CHAPTER 4

CHAPTER 4

NEUTROSOPHIC ALMOST BITOPOLOGICAL GROUP

In this chapter, to study the NABTG, the definitions of the NSOS, NSCoS, NROS, and NRCoS are introduced and some of their properties are proved. Here, in this chapter i, j means for each $i = j = 1, 2$.

Definition 4.0.1
Let $\mathcal{A}$ be a NS of a NBTS $(X, \daleth_i^X, \daleth_j^X)$, then $\mathcal{A}$ is called a NSOS of X if $\exists$ a $\mathcal{B} \in (X, \daleth_i^X, \daleth_j^X)$ such that $\mathcal{A} \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{B})$.

Definition 4.0.2
Let $\mathcal{A}$ be a NS of a NBTS $(X, \daleth_i^X, \daleth_j^X)$, then $\mathcal{A}$ is called a NSCoS of X if $\exists$ a $\mathcal{B}^c \in (X, \daleth_i^X, \daleth_j^X)$ such that $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{B}) \subseteq \mathcal{A}$.

Definition 4.0.3
Let $\mathcal{A}$ be a NS of a NBTS $(X, \daleth_i^X, \daleth_j^X)$ is said to be a NROS, if $(\daleth_i{}^{\mathcal{G}}, \daleth_j{}^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i{}^{\mathcal{G}}, \daleth_j{}^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})\Big) = \mathcal{A}$, for each $i = j = 1, 2$.

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Definition 4.0.4

Let $\mathcal{A}$ be a NS of a NBTS $(X, \daleth_i^X, \daleth_j^X)$ is said to be a NRCoS, if $({\daleth_i}^{\mathcal{G}}, {\daleth_j}^{\mathcal{G}})\mathcal{N} \sim Cl\Big(({\daleth_i}^{\mathcal{G}}, {\daleth_j}^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})\Big) = \mathcal{A}$.

Definition 4.0.5

A mapping $\phi : (X, {\daleth_i}^X, {\daleth_j}^X) \to (Y, {\daleth_i}^Y, {\daleth_j}^Y)$ is said to be a NACM, if $\phi^{-1}(\mathcal{A}) \in (X, {\daleth_i}^X, {\daleth_j}^X)$ for each NROS $\mathcal{A}$ of $(Y, {\daleth_i}^Y, {\daleth_j}^Y)$.

Definition 4.0.6

Let $\mathcal{G}$ be a NG on a group X. Let $\daleth_i^{\mathcal{G}}$ be a NT on $\mathcal{G}$, then $(\mathcal{G}, {\daleth_1}^{\mathcal{G}}, {\daleth_2}^{\mathcal{G}})$ is said to be a NABTG if the following conditions are satisfied:

(i) A mapping $\lambda : (\mathcal{G}, \daleth_i^{\mathcal{G}}) \times (\mathcal{G}, \daleth_i^{\mathcal{G}}) \to (\mathcal{G}, \daleth_i^{\mathcal{G}}) : \lambda(x, y) = xy$ is neutrosophic almost i-continuous mapping.

(ii) A mapping $\mu : (\mathcal{G}, \daleth_i^{\mathcal{G}}) \to (\mathcal{G}, \daleth_i^{\mathcal{G}}) : \mu(x) = x^{-1}$ is neutrosophic almost i-continuous mapping.

Remark 4.0.1

$(\mathcal{G}, \daleth_i^{\mathcal{G}})$ is a NABTG, if following conditions hold good:

(a) for $g_1, g_2 \in \mathcal{G}$ and for every $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NROS $\mathcal{U}$ containing g_1g_2 in $\mathcal{G}$, $\exists\ \daleth_i^{\mathcal{G}}-$ neutrosophic open nbhds $\mathcal{P}$ and $\mathcal{Q}$ of g_1 and g_2 respectively in $\mathcal{G}$ so that $\mathcal{P} * \mathcal{Q} \subseteq \mathcal{U}$ and

(b) for $g \in \mathcal{G}$ and every $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NROS $\mathcal{Q}$ in $\mathcal{G}$ containing g^{-1}, $\exists\ \daleth_i^{\mathcal{G}}-$ neutrosophic open nbhd $\mathcal{P}$ of g in $\mathcal{G}$ such that $\mathcal{P}^{-1} \subseteq \mathcal{Q}$.

