Chapter-4

Correspondence between the Scalar fields and the Polytropic gas dark energy model

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4.1 Introduction

Cosmologist's belief that our Universe expands under an accelerated expansion (Perlmutter, 2003; Riess et al., 2004 & 2007; Caldwell & Doran, 2004; Koivisto & Mota, 2006; Daniel et al., 2008; Spergel et al., 2003; Tegmark et al., 2004). This expansion is due to a new energy of negative pressure, called dark energy (DE) (Overduin & Cooperstok, 1998; Sahni & Starobinsky, 2009). The first candidate for dark energy is the cosmological constant (Weinberg, 1989). Polytropic gas is considered as dark energy where the pressure is a function of energy density (Mukhopadhyay et al., 2008; Malekjani, 2013; Das & Basak, 2018a). There are several scalar fields such as Quintessence, K-essence, and Tachyon etc. The Quintessence (Doran & Wetterich, 2003; Zlatev et al., 1999; Barreiro et al., 2000; Capozziello, 2002), Dilaton (Farooq et al., 2010; Gasperini et al., 2002; Piazza & Tsujikawa, 2004), K-essence (Chiba, 2002) and Tachyon (Sen, 2002; Bagla et al., 2003) scalar fields are considered as a source of dark energy. Here, we have established a correspondence of the Quintessence, K-essence and Tachyon scalar fields with the Polytropic gas dark energy model and from this correspondence, we have reconstructed the dynamics and potential of the scalar fields in the context of Polytropic gas dark energy model.

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4.2 Model of Polytropic gas

The Polytropic gas EOS (Das & Singh, 2020a, Karami et al., 2009) is

$$P_\Lambda = K\rho_\Lambda^{1+\frac{1}{n}} \tag{4.1}$$

The pressure, energy density, Polytropic constant and index are denoted by P_Λ,ρ_Λ,K, and n respectively (Setare & Darabi, 2013; Das & Singh, 2020a).

The equation of dark energy conservation is

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + P_\Lambda) = 0 \tag{4.2}$$

Solving (4.1) & (4.2) and integrating we get

$$\rho_\Lambda = \left[Ba^{3/n} - K\right]^{-n} \tag{4.3}$$

Here B is a positive constant of integration and a (t) is the Universe's time scale factor (Karami et al., 2009; Das & Singh, 2020a).

Also the pressure is

$$P_\Lambda = K\left[Ba^{3/n} - K\right]^{-n-1} \tag{4.4}$$

The Polytropic gas EOS parameter is

$$\omega_\Lambda = \frac{P_\Lambda}{\rho_\Lambda} = -1 + \frac{Ba^{3/n}}{Ba^{3/n} - K} \tag{4.5}$$

If $K > Ba^{3/n}$, then (4.5) implies $\omega_\Lambda < -1$ that leads to a Phantom field dominated Universe.

4.3 Polytropic quintessence model

The action for the Quintessence scalar field φ is given by (Barreiro et al., 2000; Capozziello, 2002; Das & Basak, 2019)

$$S = \int \left[\frac{1}{2}g^{ij}\partial_i\varphi\partial_j\varphi - V(\varphi)\right]\sqrt{-g}\, d^4x \tag{4.6}$$

Where $\varphi is$the Quintessence field with the potential V(φ)and g is the determinant of g_{ij}

The pressure and energy density for the Quintessence scalar field φ are given by

$$P_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \tag{4.7}$$

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \tag{4.8}$$

The scalar potential and the kinetic energy terms of the Polytropic gas are

$$V(\varphi) = \frac{\frac{B}{2}a^{3/n} - K}{\left(Ba^{3/n} - K\right)^{n+1}} \tag{4.9}$$

$$\dot{\varphi}^2 = \frac{Ba^{3/n}}{\left(Ba^{3/n} - K\right)^{n+1}} \tag{4.10}$$

The equation of state for the quintessence scalar field φ is given by

$$\omega_\varphi = \frac{P_\varphi}{\rho_\varphi} = \frac{\frac{1}{2}\dot{\varphi}^2 - V(\varphi)}{\frac{1}{2}\dot{\varphi}^2 + V(\varphi)} = \frac{\dot{\varphi}^2 - 2V(\varphi)}{\dot{\varphi}^2 + 2V(\varphi)} \tag{4.11}$$

So that $-1 \le \omega_\varphi \le 1$

If the kinetic term $\dot{\varphi}^2$ dominates then $\omega_\varphi \approx 1$ and if the potential term $V(\varphi)$ dominates then $\omega_\varphi \approx -1$

The equation of motion for the field $\varphi = \varphi(t)$ on a FLRW background is given by the Klein Gordon equation

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0 \tag{4.12}$$

Where $H = \frac{\dot{a}}{a}$ and $V'(\varphi) = \frac{dV}{d\varphi}$

The Friedmann equation and acceleration equation are given by

$$H^2 = \frac{1}{3}\left[\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right] \tag{4.13}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{3}\left[\dot{\varphi}^2 - V(\varphi)\right] \tag{4.14}$$

If $\dot{\varphi}^2 < V(\varphi)$ then $\ddot{a} > 0$ this corresponds to the accelerated expansion of the Universe.

