Chapter 4

THREE-DIMENSIONAL IMPRECISE NUMBERS AND ITS APPLICATION

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4.1. Introduction

Study of three-dimensional form of object is very important for the practical purposes. To get more effective solution of any physical problem we need a solution of the whole dimension of object. For the study of the effect of volume of the object, three-dimensional imprecise numbers take a part to define how much membership is effecting along the three axes. So, to discuss about the participation of membership value in the three-dimensional object, this chapter is coming out.

Here, the concept of two-dimensional imprecise number is extended into three dimensions imprecise numbers so that we can study the effect of fuzziness of membership in each and every part of the occupied body. Identification of the effect of fuzziness characters in a specific dimension will help to solve many difficult practical problems. For examples how must be attractive a drop of red light among the groups of decoration of color light is one of the important application of signal apply for different purpose in various institutions. As red light has the longest wavelength, among the same weightiest of different colors, it will certainly focus on eye in comparison to the remaining colors of light. Along with this, complement of three-dimensional imprecise number helps us to identify about the reflection of dullness colors among the colors of light.

We introduce the definition of intersection and union of three-dimensional imprecise numbers with the help of maximum and minimum operators. Using these definitions all the properties of classical set occurred under the intersection and union operations are proposed for the three-dimensional imprecise numbers as well as proves of those properties are discussed with the assistance of numerical examples.

4.2. Three-dimensional Imprecise Numbers

It is mentioned in the definition of two-dimensional imprecise numbers that the effecting paths of a body of impreciseness is along the two axes and all others are already fully membership. Roughly the physical problems can be represented in the three dimensions form. So to study the effect of fuzziness of a body along the length, breadth and height it may be introduced three-dimensional imprecise numbers. Three-dimensional imprecise numbers is expressible in XYZ-solid geometry comprising of three different faces namely XY plane, YZ plane and ZX plane. Here, imprecise number is defined in the three-dimensional form such a way that full membership along the x-axis, the y-axis and the z-axis respectively is considered as a membership value one. For example at any instant travelling of solitary wave up to what distance is the x-axis and the height of the tide occur during their rise is the y-axis and thickness is the z-axis.

4.2.1. Definition: A three dimensional imprecise number

$$N_{XYZ} = [(\alpha_x, \alpha_y \alpha_z); (\beta_x, \beta_y, \beta_y); (\gamma_x, \gamma_y, \gamma_z)] \quad \ldots\ldots(4.1)$$

is divided into sub intervals with a partial element is presence in both the intervals. Where all the points in this interval are element of Cartesian product of sets $X \times Y \times Z$ such that X, Y and Z are imprecise numbers.

4.2.2. Definition: An element of partial presence of the three-dimensional imprecise number $N_{XYZ} = [(\alpha_x, \alpha_y, \alpha_z); (\beta_x, \beta_y, \beta_z); (\gamma_x, \gamma_y, \gamma_Z)]$ is described by the present level indicator function $p(x, y, z)$ which is counted from reference function $r(x, y, z)$ such that present level indicator for any (x, y, z), $(\alpha_x, \alpha_y, \alpha_z) \leq (x, y, z) \leq (\gamma_x, \gamma_y, \gamma_z)$ is $(p(x, y, z) - r(x, y, z))$, where $(0,0,0) \leq r(x, y, z) \leq p(x, y, z) \leq (1,1,1)$.

4.2.3. Definition: Indicator function of three-dimensional imprecise number,

$N_{XYZ} = [(\alpha_x, \alpha_y \alpha_z); (\beta_x, \beta_y, \beta_y); (\gamma_x, \gamma_y, \gamma_z)]$ is represented and defined by

$$\rho_{N_{XYZ}}(x, y, z) = \begin{cases} \rho_{XYZ1}(x, y, z), & (\alpha_x, \alpha_y, \alpha_z) \leq (x, y, z) \leq (\beta_x, \beta_y, \beta_z) \\ \rho_{XYZ2}(x, y, z), & (\beta_x, \beta_y, \beta_z) \leq (x, y, z) \leq (\gamma_x, \gamma_y, \gamma_z) \\ 0 & otherwise \end{cases} \quad \ldots\ldots(4.2)$$

Such that $\rho_{XYZ1}(\alpha_x, \alpha_y, \alpha_z) = \rho_{XYZ2}(\gamma_x, \gamma_y, \gamma_z) = (0,0,0)$

and $\rho_{XYZ1}(\beta_x, \beta_y, \beta_Z) = \rho_{XYZ2}(\beta_x, \beta_y, \beta_Z)$.