Remark 4.0.2

For any $\mathcal{U}, \mathcal{V} \subseteq \mathcal{G}$, we denote $\mathcal{U} * \mathcal{V}$ by $\mathcal{U}\mathcal{V}$ and defined as $\mathcal{U}\mathcal{V} = \{gh : g \in \mathcal{U}, h \in \mathcal{V}\}$ and $\mathcal{U}^{-1} = \{g^{-1} : g \in \mathcal{U}\}$. If $\mathcal{U} = \{a\}$ for each $a \in \mathcal{G}$, we denote $\mathcal{U} * \mathcal{V}$ by $a\mathcal{V}$ and $\mathcal{V} * \mathcal{U}$ by $\mathcal{U}a$.

Theorem 4.0.1

Let $(\mathcal{G}, {\daleth_i}^{\mathcal{G}})$ be a NABTG and let $a \in \mathcal{G}$ be any element of $\mathcal{G}$. Then *(i)* $\pi_a : (\mathcal{G}, \daleth_i^{\mathcal{G}}) \to (\mathcal{G}, \daleth_i^{\mathcal{G}}) : \pi_a(x) = ax$, *for all* $x \in \mathcal{G}$, *is neutrosophic almost* i*-continuous mapping for each* $i = 1, 2$.

(ii) $\sigma_a : (\mathcal{G}, \daleth_i^{\mathcal{G}}) \to (\mathcal{G}, \daleth_i^{\mathcal{G}}) : \sigma_a(x) = xa$, *for all* $x \in \mathcal{G}$, *is neutrosophic almost* i*-continuous mapping for each* $i = 1, 2$.

Proof

(i) Let $p \in \mathcal{G}$ and let $\mathcal{W}$ be a $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NROS, $i = j = 1, 2$; containing ap in $\mathcal{G}$. By definition 4.0.6, $\exists\ \daleth_i^{\mathcal{G}}-$ neutrosophic open, $i = 1, 2$ nbhds $\mathcal{U}, \mathcal{V}$ of a, p in $\mathcal{G}$ so that $\mathcal{U}\mathcal{V} \subseteq \mathcal{W}$. Especially, $a\mathcal{V} \subseteq \mathcal{W}$ that is $\pi_a(\mathcal{V}) \subseteq \mathcal{W}$. This shows that π_a is NACM at p and therefore π_a is NACM.

(ii) Suppose $p \in \mathcal{G}$ and $\mathcal{W} \in (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NROS$(\mathcal{G})$ containing pa. Then $\exists\ \daleth_i^{\mathcal{G}}-$NOSs, $i = 1, 2$; $p \in \mathcal{U}$ and $a \in \mathcal{V}$ in $\mathcal{G}$ so that $\mathcal{U}\mathcal{V} \subseteq \mathcal{W}$. This shows $\mathcal{U}_a \subseteq \mathcal{W}$. This implies σ_a is NACM at p. As arbitrary element p is in $\mathcal{G}, \sigma_a$ is NACM.

Theorem 4.0.2
Let $\mathcal{U}$ *be* $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$*NROS for each* $i = j = 1, 2$ *in a NABTG* $(\mathcal{G}, \daleth_i^{\mathcal{G}})$. *Then the following conditions hold good:*

(a) $a\mathcal{U} \in (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$*NROS*$(\mathcal{G})$, *for all* $a \in \mathcal{G}$.

(b) $\mathcal{U}a \in (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$*NROS*$(\mathcal{G})$, *for all* $a \in \mathcal{G}$.

(c) $\mathcal{U}^{-1} \in (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$*NROS*$(\mathcal{G})$.

Proof

(a) First, we have to prove that $a\mathcal{U} \in \daleth_i^{\mathcal{G}}$, $i = 1, 2$. Let $p \in a\mathcal{U}$. Then from definition 4.0.6 of NABTGs, $\exists\ \daleth_i^{\mathcal{G}}-$NOSs, $i = 1, 2$; $a^{-1} \in \mathcal{W}_1$ and $p \in \mathcal{W}_2$ in $\mathcal{G}$ so that $\mathcal{W}_1\mathcal{W}_2 \subseteq \mathcal{U}$. Especially, $a^{-1}\mathcal{W}_2 \subseteq \mathcal{U}$. i.e., equivalently $\mathcal{W}_2 \subseteq a\mathcal{U}$. This indicates that $p \in$

$(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(a\mathcal{U})$ and thus, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(a\mathcal{U}) = a\mathcal{U}$. i.e., $a\mathcal{U} \in \daleth_i^{\mathcal{G}}, i = 1, 2$. Consequently, $a\mathcal{U}(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big\{(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(a\mathcal{U})\Big\}$.

