Chapter-8
Polytropic gas may avoid Big Rip singularity

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[8]

8.1 Introduction

Several cosmological studies and findings, implies that with acceleration, the current Universe is expanding. A new energy with negative pressure and positive energy density, called dark energy (DE), is responsible for this expansion in the standard cosmology of Friedman Lemaitre Robertson Walker (FLRW) (Overduin & Cooperstok, 1998; Sahni, & Starobinsky, 2009). The behavior of the DE is still uncertain and theorists in this area have suggested different theories. The DE comprises around 68.3 percent of the current energy of the Universe. The first and easiest candidate for dark energy is the cosmological constant with the time-independent state equation (Weinberg, 1989). The dark energy is usually described by an equation of state parameter $\omega = \frac{P}{\rho}$, where P is the pressure and ρ is the energy density (Corrol et al., 2003). For the different values ofω, different types of dark energy models are obtained. If $\omega < -1$ then it is phantom dark energy model and it is Quintessence for $-1 < \omega < -\frac{1}{3}$. When the parameter ω crosses the Phantom divide line $\omega = -1$ then it is termed as Quintom. Moreover from the observations of the Wilkinon Microwave Anisotropy Probe (WMAP) it is clear that the value of the equation of state parameter $\omega \equiv -1.10$ and it means that our Universe is dominated by Phantom energy ($\omega < -1$) (Caldwell, 2002; Caldwell et al., 2003; Frampton & Takahashi, 2003). The Phantom dominated Universe ends up with a finite time future singularity called Big Rip (Nojiri et al., 2005).

The existence of this future singularity is often considered as a negative feature of Phantom dark energy models and therefore the researchers are trying to construct

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various models with $\omega < -1$ to avoid this future singularity (Onemli & Woodard, 2002 & 2004; Srivastava, 2005; Jamil & Farooq, 2010). In this chapter, we consider a model where the dark energy behaves like a fluid with the equation of state parameter $P_\Lambda = \omega_\Lambda \rho_\Lambda$, $\omega_\Lambda < -1$ as well as Polytropic gas (Mukhopadhyay et al., 2008; Karami, 2009; Christensen-Dalsgard, 2004; Das & Basak, 2018a) to avoid the big rip singularity.

8.2 Solution of the Field equation

The Polytropic gas equation of state is given by (Das & Basak, 2019; Karami et al., 2009)

$$P_\Lambda = K\rho_\Lambda^{1+\frac{1}{n}} \tag{8.1}$$

The pressure, energy density, Polytropic constant and index are denoted by P_Λ,ρ_Λ,K, and n respectively (Das & Basak, 2019; Setare & Darabi, 2013).

We consider the line element for the spatially homogeneous flat FRW Universe

$$ds^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2] \tag{8.2}$$

Here x, y, z are spaces coordinate, t is the time component and $a(t)$ is the scale factor.

With this line element, the field equations can be written as

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho_\Lambda}{3} \tag{8.3}$$

$$\text{And } 2\dot{H} + 3H^2 = -P_\Lambda \tag{8.4}$$

The equation of dark energy conservation for the FRW Universe is

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + P_\Lambda) = 0 \tag{8.5}$$

Solving (8.1) & (8.2) and integrating we get

$$\rho_\Lambda = \left[Ba^{3/n} - K\right]^{-n} \tag{8.6}$$

Here B is a positive constant of integration and a(t) is the Universe's time scale factor (Das & Singh, 2020a).

And the pressure is

$$P_{\Lambda} = K\left[Ba^{3/n} - K\right]^{-n-1} \tag{8.7}$$

In this model it is assumed that the dark energy behaves like Polytropic gas obeying equation (8.1) as well as fluid with equation of state

$$P_{\Lambda} = \omega_{\Lambda}\rho_{\Lambda} \text{ With } \omega_{\Lambda} < -1 \tag{8.8}$$

The EOS parameter for the Polytropic gas is

$$\omega_{\Lambda} = \frac{P_{\Lambda}}{\rho_{\Lambda}} = -1 + \frac{Ba^{3/n}}{Ba^{3/n} - K}, \qquad K > Ba^{3/n} \tag{8.9}$$

At the present time $t = t_0$, the equation (8.9) gives

$$K = \frac{B\omega_0\, a_0^{\frac{n}{3}}}{\omega_0 + 1} \tag{8.10}$$

Where $a_0 = a(t_0),\ \ \omega_0 = \omega(t_0)$

From (8.6) and (8.10) we get

$$\rho_{\Lambda} = \left[Ba^{\frac{3}{n}} - \frac{B\omega_0\, a_0^{\frac{n}{3}}}{\omega_0 + 1}\right]^{-n} \tag{8.11}$$

The energy density and pressure of the scalar field $\varphi(t)$ and potential $V(\varphi)$ in the homogeneous Universe are given by

$$\rho_{\varphi} = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \tag{8.12}$$

$$p_{\varphi} = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \tag{8.13}$$

Here $\frac{1}{2}\dot{\varphi}^2$ is the kinetic energy and $V(\varphi)$ is the potential energy of the scalar field φ