Where $\rho_{XYZ1}(x, y, z)$ is increasing function over the interval $[(\alpha_x, \alpha_y, \alpha_Z), (\beta_x, \beta_y, \beta_Z)]$ and $\rho_{XYZ2}(x, y, z)$ is decreasing function over the interval $[(\beta_x, \beta_y, \beta_z), (\gamma_x, \gamma_y, \gamma_z)]$. Then,

Case I: Three-dimensional normal imprecise numbers if,

$$\rho_{XYZ1}(\alpha_x, \alpha_y, \alpha_z) = \rho_{XYZ2}(\gamma_x, \gamma_y, \gamma_z) = (0,0,0)$$

and $\rho_{XYZ1}(\beta_x, \beta_y, \beta_Z) = \rho_{XYZ2}(\beta_x, \beta_y, \beta_Z) = (1,1,1,)$ ……………….(4.3)

Case II: Three-dimensional sub normal imprecise numbers if,

$$\rho_{XYZ1}(\alpha_x, \alpha_y, \alpha_z) = \rho_{XYZ2}(\gamma_x, \gamma_y, \gamma_z) = (0,0,0)$$

and $\rho_{XYZ1}(\beta_x, \beta_y, \beta_Z) = \rho_{XYZ2}(\beta_x, \beta_y, \beta_Z) \neq (1,1,1,)$....……………..(4.4)

Thus, $(\rho_{XYZ1}(x,y,z) - \rho_{XYZ2}(x,y,z)) = (\alpha_x - \beta_x) \times (\alpha_y - \beta_y) \times (\alpha_z - \beta_z)$

……………………………………(4.5)

is called membership value of the indicator function, $\rho_{N_{XYZ}}(x,y,z)$.

Where $\rho_{XYZ1}(x,y,z) = (\alpha_x, \alpha_y, \alpha_z)$ and $\rho_{XYZ2}(x,y,z) = (\beta_x, \beta_y, \beta_z)$

4.2.4. Definition: Complement of three-dimensional imprecise number

$N_{XYZ} = \{(\rho_{NYXZ}(x,y,z)), (0,0,0): (x,y,z) \in X \times Y \times Z\}$ is defined by,

$N_{XYZ}{}^{c} = \{(1,1,1), (\rho_{NXYZ}(x,y,z)): (x,y,z) \in X \times Y \times Z\}$ ……………………(4.6)

Where the membership function is equal to (1,1,1) and the reference function is $\rho_{N_{XYZ}}(x,y,z) < 1$ for $-\infty < (x,y,z) < \infty$

Three-dimensional imprecise numbers is characterized by,

$\{(\rho_{XYZ1}(x,y,z)), (\rho_{XYZ2}(x,y,z)): (x,y,z) \in X \times Y \times Z\}$.

Where, $\rho_{XYZ1}(x,y,z)$ and $\rho_{XYZ2}(x,y,z)$ are called membership function and the reference function of the indicator function $\mu_{N_{XY}}(x,y,z)$ defined above and the membership function is measured from reference function, then

$(\rho_{XYZ1}(x,y,z) - \rho_{XYZ2}(x,y,z)) = (x_1 - x_2) \times (y_1 - y_2) \times (z_1 - z_2)$………(4.7)

is called the membership value of the indicator function.

Where, $\rho_{XYZ1}(x,y,z) = (x_1, y_1, z_1)$ *and* $\rho_{XYZ2}(x,y,z) = (x_2, y_2, z_2)$ respectively.

The collection of all such elements is called three-dimensional imprecise set.

If the membership value is equal to 1, then we called it three-dimensional normal imprecise number, otherwise subnormal.