Now, we have to prove that $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big\{(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(a\mathcal{U})\Big\} \subseteq a\mathcal{U}$. Since $\mathcal{U}$ is $\daleth_i^{\mathcal{G}}-$NOS, $i = 1, 2$; $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{U}) \in (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NRCS$(\mathcal{G})$. From theorem 4.0.1, $\pi_{a^{-1}} : (\mathcal{G}, \daleth_i^{\mathcal{G}}) \to (\mathcal{G}, \daleth_i^{\mathcal{G}})$ is NACM, $i = 1, 2$ and therefore, $a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}}) \mathcal{N} \sim Cl(\mathcal{U})$ is $\daleth_i^{\mathcal{G}}-$ NCoS, $i = 1, 2$. Thus, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(a\mathcal{U})\Big) \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(a\mathcal{U}) \subseteq a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{U})$. i.e., $a^{-1}(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}}) \mathcal{N} \sim Cl(a\mathcal{U})\Big) \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{U})$. Since $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\ \mathcal{N} \sim Cl(a\mathcal{U})\Big)$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NROS, $i = j = 1, 2$, it follows that $a^{-1}(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\ \mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\ \mathcal{N} \sim Cl\ (a\mathcal{U})\Big) \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{U})\Big) = \mathcal{U}$, i.e., $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(a\mathcal{U})\Big) \subseteq a\mathcal{U}$. Thus, $a\mathcal{U} = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(a\mathcal{U})\Big)$. This shows that $a\mathcal{U} \in (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NROS$(\mathcal{G})$.

(b) Following Theorem 4.0.2 (a), the proof is straightforward.

(c) Let $x \in \mathcal{U}^{-1}$, then $\exists\ \daleth_i^{\mathcal{G}}-$NOS, $i = 1, 2; p \in \mathcal{W}$ in $\mathcal{G}$ so that $\mathcal{W}^{-1} \subseteq \mathcal{U} \Rightarrow \mathcal{W} \subseteq \mathcal{U}^{-1}$. Therefore $\mathcal{U}^{-1}$ has interior point p. Thus, $\mathcal{U}^{-1}$ is $\daleth_i^{\mathcal{G}}-$NOS, $i = 1, 2$. i.e., $\mathcal{U}^{-1} \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}}) \mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{U}^{-1})\Big)$. Now, we have to prove that $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\ \mathcal{N} \sim Cl(\mathcal{U}^{-1})\Big) \subseteq \mathcal{U}^{-1}$. Since $\mathcal{U}$ is $\daleth_i^{\mathcal{G}}-$NOS, $i = 1, 2$, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{U})$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NRCoS, $i = j = 1, 2$ and hence $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\ \mathcal{N} \sim Cl(\mathcal{U})^{-1}$ is $\daleth_i^{\mathcal{G}}-$NCoS, $i = 1, 2$ in $\mathcal{G}$. Therefore, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\ \mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}},$

$\urcorner_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{U}^{-1})\Big) \subseteq (\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})\, \mathcal{N} \sim Cl(\mathcal{U}^{-1}) \subseteq (\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})$ $\mathcal{N} \sim Cl(\mathcal{U})^{-1} \Rightarrow (\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})\, \mathcal{N} \sim Int\Big((\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{U}^{-1})\Big)$ $\subseteq \Big((\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{U})\Big)^{-1} \subseteq \mathcal{U}^{-1}$. Thus, $\mathcal{U}^{-1} = (\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})$ $\mathcal{N} \sim Int\Big((\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})\, \mathcal{N} \sim Cl(\mathcal{U}^{-1})\Big)$. This shows that $\mathcal{U}^{-1} \in$ $(\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})-$NROS $(\mathcal{G})$.

Corollary 4.0.1

Let $\mathcal{Q}$ be any $(\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})-$NRCoS in a NABTG in $\mathcal{G}, i = j = 1, 2$. Then

(i) $a\mathcal{Q} \in (\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})-$NRCS$(\mathcal{G})$, for each $a \in \mathcal{G}$.

(ii) $\mathcal{Q}^{-1} \in (\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})-$NRCS$(\mathcal{G})$.

Theorem 4.0.3

Let $\mathcal{U}$ be any $(\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})-$NROS, $i = j = 1, 2$ in a NABTG $\mathcal{G}$. Then

(a) $(\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{U}a) = (\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{U})a$, for each $a \in \mathcal{G}$.

(b) $(\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})\mathcal{N} \sim Cl(a\mathcal{U}) = a(\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{U})$, for each $a \in \mathcal{G}$.