From (8.12) & (8.13) we get

$$\dot{\varphi}^2 = \rho_{\varphi} + P_{\varphi} \tag{8.14}$$

Using (8.6), (8.7) and (8.10) in (8.14) we get

$$\dot{\varphi}^2 = \frac{Ba^{\frac{3}{n}}(\omega_0+1)^{n+1}}{\left[Ba^{\frac{3}{n}}\left\{(\omega_0+1) - \omega_0\left(\frac{a_0}{a}\right)^{\frac{3}{n}}\right\}\right]^{n+1}} \tag{8.15}$$

From the equation (8.15) we see that $\dot{\varphi}^2 > 0$ when $\omega_0 > -1$ and $\dot{\varphi}^2 < 0$ when $\omega_0 < -1$.Thus $\omega_0 > -1$ and $\omega_0 < -1$ represents a case of Quintessence and Phantom fluid dominated Universe respectively. Hence dual behavior of dark energy fluid obeying equations (8.1) and (8.8) is possible for scalars.

From the equations (8.3) and (8.11) we get

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\Omega_0 {H_0}^2}{2}\left[1 - \frac{\omega_0\left(\frac{a_0}{a}\right)^{\frac{3}{n}}}{1+\omega_0}\right]^{-n}$$

$$= \frac{\Omega_0 {H_0}^2}{2}\left[1 - \frac{|\omega_0|\left(\frac{a_0}{a}\right)^{\frac{3}{n}}}{|\omega_0|-1}\right]^{-n} \quad (8.16)$$

Here $|\omega_0| = -\omega_0$, H_0 is the present value of the Hubble constant and $\Omega_0 = \frac{\rho_0}{\rho_{cr,0}}$, with

$\rho_{cr,0} = \frac{3{H_0}^2}{8\pi G}$(G being the Newtonian gravitational constant)

Taking square root to the both sides of equation (8.16) we get

$$H = \frac{\dot{a}}{a} = \frac{\sqrt{\Omega_0}}{\sqrt{2}} H_0\left[1 - \frac{|\omega_0|\left(\frac{a_0}{a}\right)^{\frac{3}{n}}}{|\omega_0|-1}\right]^{-\frac{n}{2}} \quad (8.17)$$

Expanding right hand side of equation (8.17) and neglecting higher powers of $\frac{|\omega_0|\left(\frac{a_0}{a}\right)^{\frac{3}{n}}}{|\omega_0|-1}$ we get

$$H = \frac{\dot{a}}{a} = \frac{\sqrt{\Omega_0}}{\sqrt{2}} H_0\left[1 + \frac{n|\omega_0|\left(\frac{a_0}{a}\right)^{\frac{3}{n}}}{2(|\omega_0|-1)}\right] \quad (8.18)$$

Integrating (8.18) we get

$$a(t) = \frac{a_0}{\{2(|\omega_0|-1)\}^{\frac{n}{3}}}\left[\{n + 2(|\omega_0|-1)\}e^{\frac{\sqrt{\Omega_0}}{\sqrt{2}}H_0(t-t_0)}\right]^{\frac{3}{n}} \quad (8.19)$$

From (8.19) we see that $a(t) \to \infty$ as $t \to \infty$

For this situation, the Hubble distance is

$$H^{-1} = \frac{\sqrt{2}}{\sqrt{\Omega_0} H_0}\left[1 - \frac{n|\omega_0|\left(\frac{a_0}{a}\right)^{\frac{3}{n}}}{2(|\omega_0|-1)}\right] \tag{8.20}$$

Equation (8.20) shows the growth of Hubble's distance $H^{-1} \rightarrow \frac{\sqrt{2}}{\sqrt{\Omega_0} H_0} \neq 0$ as $t \rightarrow \infty$

It means that the galaxies will not disappear when $t \rightarrow \infty$, avoiding Big Rip singularities.

8.3 Conclusion

In the FRW Universe, we have found that if the dark energy acts concurrently as a fluid with $P_\Lambda = \omega_\Lambda \rho_\Lambda$, $\omega_\Lambda < -1$ and Polytropic gas with $P_\Lambda = K\rho_\Lambda^{1+\frac{1}{n}}$, then the ultimate explanations are as follows

i) For the positive kinetic energy i.e. $\dot{\varphi}^2 > 0$ we have $\omega_0 > -1$ which represents a Quintessence fluid dominated Universe and for the negative kinetic energy i.e. $\dot{\varphi}^2 < 0$ we have $\omega_0 < -1$ which represents a Phantom fluid dominated Universe. This results match with results obtained by Hoyle-Narlikar in C-field with negative kinetic energy for steady state theory of Universe.

ii) For $t \rightarrow \infty$, we have $a(t) \rightarrow \infty$, this indicates that the present model is free from finite time future singularity.

iii) For $t \rightarrow \infty$ we have $H^{-1} \neq 0$, this indicates that the galaxies will not disappear as $t \rightarrow \infty$, avoiding Big Rip singularities. Therefore one can conclude that when cosmic dark energy behaves like a fluid with equation of state parameter $P_\Lambda = \omega_\Lambda \rho_\Lambda$, $\omega_\Lambda < -1$ as well as Polytropic gas then the Big Rip singularity does not arise and the scale factor is found to be regular for all time.