4.2.5. Definition: *(*Intersection and union of three-dimensional imprecise numbers)

If $A(\rho_{XYZ1}(x,y,z)) = \{(\rho_{XYZ1}(x,y,z)), (\rho_{XYZ2}(x,y,z)) : (x,y,z) \in X \times Y \times Z\}$

And $B(\rho_{XYZ3}(x,y,z)) = \{(x,y,z), \rho_{XYZ3}(x,y,z), \rho_{XYZ4}(x,y,z): (x,y,z) \in X \times Y \times Z\}$ be imprecise numbers. Then, intersection and union of imprecise numbers is defined by

$$A(\rho_{XYZ1}, \mu_{XYZ2}) \cap B(\rho_{XYZ3}, \mu_{XYZ4}) = \left\{ \begin{array}{c} min(\rho_{XYZ1}(x,y,z), \rho_{XYZ3}(x,y,z)) \\ , max(\rho_{XYZ2}(x,y,z) \\ (\rho_{XYZ4}(x,y,z)); (x,y,z) \in X \times Y \times Z \end{array} \right\} \ldots(4.8)$$

$$A(\rho_{XYZ1}, \rho_{XYZ2}) \cup B(\rho_{XYZ3}, \rho_{XYZ4}) = \left\{ \begin{array}{c} max(\rho_{XYZ1}(x,y,z), \rho_{XYZ3}(x,y,z)), \\ min(\rho_{XYZ2}(x,y,z), \rho_{XYZ4}(x,y,z)); \\ (x,y,z) \in X \times Y \times Z \end{array} \right\} \ldots..(4.9)$$

4.3. Properties of Three-dimensional Imprecise Numbers

Based on the operations of intersection and union, classical set theory properties are discussed in three-dimensional imprecise numbers as follows.

4.3.1. Property(Universal Laws)

(i) $A(\rho_{XYZ}(x,y,z)) \cap A^C(\rho_{XYZ}(x,y,z)) = \emptyset(\rho_{XYZ}(x,y,z))$

and (ii) $A(\rho_{XYZ}(x,y,z)) \cup A^C(\rho_{XYZ}(x,y,z)) = \Omega(\rho_{XYZ}(x,y,z))$

where, $A^C(\rho_{XYZ}(x,y,z))$, $\emptyset((\rho_{XYZ}(x,y,z))$ and $\Omega((\rho_{XYZ}(x,y,z))$ complement, null and the universal three-dimensional imprecise numbers respectively.

Let us consider a cuboid that stands in the xyz-plane such that $\frac{1}{2}$th portion of region is filled with water. Portion of water is shown with dark region in the following figure.

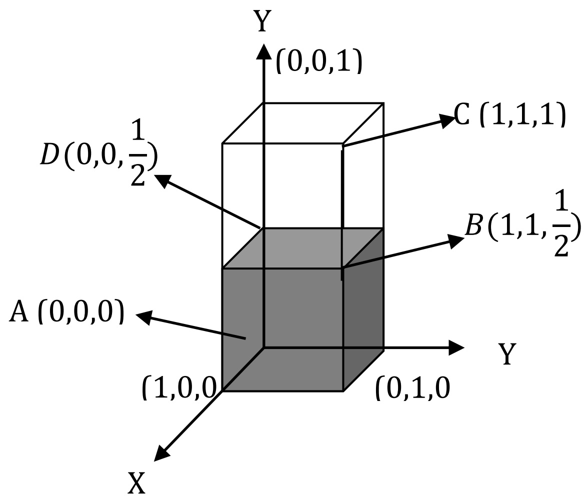

Fig.4.1. Three-dimensional imprecise numbers

Here, the membership function of cuboid is $E(\rho_{XYZ}(x,y,z)) = \left\{ \left(1,1,\frac{1}{2}\right), (0,0,0)\right\}$ with membership value $\left((1-0)\times(1-0)\times(\frac{1}{2}-0)\right) = \frac{1}{2}$ and the membership function of the complementary part is $E^C(\rho_{XYZ}(x,y,z)) = \left\{(1,1,1), \left(0,0,\frac{1}{2}\right)\right\}$ with membership value $\left((1-0)\times(1-0)\times(1-\frac{1}{2})\right) = \frac{1}{2}$. Now intersection and the union of three dimensional imprecise numbers are obtained as follows.

Intersection:

$$E\big(\rho_{XYZ}(x,y,z)\big) \cap E^C\big(\rho_{XYZ}(x,y,z)\big) = \left\{\left(1,1,\frac{1}{2}\right), (0,0,0)\right\} \cap \left\{(1,1,1), \left(0,0,\frac{1}{2}\right)\right\}$$

$$= \left\{\begin{matrix}\left(\min(1,1), min(1,1), \min(1,\frac{1}{2})\right), \\ \left(\max(0,0), \max(0,0), \max(0,\frac{1}{2})\right)\end{matrix}\right\}$$

$$= \left\{\left(1,1,\frac{1}{2}\right), (0,0,\frac{1}{2})\right\}$$

It has a membership value $\left((1-0)\times(1-0)\times(\frac{1}{2}-\frac{1}{2})\right) = 0$

So, it is a null or empty imprecise number.