(c) $(\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{U}^{-1}) = (\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{U})^{-1}$.

Proof

(a) Taking $p \in (\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{U}a)$ and consider $q = pa^{-1}$. Let $q \in \mathcal{W}$ be $\urcorner_i^{\mathcal{G}}-$NOS, $i = 1, 2$ in $\mathcal{G}$. Then $\exists\ \urcorner_i^{\mathcal{G}}-$NOSs, $i = 1, 2; a^{-1} \in \mathcal{V}_1$ and $p \in \mathcal{V}_2$ in $\mathcal{G}$, so that $\mathcal{V}_1\mathcal{V}_2 \subseteq (\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})\, \mathcal{N} \sim Cl(\mathcal{W})\Big)$. By assumption, there is $g \in \mathcal{U}a \bigcap \mathcal{V}_2 \Rightarrow ga^{-1} \in \mathcal{U} \bigcap \mathcal{V}_1\mathcal{V}_2 \subseteq \mathcal{U} \bigcap (\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})\, \mathcal{N} \sim Int\Big((\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{W})\Big) \Rightarrow \mathcal{U} \bigcap (\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})\, \mathcal{N} \sim Cl(\mathcal{W})\Big) \neq 0_N \Rightarrow \mathcal{U} \bigcap \Big((\urcorner_i^{\mathcal{G}}, \urcorner_j^{\mathcal{G}})\, \mathcal{N} \sim Cl(\mathcal{W})\Big) \neq 0_N$. Since

$\mathcal{U}$ is $\daleth_i{}^{\mathcal{G}}-$NOS, $i=1,2, \mathcal{U}\bigcap\mathcal{W}\neq 0_N$. i.e., $p\in(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\mathcal{N}\sim Cl(\mathcal{U})a$.

Conversely, let $q\in(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\mathcal{N}\sim Cl(\mathcal{U})a$. Then $q=pg$, for some $p\in(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\mathcal{N}\sim Cl(\mathcal{U})$. To prove $(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\ \mathcal{N}\sim Cl(\mathcal{U})a\subseteq(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\mathcal{N}\sim Cl(\mathcal{U}a)$. Let $pg\in\mathcal{W}$ be an $\daleth_i{}^{\mathcal{G}}-$NOS, $i=1,2$ in $\mathcal{G}$. Then $\exists\ \daleth_i{}^{\mathcal{G}}-$NOSs, $i=1,2; a\in\mathcal{V}_1$ in $\mathcal{G}$ and $p\in\mathcal{V}_2$ in $\mathcal{G}$ so that $\mathcal{V}_1\mathcal{V}_2\subseteq(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\ \mathcal{N}\sim Int\Big((\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\ \mathcal{N}\sim Cl(\mathcal{W})\Big)$. Since $p\in(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\ \mathcal{N}\sim Cl(\mathcal{U}), \mathcal{U}\bigcap\mathcal{V}_2\neq 0_N$. There is $g\in\mathcal{U}\bigcap\mathcal{V}_2$. This gives $ga\in(\mathcal{U}a)\bigcap(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\mathcal{N}\sim Int\Big((\daleth_i{}^{\mathcal{G}},\ \daleth_j{}^{\mathcal{G}})\mathcal{N}\sim Cl(\mathcal{W})\Big)\Rightarrow(\mathcal{U}a)\bigcap\Big((\daleth_i{}^{\mathcal{G}},\ \daleth_j{}^{\mathcal{G}})\ \mathcal{N}\sim Cl(\mathcal{W})\Big)\neq 0_N$. From Theorem 4.0.2, $\mathcal{U}a$ is $\daleth_i{}^{\mathcal{G}}-$ NOS, $i=1,2$ and thus $(\mathcal{U}a)\bigcap\mathcal{W}\neq 0_N$, therefore, $q\in(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\ \mathcal{N}\sim Cl(\mathcal{U}a)$. Therefore, $(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\ \mathcal{N}\sim Cl(\mathcal{U}a)=(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\mathcal{N}\sim Cl(\mathcal{U})a$.

(b) Following Theorem 4.0.3 (a), the proof is straightforward, therefore $(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\ \mathcal{N}\sim Cl(a\mathcal{U})=a(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\mathcal{N}\sim Cl(\mathcal{U})$.

