Chapter-3

Universe dominated by the Polytropic gas as dark energy

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3.1 Introduction

Several cosmological research and discoveries, reveals that with acceleration, the present Universe expands. A new energy of negative pressure, called dark energy (DE), is responsible for this expansion in the standard cosmology of Friedman Lemaitre Robertson Walker (FLRW) (Overduin & Cooperstok, 1998; Sahni & Starobinsky, 2009). The DE's behavior is still unclear and numerous theories in this field have been proposed by researchers. The cosmological constant is the first candidate for dark energy (Weinberg, 1989). There are several dark energy models with the time-dependent state equation, in addition to the cosmological constant, that have been proposed to describe cosmic acceleration. Polytropic gas is one of such models of dark energy that describes the Universe's cosmic acceleration (Mukhopadhyay et al., 2008; Karami et al., 2009; Christensen-Dalsgard, 2004; Das & Basak, 2018a).

3.2 Formulation of the topic

The Polytropic gas EOS (Karami et al., 2009; Das & Basak, 2018b) is

$$P_\Lambda = K\rho_\Lambda^{1+\frac{1}{n}} \tag{3.1}$$

The pressure, energy density, Polytropic constant and index are denoted by P_Λ,ρ_Λ,K, and n respectively (Setare & Darabi, 2013; Das & Basak, 2018b; Das & Singh, 2020a).

The equation of dark energy conservation is

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + P_\Lambda) = 0 \tag{3.2}$$

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Solving (3.1) & (3.2) and integrating we get

$$\rho_\Lambda = \left[-K + Ba^{3/n}\right]^{-n} \tag{3.3}$$

Here B is a positive constant of integration and a (t) is the Universe's time scale factor (Karami et al., 2009; Das & Singh, 2020a).

When $K < Ba^{3/n}$, we see that $\rho_\Lambda > 0$ for any arbitrary value of n; when $K > Ba^{3/n}$, we see that $\rho_\Lambda > 0$ for even value of n. Also when $K = Ba^{3/n}$, we see that $\rho_\Lambda \to \infty$ and so at $a_s = \left(\frac{K}{B}\right)^{n/3}$, Polytropic gas has the singularity of a finite time.

The Polytropic gas EOS parameter (Das & Basak, 2018b; Das & Singh, 2020a) is

$$\omega_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} = -1 + \frac{Ba^{3/n}}{Ba^{3/n} - K} \tag{3.4}$$

If $K > Ba^{3/n}$, then $\omega_\Lambda < -1$ and so the Universe might be dominated by the Phantom dark energy; if $K < Ba^{3/n}$, then we see that $\omega_\Lambda > -1$ which corresponds to a Quintessence like accelerated Universe; also when $K = Ba^{3/n}$, we see that $\omega_\Lambda \to \infty$ which corresponds to a singularity at $a_s = \left(\frac{K}{B}\right)^{n/3}$.

The energy density and pressure of the scalar field $\varphi(t)$ and potential $V(\varphi)$ are given by

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \tag{3.5}$$

$$P_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \tag{3.6}$$

Where $\frac{1}{2}\dot{\varphi}^2$ is the kinetic energy and $V(\varphi)$ is the potential energy of the scalar field φ

For the Polytropic gas, the scalar potential and kinetic energy terms are

$$V(\varphi) = \frac{\frac{B}{2}a^{3/n} - K}{\left(Ba^{3/n} - K\right)^{n+1}} \tag{3.7}$$

$$\dot{\varphi}^2 = \frac{Ba^{3/n}}{\left(Ba^{3/n} - K\right)^{n+1}} \tag{3.8}$$

When$K > Ba^{3/n}$, we see that $\dot{\varphi}^2 < 0$(negative kinetic energy), therefore the scalar field is a Phantom field. The Phantom field lead to super accelerated expansion of the Universe. When $K < Ba^{3/n}$, we see that, $\dot{\varphi}^2 > 0$(positive kinetic energy), therefore the scalar is a Quintessence field.

Using equations (3.5) & (3.6), the EOS parameter for the scalar fields is

$$\omega_\varphi = \frac{P_\varphi}{\rho_\varphi} = \frac{\frac{1}{2}\dot{\varphi}^2 - V(\varphi)}{\frac{1}{2}\dot{\varphi}^2 + V(\varphi)} = \frac{\dot{\varphi}^2 - 2V(\varphi)}{\dot{\varphi}^2 + 2V(\varphi)}$$

$$= -1 + \frac{2\dot{\varphi}^2}{\dot{\varphi}^2 + 2V(\varphi)} \tag{3.9}$$

When $\dot{\varphi}^2 = 0$, then equation (3.9) gives $\omega_\varphi = -1$, when $V(\varphi) = 0$, then equation (3.9) gives $\omega_\varphi = 1$. Here$\omega_\varphi = -1$and $\omega_\varphi = 1$ representing the vacuum fluid and stiff fluid dominated Universe respectively. When $V(\varphi) > \frac{1}{2}\dot{\varphi}^2$, then equation (3.9) gives $\omega_\varphi > -1$ which corresponds a Quintessence dominated Universe.

Considering negative kinetic energy in the equations (3.5) & (3.6), we get as follows

$$\rho_\varphi = -\frac{1}{2}\dot{\varphi}^2 + V(\varphi) \tag{3.10}$$

$$P_\varphi = -\frac{1}{2}\dot{\varphi}^2 - V(\varphi) \tag{3.11}$$

Using equations (3.10) & (3.11), the EOS of parameter for the scalar field is

$$\omega_\varphi = \frac{P_\varphi}{\rho_\varphi} = \frac{-\frac{1}{2}\dot{\varphi}^2 - V(\varphi)}{-\frac{1}{2}\dot{\varphi}^2 + V(\varphi)} = \frac{\dot{\varphi}^2 + 2V(\varphi)}{\dot{\varphi}^2 - 2V(\varphi)}$$

$$= -1 + \frac{2\dot{\varphi}^2}{\dot{\varphi}^2 - 2V(\varphi)} = -1 + \frac{\dot{\varphi}^2}{\frac{1}{2}\dot{\varphi}^2 - V(\varphi)} \tag{3.12}$$

When $\dot{\varphi}^2 = 0$, then equation (3.12) gives $\omega_\varphi = -1$; when $V(\varphi) = 0$, then equation (3.12) gives $\omega_\varphi = 1$,When $V(\varphi) > \frac{1}{2}\dot{\varphi}^2$, then equation (3.12) gives $\omega_\varphi < -1$ which corresponds a Phantom field. The Phantom field lead to super accelerated expansion of the Universe.

3.3 Conclusion

The Universe may be dominated by the Phantom dark energy or Quintessence dark energy according as $\omega_\Lambda \left(=\frac{p_\Lambda}{\rho_\Lambda}\right) \lesseqgtr -1$ in presence of Polytropic gas. Also for $V(\varphi) > \frac{1}{2}\dot{\varphi}^2$, the Universe may be dominated by Phantom dark energy or Quintessence dark energy according as negative kinetic energy$(-\frac{1}{2}\dot{\varphi}^2)$ or positive kinetic energy$(\frac{1}{2}\dot{\varphi}^2)$.