Union: $E\big(\rho_{XYZ}(x,y,z)\big) \cup E^C\big(\rho_{XYZ}(x,y,z)\big)$

$$= \left\{\left(1,1,\frac{1}{2}\right), (0,0,0)\right\} \cup \left\{(x,y,z), (1,1,1), \left(0,0,\frac{1}{2}\right)\right\}$$

$$= \left\{\begin{matrix}\left(\max(1,1), max(1,1), \max(1,\frac{1}{2})\right), \\ \left(\min(0,0), \min(0,0), \min(0,\frac{1}{2}\right)\end{matrix}\right\}$$

$$= \{(1,1,1), (0,0,0)\}$$

It has a membership value $\big((1-0)\times(1-0)\times(1-0)\big) = 1$

So, it is the universal imprecise number.

4.3.2. Property (Commutative laws)

Let $A(\rho_{XYZ}(x,y,z)) = \{(\rho_{XYZ1}(x,y,z)), (\rho_{XYZ2}(x,y,z)) : (x,y,z) \in X \times Y \times Z\}$ and $B(\rho_{XYZ}(x,y,z)) = \{(\rho_{XYZ3}(x,y,z)),\ (\rho_{XYZ4}(x,y,z)) : (x,y,z) \in X \times Y \times Z\}$ be three-dimensional imprecise numbers. Then

(i) $A\big(\rho_{XYZ1}(x,y,z)\big) \cup B\big(\rho_{XYZ2}(x,y,z)\big)$

$$= B\big(\rho_{XYZ2}(x,y,z)\big) \cup A\big(\rho_{XYZ1}(x,y,z)\big)$$

(ii) $A(\rho_{XYZ1}(x,y,z)) \cap B(\rho_{XYZ2}(x,y,z))$

$$= B(\rho_{XYZ2}(x,y,z)) \cap A(\rho_{XYZ1}(x,y,z))$$

If $A(\rho_{XYZ1}(x,y,z)) = \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{4},\frac{1}{4},\frac{1}{4}\right)\right\}$ and

$B(\rho_{XYZ2}(x,y,z)) = \left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right)\right\}$ be three-dimensional imprecise numbers. Then,

Proof:

(i) $A(\rho_{XYZ1}(x,y,z)) \cup B(\rho_{XYZ2}(x,y,z))$

$$= \left\{\begin{matrix}\left(\max\left(\frac{1}{2},\frac{1}{3}\right),\max\left(\frac{1}{2},\frac{1}{3}\right),max\left(\frac{1}{2},\frac{1}{3}\right)\right),\\ \left(min\left(\frac{1}{4},\frac{1}{6}\right),min\left(\frac{1}{4},\frac{1}{6}\right),min\left(\frac{1}{6},\frac{1}{6}\right)\right)\end{matrix}\right\}$$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right)\right\}$$

And $B(\rho_{XYZ2}(x,y,z)) \cup A(\rho_{XYZ1}(x,y,z)) = \left\{\begin{matrix}\left(\max\left(\frac{1}{3},\frac{1}{2}\right),\max\left(\frac{1}{3},\frac{1}{2}\right),\max\left(\frac{1}{3},\frac{1}{2}\right)\right),\\ \left(min\left(\frac{1}{6},\frac{1}{4}\right),min\left(\frac{1}{6},\frac{1}{4}\right),\min\left(\frac{1}{6},\frac{1}{4}\right)\right)\end{matrix}\right\}$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right)\right\}$$

Hence proved

(ii) $A(\rho_{XYZ1}(x,y,z)) \cap B(\rho_{XYZ2}(x,y,z))$

$$= \left\{\begin{matrix}\left(\min\left(\frac{1}{2},\frac{1}{3}\right),\min\left(\frac{1}{2},\frac{1}{3}\right),min\left(\frac{1}{2},\frac{1}{3}\right)\right),\\ \left(max\left(\frac{1}{4},\frac{1}{6}\right),max\left(\frac{1}{4},\frac{1}{6}\right),max\left(\frac{1}{4},\frac{1}{6}\right)\right)\end{matrix}\right\}$$

$$= \left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{4},\frac{1}{4},\frac{1}{4}\right)\right\}$$