(c) Since $(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\mathcal{N}\sim Cl(\mathcal{U})$ is $(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})-$NRCoS, $i=j=1,2$; $(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\mathcal{N}\sim Cl(\mathcal{U})^{-1}$ is $\daleth_i{}^{\mathcal{G}}-$NCoS, $i=1,2$ in $\mathcal{G}$. So, $\mathcal{U}^{-1}\subseteq(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\mathcal{N}\sim Cl(\mathcal{U})^{-1}$ this implies $(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\mathcal{N}\sim Cl(\mathcal{U}^{-1})\subseteq(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\mathcal{N}\sim Cl(\mathcal{U})^{-1}$. Next, let $q\in(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\mathcal{N}\sim Cl(\mathcal{U})^{-1}$. Then $q=p^{-1}$, for some $p\in(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\ \mathcal{N}\sim Cl(\mathcal{U})$. Let $q\in\mathcal{V}$ be any $\daleth_i{}^{\mathcal{G}}-$NOS, $i=1,2$ in $\mathcal{G}$. Then $\exists\ \daleth_i{}^{\mathcal{G}}-$NOS, $i=1,2;\mathcal{U}$ in $\mathcal{G}$ so that $p\in\mathcal{U}$ with $\mathcal{U}^{-1}\subseteq(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\ \mathcal{N}\sim Int\Big((\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\ \mathcal{N}\sim Cl(\mathcal{V})\Big)$. Also, there is $a\in\mathcal{A}\bigcap\mathcal{U}$ which implies $a^{-1}\in\mathcal{A}^{-1}\bigcap(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\ \mathcal{N}\sim Int\Big((\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\ \mathcal{N}\sim Cl(\mathcal{V})\Big)$. That is, $\mathcal{A}^{-1}\bigcap(\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\ \mathcal{N}\sim Int\Big((\daleth_i{}^{\mathcal{G}},\daleth_j{}^{\mathcal{G}})\ \mathcal{N}\sim Cl(\mathcal{V})\Big)\neq 0_N\Rightarrow\mathcal{U}^{-1}\bigcap(\daleth_i{}^{\mathcal{G}},\ \daleth_j{}^{\mathcal{G}})\mathcal{N}\sim Cl(\mathcal{V})\neq 0_N\Rightarrow\mathcal{A}^{-1}$

$\bigcap \mathcal{V} \neq 0_N$, since $\mathcal{U}^{-1}$ is $\daleth_i^{\mathcal{G}}-$NOS, $i = 1, 2$. Therefore, $q \in (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{U})^{-1}$. Hence $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{U}^{-1}) \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\, \mathcal{N} \sim Cl(\mathcal{U})^{-1}$.

Theorem 4.0.4

Let $\mathcal{Q}$ be $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$ neutrosophic regularly closed subset in a NABTG $\mathcal{G}, i = j = 1, 2$. Then the following statements are satisfied:

(a) $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(a\mathcal{Q}) = a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{Q})$, *for all* $a \in \mathcal{G}$.

(b) $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{Q}a) = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{Q})a$, *for all* $a \in \mathcal{G}$.

(c) $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{Q}^{-1}) = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{Q})^{-1}$.

Proof

(a) Since $\mathcal{Q}$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NRCoS, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\, \mathcal{N} \sim Int(\mathcal{Q})$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NROS in $\mathcal{G}, i = j = 1, 2$. Consequently, $a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{Q}) \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\, \mathcal{N} \sim Int(a\mathcal{Q})$. Conversely, let q be an arbitrary element of $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\, \mathcal{N} \sim Int(a\mathcal{Q})$. Assume that $q = ap$, for some $p \in \mathcal{Q}$. By assumption, this shows $a\mathcal{Q}$ is $\daleth_i^{\mathbb{G}}-$NCoS, $i = 1, 2$ and that is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(a\mathcal{Q})$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NROS in $\mathcal{G}, i = j = 1, 2$. Suppose $a \in \mathcal{U}$ and $p \in \mathcal{V}$ be $\daleth_i^{\mathcal{G}}-$NOSs, $i = 1, 2$ in $\mathcal{G}$, so that $\mathcal{U}\mathcal{V} \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\, \mathcal{N} \sim Int(a\mathcal{Q})$. Then $a\mathcal{V} \subseteq a\mathcal{Q}$, which follows that $a\mathcal{V} \subseteq a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{Q})$. Thus, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\, \mathcal{N} \sim Int(a\mathcal{Q}) \subseteq a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{Q})$. Hence the statement follows.

(b) Following Theorem 4.0.4 (a), the proof is straightforward, therefore $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{Q}a) \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{Q})a$.

