{\phi}\left[\frac{(1-kr^2)}{R^2}\phi'' - \frac{(2kr^2-1)}{R^2 r}\phi' - \frac{2\dot{R}\dot{\phi}}{R} - \ddot{\phi}\right], \quad (5)$$

$$3\left(\frac{k}{R^2} + \frac{\dot{R}^2}{R^2}\right) - \Lambda = \frac{\omega}{2\phi^2}\left[\dot{\phi}^2 + \frac{(1-kr^2)}{R^2}\phi'^2\right] + \frac{1}{\phi}\left[\frac{(1-kr^2)}{R^2}\phi'' - \frac{(3kr^2-2)}{R^2 r}\phi' - \frac{3\dot{R}\dot{\phi}}{R}\right], \quad (6)$$

and

$$\frac{\omega}{\phi^2}\phi'\dot{\phi} + \frac{\dot{\phi}'}{\phi} - \frac{\dot{R}\phi'}{R\phi} = 0. \quad (7)$$

From Eq. (2), we get

$$\left[-\frac{(1-kr^2)}{R^2}\phi'' + \frac{(3kr^2-2)}{R^2 r}\phi' + \frac{3\dot{R}\dot{\phi}}{R} + \ddot{\phi}\right] = \frac{4\Lambda}{(3+2\omega)}, \quad (8)$$

where a dot (.) and dash ($'$) denote the differentiation with respect to the time t and r, respectively. From Eqs. (4) and (5), we obtain the relation

$$\frac{\phi''}{\phi'} + \omega\frac{\phi'}{\phi} = \frac{1}{r} + \frac{kr}{1-kr^2} \quad (9)$$

under the conditions $\phi' \neq 0, 1-kr^2 \neq 0$. Integrating Eq. (9), we get

$$\phi^{\omega+1} = B\sqrt{1-kr^2} + D \quad (10)$$

provided $k \neq 0$, where B and D are arbitrary functions of time t.

Using (10) in (4) and (5), we obtain

$$\frac{k}{R^2} + \frac{\dot{R}^2}{R^2} + \frac{2\ddot{R}}{R} - \Lambda = -\frac{\dot{L}}{(1+\omega)\phi^{1+\omega}} - \frac{\omega B^2k^2r^2}{2R^2(1+\omega)^2(\phi^{1+\omega})^2} + \frac{\omega L^2}{2(1+\omega)^2(\phi^{1+\omega})^2} - \frac{2Bk\sqrt{1-kr^2}}{R^2(1+\omega)\phi^{1+\omega}} - \frac{2\dot{R}L}{R(1+\omega)\phi^{1+\omega}}, \quad (11)$$

where $L = \dot{B}\sqrt{1-kr^2} + \dot{D}$, $\dot{L} = \ddot{B}\sqrt{1-kr^2} + \ddot{D}$.

Using (10) in (6), we obtain

$$3\left(\frac{k}{R^2} + \frac{\dot{R}^2}{R^2}\right) - \Lambda = \frac{\omega L^2}{2(1+\omega)(\phi^{1+\omega})^2} - \frac{\omega B^2k^2r^2}{2R^2(1+\omega)^2(\phi^{1+\omega})^2} - \frac{3Bk\sqrt{1-kr^2}}{R^2(1+\omega)\phi^{1+\omega}} - \frac{3\dot{R}L}{R(1+\omega)\phi^{1+\omega}}. \quad (12)$$

Using (10) in (8), we obtain

$$\left[\frac{3Bk\sqrt{1-kr^2}}{R^2(1+\omega)\phi^{1+\omega}} + \frac{\omega B^2k^2r^2}{R^2(1+\omega)^2(\phi^{1+\omega})^2} + \frac{3\dot{R}L}{R(1+\omega)\phi^{1+\omega}} - \frac{\omega L^2}{(1+\omega)^2(\phi^{1+\omega})^2} + \frac{\dot{L}}{(1+\omega)\phi^{1+\omega}}\right] = \frac{4\Lambda}{(3+2\omega)}. \quad (13)$$

Using (10) in (7), we obtain

$$\frac{\dot{B}}{B} = \frac{\dot{R}}{R}. \tag{14}$$

Now, we shall determine the values of five unknowns B, ω, R, Λ, and D, by using four equations (11), (12), (13), and (14). Since the number of unknowns is more than the number of equations, this is a case of underdeterminacy, so it is reasonable to assume a physical relation to solve the field equations. Now, we try to solve the field equations under different physical situations.