If $\frac{1}{2}\dot{\varphi}^2 \ll V(\varphi)$ (Slow roll approximation) then $H^2 \approx \frac{1}{3}V(\varphi)$ and $\omega_\varphi \approx -1$ (like cosmological constant) which represents a potential dominated scalar field. In the slow roll approximation as the term $\dot{\varphi}^2$ is considered negligible and the potential $V(\varphi)$ can be considered to fulfill

$$3H\dot{\varphi} \approx -V'(\varphi) \tag{4.15}$$

Therefore this slow rolling potential dominated scalar field can accelerated the expansion of the universe and act as a dark energy candidate.

4.4 Polytropic K-essence model

The scalar field of K-essence (Malekjani, 2013; Das & Basak, 2018a) is

$$S = \int d^4x\sqrt{-g}\,P(\varphi,\chi) \tag{4.16}$$

Pressure and energy density for the scalar field of K-essence are

$$P_k = f(\varphi)(-\chi + \chi^2) \tag{4.17}$$

$$\rho_k = f(\varphi)(-\chi + 3\chi^2) \tag{4.18}$$

The K-essence scalar field EOS parameter (Malekjani, 2013; Das & Basak, 2018a) is

$$\omega_k = \frac{P_k}{\rho_k} = \frac{\chi-1}{3\chi-1} \tag{4.19}$$

When $\frac{1}{3} < \chi < \frac{1}{2}$, then $\omega_k < -1$ which represents Phantom energy and hence the K-essence scalar field can interpret the accelerated expansion.

Using (4.5) in (4.19) we get

$$\omega_\Lambda = -1 + \frac{Ba^{3/n}}{Ba^{3/n} - K} = \frac{\chi-1}{3\chi-1} \tag{4.20}$$

The parameter χ can be obtained as

$$\chi = \frac{2 + \frac{Ba^{3/n}}{K - Ba^{3/n}}}{4 + 3\frac{Ba^{3/n}}{K - Ba^{3/n}}} \tag{4.21}$$

Using $2\chi = \dot{\varphi}^2$ in (4.21) we get

$$\dot{\varphi}^2 = \frac{4 + 2\frac{Ba^{3/n}}{K - Ba^{3/n}}}{4 + 3\frac{Ba^{3/n}}{K - Ba^{3/n}}} \tag{4.22}$$

When $k > Ba^{3/n}$, then from (4.22) we see that $\dot{\varphi}^2 > 0$, (positive kinetic energy), therefore the K-essence scalar field is a Quintessence field. When $K < Ba^{3/n}$, then from (4.22) we see that $\dot{\varphi}^2 < 0$(negative kinetic energy), therefore the K-essence scalar field is a Phantom field.

4.5 Polytropic Tachyon model

The energy density and pressure for the Tachyon scalar field $\varphi$are (Das & Basak, 2018c)

$$\rho_t = \frac{V(\varphi)}{\sqrt{1-\dot{\varphi}^2}} \tag{4.23}$$

$$P_t = -V(\varphi)\sqrt{1-\dot{\varphi}^2} \tag{4.24}$$

The EOS parameter of the Tachyon field is given by

$$\omega_t = \frac{P_t}{\rho_t} = \dot{\varphi}^2 - 1 \tag{4.25}$$

If $\dot{\varphi}^2 < 0$, then (4.25), give $\omega_t < -1$, a Phantom field. Therefore Tachyon field shows that the Universe expands with acceleration in the context of Polytropic gas.

Using equation (4.5) in (4.25) and (4.3) in (4.23) we get

$$\omega_\varphi = -1 + \frac{Ba^{3/n}}{Ba^{3/n} - K} = \dot{\varphi}^2 - 1 \tag{4.26}$$

$$\rho_\Lambda = \left[Ba^{3/n} - K\right]^{-n} = \frac{V(\varphi)}{\sqrt{1-\dot{\varphi}^2}} \tag{4.27}$$

From the equations (4.26) and (4.27), we can find the dynamics and potential of the Tachyon field in the context of Polytropic gas dark energy model as follows

$$\dot{\varphi}^2 = \frac{Ba^{3/n}}{Ba^{3/n} - K} \tag{4.28}$$

$$V(\varphi) = \left[Ba^{3/n} - K\right]^{-n}\sqrt{1-\dot{\varphi}^2} \tag{4.29}$$

If $K > Ba^{3/n}$, then from the equation (4.28), we see that $\dot{\varphi}^2 < 0$ which represents a Phantom behavior Tachyon field and if $K < Ba^{3/n}$, then from the equation (4.29), we see that $\dot{\varphi}^2 > 0$, which represents a Quintessence behavior Tachyon field.

4.6 Conclusion

In this chapter, we have established a correspondence of the Polytropic gas with the Quintessence, K-essence and Tachyon scalar fields. The Polytropic quintessence model indicates a potential dominated scalar field Universe in the slow roll approximation that corresponds to the accelerated expansion of the Universe. The Polytropic K-essence model interprets that the Universe expand with acceleration expansion when $\frac{1}{3} < \chi < \frac{1}{2}$ and may be dominated by Phantom field or Quintessence field according as $K < Ba^{3/n}$ or $K > Ba^{3/n}$. The Polytropic Tachyon model represents that the Universe may be dominated by Phantom field or Quintessence field according as $K > Ba^{3/n}$ or $K < Ba^{3/n}$. Also from this correspondence, we have reformed the dynamics and potential of the Quintessence, K-essence and Tachyon scalar fields in the context of Polytropic gas dark energy model.