(c)) Since $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{Q})$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NROS, $i = j = 1, 2$; so, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{Q})^{-1}$ is $\daleth_i^{\mathcal{G}}-$NOS in $\mathcal{G}$, $i = 1, 2$. Therefore, $\mathcal{Q}^{-1} \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{Q})^{-1}$ implies that $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{Q}^{-1}) \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\ \mathcal{N} \sim Int(\mathcal{Q})^{-1}$. Next, let q be an arbitrary element of $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\ \mathcal{N} \sim Int(\mathcal{Q})^{-1}$. Then $q = p^{-1}$, for some $p \in (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\ \mathcal{N} \sim Int(\mathcal{Q})$. Let $q \in \mathcal{V}$ be $\daleth_i^{\mathcal{G}}-$NOS, $i = 1, 2$ in $\mathcal{G}$. Then $\exists$ $\daleth_i^{\mathcal{G}}-$NOS, $i = 1, 2; \mathcal{U}$ is in $\mathcal{G}$ so that $p \in \mathcal{U}$ with $\mathcal{U}^{-1} \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\ \mathcal{N} \sim Cl\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\ \mathcal{N} \sim Int(\mathcal{V})\Big)$. Also, there is $g \in \mathcal{Q} \bigcap \mathcal{U}$ which implies $g^{-1} \in \mathcal{Q}^{-1} \bigcap (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\ \mathcal{N} \sim Cl\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\ \mathcal{N} \sim Int(\mathcal{V})\Big)$. That is $\mathcal{Q}^{-1} \bigcap (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\ \mathcal{N} \sim Cl\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\ \mathcal{N} \sim Int(\mathcal{V})\Big) \neq 0_N$, since $\mathcal{Q}^{-1}$ is $\daleth_i^{\mathcal{G}}-$NCoS, $i = 1, 2$. Hence $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\ \mathcal{N} \sim Int(\mathcal{Q}^{-1}) = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{Q})^{-1}$.

Theorem 4.0.5

Let $\mathcal{A}$ be any $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NSOS in a NABTG $\mathcal{G}, i = j = 1, 2$. Then

(a) $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(a\mathcal{A}) \subseteq a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\ \mathcal{N} \sim Cl(\mathcal{A})$*, for all* $a \in \mathcal{G}$.

(b) $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A}a) \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\ \mathcal{N} \sim Cl(\mathcal{A})a$*, for all* $a \in \mathcal{G}$.

(c) $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A}^{-1}) \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\ \mathcal{N} \sim Cl(\mathcal{A})^{-1}$.

Proof

(a) As $\mathcal{A}$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NSOS; $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\ \mathcal{N} \sim Cl(\mathcal{A})$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NRCoS, $i = j = 1, 2$. From Theorem 4.0.1, $\pi_{a^{-1}} : (\mathcal{G}, \daleth_i^{\mathcal{G}}) \to (\mathcal{G}, \daleth_i^{\mathcal{G}})$ is NACM, for each $i = 1, 2$. So, $a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})$ is $\daleth_i^{\mathcal{G}}-$NCoS, $i = 1, 2$. Hence $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(a\mathcal{A}) \subseteq a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})$.

(b) As $\mathcal{A}$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NSOS; $i = j = 1, 2$; $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\,\mathcal{N} \sim Cl(\mathcal{A})$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NRCoS, $i = j = 1, 2$. From Theorem 4.0.1, $\sigma_{a^{-1}} : (\mathcal{G}, \daleth_i^{\mathcal{G}}) \to (\mathcal{G}, \daleth_i^{\mathcal{G}})$ is NACM, for each $i = 1, 2$. So, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})a$ is $\daleth_i^{\mathcal{G}}-$NCoS, $i = 1, 2$. Thus, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A}a) \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})a$.

(c) Since $\mathcal{A}$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NSOS, $i = j = 1, 2$; so $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NRCoS, $i = j = 1, 2$ and hence $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})^{-1}$ is $\daleth_i^{\mathcal{G}}-$NCoS, $i = 1, 2$. Consequently, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A}) \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})^{-1}$.

Theorem 4.0.6

Let $\mathcal{A}$ be both $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NSOS and $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NSCoS subset of a NABTG, $i = j = 1, 2$. Then the following statements hold:

(a) $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(a\mathcal{A}) = a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})$*, for each* $a \in \mathcal{G}$.

(b) $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A}a) = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})a$*, for each* $a \in \mathcal{G}$.

(c) $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A}^{-1}) = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})^{-1}$.