And

$$B(\rho_{XYZ2}(x,y,z)) \cap A(\rho_{XYZ1}(x,y,z)) = \left\{\begin{matrix}\left(\min\left(\frac{1}{3},\frac{1}{2}\right),\min\left(\frac{1}{3},\frac{1}{2}\right),\min\left(\frac{1}{3},\frac{1}{2}\right)\right),\\ \left(max\left(\frac{1}{6},\frac{1}{4}\right),max\left(\frac{1}{6},\frac{1}{4}\right),\max\left(\frac{1}{6},\frac{1}{4}\right)\right)\end{matrix}\right\}$$

$$= \left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{4},\frac{1}{4},\frac{1}{4}\right)\right\}$$

Hence proved

4.3.3. Property (Distributive Laws)

If $A(\rho_{XYZ}(x,y,z)) = \{(\rho_{XYZ1}(x,y,z)),(\rho_{XYZ2}(x,y,z)) : (x,y,z) \in X \times Y \times Z\}$

$B(\rho_{XYZ}(x,y,z)) = \{(\rho_{XYZ3}(x,y,z)),\ (\rho_{XYZ4}(x,y,z)) : (x,y,z) \in X \times Y \times Z\}$ and
$C(\rho_{XYZ}(x,y,z)) = \{(\rho_{XYZ5}(x,y,z)), (\ \rho_{XYZ6}(x,y,z)) : (x,y,z) \in X \times Y \times Z\}$ be three-dimensional imprecise numbers. Then,

(i) $A(\rho_{XYZ1}(x,y,z)) \cap \big(B(\rho_{XYZ2}(x,y,z)) \cup C(\rho_{XYZ3}(x,y,z))\big)$
$= \big(A(\rho_{XYZ1}(x,y,z)) \cap B(\rho_{XYZ2}(x,y,z))\big) \cup \big(A(\rho_{XYZ1}(x,y,z)) \cap C(\rho_{XYZ3}(x,y,z))\big)$

(ii) $A(\rho_{XYZ1}(x,y,z)) \cup \big(B((\rho_{XYZ2}(x,y,z)) \cap C(\rho_{XYZ3}(x,y,z))\big)$
$= \big(A(\rho_{XYZ1}(x,y,z)) \cup B(\rho_{XYZ2}(x,y,z))\big) \cap \big(A(\rho_{XYZ1}(x,y,z)) \cup C(\rho_{XYZ3}(x,y,z))\big)$

Let us prove the property (i) and (ii) with example.

If $A(\rho_{XYZ1}(x,y,z)) = \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{4},\frac{1}{4},\frac{1}{4}\right)\right\}$, $B(\rho_{XYZ2}(x,y,z)) = \left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{5},\frac{1}{5},\frac{1}{5}\right)\right\}$ and $C(\rho_{XYZ3}(x,y,z)) = \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{7},\frac{1}{7},\frac{1}{7}\right)\right\}$. Then,

Proof:

(i) $A(\rho_{XYZ1}(x,y,z)) \cap \big(B(\rho_{XYZ2}(x,y)) \cup C(\rho_{XYZ3}(x,y,z))\big)$

$$= \left(\left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{4},\frac{1}{4},\frac{1}{4}\right)\right\} \cap \left\{\begin{matrix}\left(\max\left(\frac{1}{3},\frac{1}{6}\right),\max\left(\frac{1}{3},\frac{1}{6}\right),\max\left(\frac{1}{3},\frac{1}{6}\right)\right),\\ \left(min\left(\frac{1}{5},\frac{1}{7}\right),min\left(\frac{1}{5},\frac{1}{7}\right),\min\left(\frac{1}{5},\frac{1}{7}\right)\right)\end{matrix}\right\}\right)$$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{4},\frac{1}{4},\frac{1}{4}\right)\right\} \cap \left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{7},\frac{1}{7},\frac{1}{7}\right)\right\}$$

$$= \left\{\begin{matrix}\left(\min\left(\frac{1}{2},\frac{1}{3}\right),\min\left(\frac{1}{2},\frac{1}{3}\right),\min\left(\frac{1}{2},\frac{1}{3}\right)\right),\\ \left(max\left(\frac{1}{4},\frac{1}{7}\right),max\left(\frac{1}{4},\frac{1}{7}\right),max\left(\frac{1}{4},\frac{1}{7}\right)\right)\end{matrix}\right\}$$

$$= \left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{4},\frac{1}{4},\frac{1}{4}\right)\right\}$$