Case I: Taking the arbitrary constant $D = 0$ and using Eq. (14) in (11), (12), and (13), we obtain the relations

$$\left(\frac{\dot{R}}{R}\right)^2 \left[\frac{2(1+\omega)^2 + 3\omega + 4}{2(1+\omega)^2}\right] + \frac{3+2\omega}{1+\omega}\frac{\ddot{R}}{R} = \\ = \Lambda - \frac{k}{R^2}\left[\frac{\omega k r^2}{2(1+\omega)^2(1-kr^2)} + \frac{3+\omega}{1+\omega}\right], \tag{15}$$

$$\left(\frac{\dot{R}}{R}\right)^2 \left[3 - \frac{\omega}{2(1+\omega)^2} + \frac{3}{1+\omega}\right] - \Lambda = - \\ = -\frac{k}{R}\left[3 + \frac{\omega k r^2}{2(1+\omega)^2(1-kr^2)} + \frac{3}{1+\omega}\right], \tag{16}$$

$$\left[\frac{3k}{R^2(1+\omega)} + \frac{\omega k^2 r^2}{R^2(1+\omega)^2(1-kr^2)} + \\ + \frac{3+2\omega}{(1+\omega)^2}\left(\frac{\dot{R}}{R}\right)^2 + \frac{\ddot{R}}{R(1+\omega)}\right] = \frac{4\Lambda}{(3+2\omega)}. \tag{17}$$

To obtain the exact solutions from Eqs. (15), (16), and (17), we consider a case where the coupling constant $\omega = 0$. Then Eqs. (15), (16), and (17) are reduced to the following forms:

$$3\left(\frac{\dot{R}}{R}\right)^2 + 3\frac{\ddot{R}}{R} - \Lambda = -\frac{3k}{R^2}, \tag{18}$$

$$3\left(\frac{\dot{R}}{R}\right)^2 - \frac{\Lambda}{2} = -\frac{3k}{R^2}, \tag{19}$$

$$3\left(\frac{\dot{R}}{R}\right)^2 + \frac{\ddot{R}}{R} + \frac{3k}{R^2} = 4\Lambda. \tag{20}$$

Corresponding to $k = -1$, Eqs. (18), (19), and (20) imply that $\Lambda = 0$ and $R = t$. In this case, the value of ϕ from Eqs. (10) is given by

$$\phi = t\sqrt{1+r^2}. \tag{21}$$

From Eqs. (14) and (21), we observe that the expansion parameter is purely a function of the time t, while the B–D scalar ϕ is a function of both r and t. Here, $r \to \infty$, $\phi \to \infty$, while R remains finite. However, as $t \to \infty$, both ϕ and R tends to ∞. We can further conclude that, corresponding to $k = -1$ and $\omega = 0$, the B–D scalar ϕ is an increasing function of both r and t, since the B–D scalar ϕ and the gravitational variable G [18] are related by the relation

$$G = \frac{1}{\phi}\left(\frac{4+2\omega}{3+2\omega}\right). \tag{22}$$

So, the gravitational variable

$$G \propto \frac{1}{\phi}, \tag{23}$$

i.e., G decreases, as t (or r) increases. From Eq. (14), we further observe that, at the initial stage (i.e., when $t = 0$), the radius of the Universe is zero, thereby showing that the Universe was concentrated to a mass point and expands gradually till it becomes infinitely large, which supports the present finding for the accelerated expansion of the Universe. This is in conformity with the steady state theory of the cosmological Universe. The corresponding deceleration parameter is zero.

Case II: Taking $\phi' = 0$ and $3 + 2\omega \neq 0$ in the field equations, we obtain

$$3\frac{\dot{R}}{R}\dot{\phi} + \ddot{\phi} = \frac{4\Lambda}{3+2\omega}, \tag{24}$$

$$\frac{k}{R^2} + \left(\frac{\dot{R}}{R}\right)^2 + 2\frac{\ddot{R}}{R} - \Lambda = -\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - 2\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} - \frac{\ddot{\phi}}{\phi}, \tag{25}$$