Proof

(a) Since $\mathcal{A}$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NSOS, $i = j = 1, 2$; $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NRCoS, from which it follows that $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(a\mathcal{A}) \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(a\mathcal{A})$. Further, $\daleth_i^{\mathcal{G}}-$neutrosophic semi-openness of $\mathcal{A}$, $i = 1, 2$ implies $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A}) = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})\Big) \Rightarrow (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A}) = a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})\Big)$. As $\mathcal{A}$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NSCoS; $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NROS in $\mathcal{G}$. From Theorem 4.0.5, $a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim$

$Cl(\mathcal{A}) = a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})\Big) = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl\Big(a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A}\Big) \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(a\mathcal{A})$. Hence $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(a\mathcal{A}) = a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})$.

(b) Following Theorem 4.0.6 (a), the proof is straightforward.

(c) By assumption, this shows $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NRCoS, $i = j = 1, 2$ and therefore $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})^{-1}$ is $\daleth_i^{\mathcal{G}}-$NCoS, $i = 1, 2$. Consequently, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A}^{-1}) \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})^{-1}$. Next, as $\mathcal{A}$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NSOS; $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A}) = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})\Big) \Rightarrow (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})^{-1} = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})\Big)$. Also, as $\mathcal{A}$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NSCoS, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$ NROS, $i = j = 1, 2$. From Theorem 4.0.3, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})^{-1} = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})^{-1}\Big) \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A}^{-1})$. This shows that $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A}^{-1}) = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})^{-1}$.

Corollary 4.0.2

From Theorem 4.0.6, we have the following corollaries:

(a) $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(a\mathcal{A}) = a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})$*, for each* $a \in \mathcal{G}$.

(b) $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A}a) = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})a$*, for each* $a \in \mathcal{G}$.

(c) $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A}^{-1}) = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})^{-1}$.

Proof

(a) As $\mathcal{A}$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NSCoS, $i = j = 1, 2$; $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NROS, $i = j = 1, 2$. From Theorem 4.0.1, $\pi_{a^{-1}}$: $(\mathcal{G}, \daleth_i^{\mathcal{G}} \to (\mathcal{G}, \daleth_i^{\mathcal{G}})$ is NACM, for each $i = 1, 2$. So, $\pi^{-1}{}_{a^{-1}}\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})\Big) = a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})$ is $\daleth_i^{\mathcal{G}}-$NOS, $i = 1, 2$. Thus, $a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A}) \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(a\mathcal{A})$. Next, by hypothesis, it follows that $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A}) = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})\Big) \Rightarrow a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\,(\mathcal{A}) = a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})\Big)$. As $\mathcal{A}$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NSOS, $i = j = 1, 2$; $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(A)$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NRCoS, $i = j = 1, 2$. From Theorem 4.0.4, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})\Big) = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big(a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})\Big) \supseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(a\mathcal{A})$. That is, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(a\mathcal{A}) \subseteq a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})$. Therefore, we have, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(a\mathcal{A}) = a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})$. Hence proved.

(b) As $\mathcal{A}$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NSCoS, $i = j = 1, 2$; $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NROS, $i = j = 1, 2$. From Theorem 4.0.1, $\pi_{a^{-1}}$: $(\mathcal{G}, \daleth_i^{\mathcal{G}}) \to (\mathcal{G}, \daleth_i^{\mathcal{G}})$ is NACM, for each $i = 1, 2$. So, $\sigma^{-1}{}_{a^{-1}}\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})\Big) = a\mathcal{N} \sim Int(\mathcal{A})$ is $\daleth_i^{\mathcal{G}}-$NOS, $i = 1, 2$. Thus, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})a \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A}a)$. Next, by hypothesis, this shows that $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A}) = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A}\Big) \Rightarrow (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})a = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})\Big)a$. Since $\mathcal{A}$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NSOS, $i = j = 1, 2$; $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NRCoS, $i = j = 1, 2$. From Theorem 4.0.4, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})\Big)a = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})a\Big) \supseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim$

$Int(\mathcal{A}a)$. That is, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A}a) \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})a$. Therefore, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A}a) = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})a$. Hence proved.

(c) From hypothesis, this shows that $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NROS, $i = j = 1, 2$ and therefore $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})^{-1}$ is $\daleth_i^{\mathcal{G}}-$NOS, $i = 1, 2$. Consequently, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A}^{-1}) \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})^{-1}$. Next, as $\mathcal{A}$ is $(\daleth_i^{\mathcal{G}}\ \daleth_j^{\mathcal{G}})-$NSCoS, $i = j = 1, 2$; $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A}) = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})\Big) \Rightarrow (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})^{-1} = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})\Big)^{-1}$. Also, as $\mathcal{A}$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NSOS, $i = j = 1, 2$; $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})$ is $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})-$NRCoS, $i = j = 1, 2$. From Theorem 4.0.4, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})^{-1} = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})^{-1}\Big) \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A}^{-1})$. This proves that $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A}^{-1}) = (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})^{-1}$.