And $\big(A(\rho_{XYZ1}(x,y,z)) \cap B(\rho_{XYZ2}(x,y,z))\big) \cup \big(A(\rho_{XYZ1}(x,y,z)) \cap C(\rho_{XYZ3}(x,y,z))\big)$

$$= \left\{\begin{matrix}\left(\min\left(\frac{1}{2},\frac{1}{3}\right),\min\left(\frac{1}{2},\frac{1}{3}\right),min\left(\frac{1}{2},\frac{1}{3}\right)\right),\\ \left(max\left(\frac{1}{4},\frac{1}{5}\right),max\left(\frac{1}{4},\frac{1}{5}\right),max\left(\frac{1}{4},\frac{1}{5}\right)\right)\end{matrix}\right\}$$

$$\cup \left\{\begin{matrix}\left(\min\left(\frac{1}{2},\frac{1}{6}\right),\min\left(\frac{1}{2},\frac{1}{6}\right),min\left(\frac{1}{2},\frac{1}{6}\right)\right),\\ \left(max\left(\frac{1}{4},\frac{1}{7}\right),max\left(\frac{1}{4},\frac{1}{7}\right),max\left(\frac{1}{4},\frac{1}{7}\right)\right)\end{matrix}\right\}$$

$$= \left\{\begin{matrix}\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\\ \left(\frac{1}{4},\frac{1}{4},\frac{1}{4}\right)\end{matrix}\right\} \cup \left\{\begin{matrix}\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\\ \left(\frac{1}{4},\frac{1}{4},\frac{1}{4}\right)\end{matrix}\right\}$$

$$= \begin{Bmatrix} \left(\max\left(\frac{1}{3},\frac{1}{6}\right), \max\left(\frac{1}{3},\frac{1}{6}\right), max\left(\frac{1}{3},\frac{1}{6}\right)\right), \\ \left(min\left(\frac{1}{4},\frac{1}{4}\right), min\left(\frac{1}{4},\frac{1}{4}\right), min\left(\frac{1}{4},\frac{1}{4}\right)\right) \end{Bmatrix}$$

$$= \left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right), \left(\frac{1}{4},\frac{1}{4},\frac{1}{4}\right)\right\}$$

Hence proved

(ii) $A(\rho_{XYZ1}(x,y,z)) \cup \left(B(\rho_{XYZ2}(x,y,z)) \cap C(\rho_{XYZ3}(x,y,z))\right)$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right), \left(\frac{1}{4},\frac{1}{4},\frac{1}{4}\right)\right\} \cup \begin{Bmatrix} \left(\min\left(\frac{1}{3},\frac{1}{6}\right), \min\left(\frac{1}{3},\frac{1}{6}\right), \min\left(\frac{1}{3},\frac{1}{6}\right)\right), \\ \left(max\left(\frac{1}{5},\frac{1}{7}\right), max\left(\frac{1}{5},\frac{1}{7}\right), max\left(\frac{1}{5},\frac{1}{7}\right)\right) \end{Bmatrix}$$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right), \left(\frac{1}{4},\frac{1}{4},\frac{1}{4}\right)\right\} \cup \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right), \left(\frac{1}{5},\frac{1}{5},\frac{1}{5}\right)\right\}$$

$$= \begin{Bmatrix} \left(\max\left(\frac{1}{2},\frac{1}{6}\right), \max\left(\frac{1}{2},\frac{1}{6}\right), max\left(\frac{1}{2},\frac{1}{6}\right)\right), \\ \left(min\left(\frac{1}{4},\frac{1}{5}\right), min\left(\frac{1}{4},\frac{1}{5}\right), min\left(\frac{1}{4},\frac{1}{5}\right)\right) \end{Bmatrix}$$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right), \left(\frac{1}{5},\frac{1}{5},\frac{1}{5}\right)\right\}$$