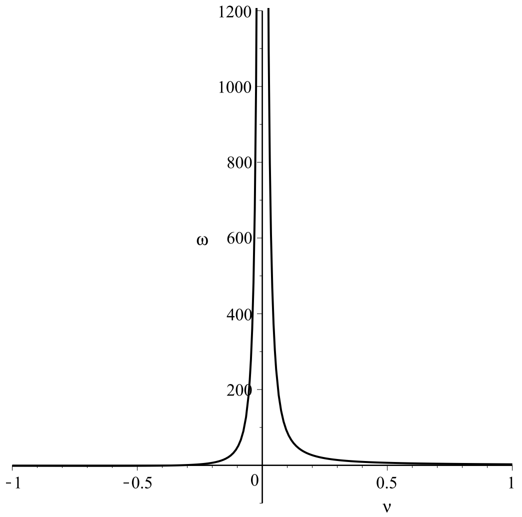
and

$$\frac{3k}{R^2} + 3\left(\frac{\dot{R}}{R}\right)^2 - \Lambda = \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 - 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi}. \tag{26}$$

Under the conditions $\Lambda = 0$ and $k = 0$, relations (24)–(26) become

$$\frac{\dot{R}}{R} = -\frac{1}{3}\frac{\ddot{\phi}}{\dot{\phi}}, \tag{27}$$

$$\left(\frac{\dot{R}}{R}\right)^2 + 2\frac{\ddot{R}}{R} = -\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - 2\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} - \frac{\ddot{\phi}}{\phi}, \tag{28}$$



Variation of ω for various values of ν according to (37)

$$3\left(\frac{\dot{R}}{R}\right)^2 = \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 - 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi}. \tag{29}$$

Adding (28) and (29), we get

$$4\left(\frac{\dot{R}}{R}\right)^2 + 2\frac{\ddot{R}}{R} = -5\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} - \frac{\ddot{\phi}}{\phi}. \tag{30}$$

Integrating (27) and (30), we get

$$R^3\dot{\phi} = a = \text{const}, \tag{31}$$

$$\phi\frac{d}{dt}(R^3) = b = \text{const}. \tag{32}$$

The sum of Eqs. (31) and (32) becomes

$$\frac{d}{dt}(\phi R^3) = a + b = c = \text{const}. \tag{33}$$

Integrating, we get

$$\phi = \frac{ct + l}{R^3}, \tag{34}$$

where c and l are constants. Moreover, from (31) and (32), we get

$$\frac{\dot{\phi}}{\phi} = 3\nu\frac{\dot{R}}{R}, \tag{35}$$

where $\nu = \frac{a}{b} = \text{const}$.

Using (35) in (29), we have

$$\left(1 + 3\nu - \frac{3}{2}\omega\nu^2\right)\left(\frac{\dot{R}}{R}\right)^2 = 0. \tag{36}$$

Since $\frac{\dot{R}}{R} \neq 0$, Eq. (36) becomes

$$\omega = \frac{2}{3}\left(\frac{1 + 3\nu}{\nu^2}\right). \tag{37}$$

The variation of ω according to (37) for variuos values of ν has been shown in Figure.

For $\nu = -\frac{1}{3}$ and $\omega = 0$, we get that there is neither expansion nor contraction of the Universe, where the B–D scalar ϕ decreases with time, till it vanishes as $t \to \infty$.

In addition, when $\nu = -1$ and $\nu = -\frac{1}{2}$, we get $\omega = -\frac{4}{3}$, which implies that the B–D scalar ϕ and the gravitational variable G will remain finite for all finite values of the time t. Here, corresponding to $\omega = 0$ and $\omega = -\frac{4}{3}$ from Eq. (22), we find that $G \propto \frac{1}{\phi}$, as in Eq. (23), which implies that ϕ and G will remain finite for all finite values of time t, and the gravitational variable G will be an increasing function of the time.

3. Conclusion

Here, we have seen that the role played by the scalar ϕ relating to the contraction and the expansion of the Universe consists in that the B–D scalar ϕ, which is an increasing function of the time, can be treated as something reflecting the contraction of the Universe, while the B–D scalar ϕ which is a decreasing function of the time may be treated as something reflecting the expansion of the Universe.

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СФЕРИЧНО-СИМЕТРИЧНИЙ
Р/У ВСЕСВІТ, ВЗАЄМОДІЮЧИЙ
З ВАКУУМНИМ Б–Д СКАЛЯРНИМ ПОЛЕМ

Р е з ю м е

Розглянуто сферично-симетричну вакуумну космологічну модель Всесвіту, взаємодіючу зі скалярним Бранса–Діке (Б–Д) полем в метриці Робертсона–Уолкера (Р/У). Отримано точні залежні від часу рішення Б–Д вакуумних польових рівнянь для двох різних випадків. Докладно обговорюються фізичні і динамічні властивості моделі.