Theorem 4.0.7

Let $\mathcal{A}$ be $\daleth_i^{\mathcal{G}}-$NOS in a NABTG $\mathcal{G}, i = 1, 2$. Then $a\mathcal{A} \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big(a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})\Big)\Big)$ for $a \in \mathcal{G}$.

Proof

Since $\mathcal{A}$ is $\daleth_i^{\mathcal{G}}-$NOS, $i = 1, 2$, so $\mathcal{A} \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})\Big) \Rightarrow a\mathcal{A} \subseteq a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})\Big)$. From Theorem 4.0.2, $a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})\Big)$ is $\daleth_i^{\mathcal{G}}-$NOS, $i = 1, 2$ $\Big($in fact, $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}}) -$ NROS, $i = j = 1, 2\Big)$. Hence $a\mathcal{A} \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big(a(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Int\Big((\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})\mathcal{N} \sim Cl(\mathcal{A})\Big)\Big)$.

Theorem 4.0.8

Let $\mathcal{Q}$ be any $(\daleth_i{}^{\mathcal{G}}, \daleth_j{}^{\mathcal{G}})-$ Neutrosophic closed subset in a NABTG $\mathcal{G}, i = j = 1, 2$. Then $(\daleth_i{}^{\mathcal{G}}, \daleth_j{}^{\mathcal{G}})\mathcal{N} \sim Cl\Big(a(\daleth_i{}^{\mathcal{G}}, \daleth_j{}^{\mathcal{G}})\,\mathcal{N} \sim Cl\Big((\daleth_i{}^{\mathcal{G}}, \daleth_j{}^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})\Big)\Big) \subseteq a\mathcal{Q}$ for each $a \in \mathcal{G}$.

Proof

Since $\mathcal{Q}$ is $\daleth_i{}^{\mathcal{G}}$-NCoS, $i = 1, 2$, so $\mathcal{Q} \supseteq (\daleth_i{}^{\mathcal{G}}, \daleth_j{}^{\mathcal{G}})\mathcal{N} \sim Cl\Big((\daleth_i{}^{\mathcal{G}}, \daleth_j{}^{\mathcal{G}})\ \mathcal{N} \sim Int(\mathcal{Q})\Big) \Rightarrow a\mathcal{Q} \supseteq a(\daleth_i{}^{\mathcal{G}}, \daleth_j{}^{\mathcal{G}})\ \mathcal{N} \sim Cl\Big((\daleth_i{}^{\mathcal{G}}, \daleth_j{}^{\mathcal{G}})\ \mathcal{N} \sim Int(\mathcal{Q})\Big)$. From Theorem 4.0.2, $a(\daleth_i{}^{\mathcal{G}}, \daleth_j{}^{\mathcal{G}})\mathcal{N} \sim Cl\Big((\daleth_i{}^{\mathcal{G}}, \daleth_j{}^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{Q})\Big)$ is $\daleth_i{}^{\mathcal{G}}-$NCoS, $i = 1, 2$ $\Big($in fact, $(\daleth_i{}^{\mathcal{G}}, \daleth_j{}^{\mathcal{G}}) -$ NRCoS, $i = j = 1, 2\Big)$. Therefore, $a\mathcal{Q} \supseteq (\daleth_i{}^{\mathcal{G}}, \daleth_j{}^{\mathcal{G}})\ \mathcal{N} \sim Cl\Big(a(\daleth_i{}^{\mathcal{G}}, \daleth_j{}^{\mathcal{G}})\ \mathcal{N} \sim Cl\Big((\daleth_i{}^{\mathcal{G}}, \daleth_j{}^{\mathcal{G}})\ \mathcal{N} \sim Int(\mathcal{A})\Big)\Big)$. Hence $(\daleth_i{}^{\mathcal{G}}, \daleth_j{}^{\mathcal{G}})\ \mathcal{N} \sim Cl\Big(a(\daleth_i{}^{\mathcal{G}}, \daleth_j{}^{\mathcal{G}})\ \mathcal{N} \sim Cl\Big((\daleth_i{}^{\mathcal{G}}, \daleth_j{}^{\mathcal{G}})\mathcal{N} \sim Int(\mathcal{A})\Big)\Big) \subseteq a\mathcal{Q}$.