And $\left(A(\rho_{XYZ1}(x,y,z)) \cup B(\rho_{XYZ2}(x,y,z))\right) \cap \left(A(\rho_{XYZ1}(x,y)) \cup C(\rho_{XYZ3}(x,y,z))\right)$

$$= \begin{Bmatrix} \left(\max\left(\frac{1}{2},\frac{1}{3}\right), \max\left(\frac{1}{2},\frac{1}{3}\right), max\left(\frac{1}{2},\frac{1}{3}\right)\right), \\ \left(min\left(\frac{1}{4},\frac{1}{5}\right), min\left(\frac{1}{4},\frac{1}{5}\right), min\left(\frac{1}{4},\frac{1}{5}\right)\right) \end{Bmatrix}$$

$$\cap \begin{Bmatrix} \left(\max\left(\frac{1}{2},\frac{1}{6}\right), \max\left(\frac{1}{2},\frac{1}{6}\right), max\left(\frac{1}{2},\frac{1}{6}\right)\right), \\ \left(min\left(\frac{1}{4},\frac{1}{7}\right), min\left(\frac{1}{4},\frac{1}{7}\right), min\left(\frac{1}{4},\frac{1}{7}\right)\right) \end{Bmatrix}$$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right), \left(\frac{1}{5},\frac{1}{5},\frac{1}{5}\right)\right\} \cap \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right), \left(\frac{1}{7},\frac{1}{7},\frac{1}{7}\right)\right\}$$

$$= \begin{Bmatrix} \left(\min\left(\frac{1}{2},\frac{1}{2}\right), \min\left(\frac{1}{2},\frac{1}{2}\right), min\left(\frac{1}{2},\frac{1}{2}\right)\right), \\ \left(max\left(\frac{1}{5},\frac{1}{7}\right), max\left(\frac{1}{5},\frac{1}{7}\right), max\left(\frac{1}{5},\frac{1}{7}\right)\right) \end{Bmatrix}$$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right), \left(\frac{1}{5},\frac{1}{5},\frac{1}{5}\right)\right\}$$

Hence proved

4.3.4. Property(Idempotence Laws)

(i) $A(\rho_{XYZ}(x,y,z)) \cap A\left(\rho_{XYZ}(x,y,z)\right) = A(\rho_{XYZ}(x,y,z))$

and (ii) $A(\rho_{XYZ}(x,y,z)) \cup A(\rho_{XYZ}(x,y,z)) = A(\rho_{XYZ}(x,y,z))$

Where, $A(\rho_{XYZ}(x,y,z)) = \{(\rho_{XYZ1}(x,y,z)), (\rho_{XYZ2}(x,y,z)) : (x,y,z) \in X \times Y \times Z\}$

Obviously the properties can be proved.

4.3.5. Property(Identity Laws)

(i) $A(\rho_{XYZ}(x,y,z)) \cap \emptyset(\rho_{XYZ}(x,y,z)) = \emptyset(\rho_{XYZ}(x,y.z))$

(ii) $A(\rho_{XYZ}(x,y,z)) \cup \emptyset((\rho_{XYZ}(x,y,z)) = A(\rho_{XYZ}(x,y,z))$

(iii) $A(\rho_{XYZ}(x,y,z)) \cap X(\rho_{XYZ}(x,y,z)) = A(\rho_{XYZ}(x,y,z))$

(iv) $A(\rho_{XYZ}(x,y,z)) \cup X((\rho_{XYZ}(x,y,z)) = X(\rho_{XYZ}(x,y,z))$

Where, $X((\rho_{XYZ}(x,y,z))$ is universal set and $\emptyset((\rho_{XYZ}(x,y,z))$ is null set.

To prove 4.3.5. (i) and 4.3.5.(ii) let us consider $A((\rho_{XYZ1}(x,y,z)) = \left\{\left(\frac{1}{7},\frac{1}{7},\frac{1}{7}\right),(\frac{1}{8},\frac{1}{8},\frac{1}{8})\right\}$ and $\emptyset(\rho_{XYZ2}(x,y,z)) = \left\{(0,0,0),(\frac{1}{8},\frac{1}{8},\frac{1}{8})\right\}$ be such that membership function of three dimensional imprecise number of $A(\rho_{XYZ1}(x,y,z))$ is $\left(\frac{1}{7},\frac{1}{7},\frac{1}{7}\right)$ and is measured from the reference function $(\frac{1}{8},\frac{1}{8},\frac{1}{8})$. Where $\left(\frac{1}{7},\frac{1}{7},\frac{1}{7}\right)$ and $(\frac{1}{8},\frac{1}{8},\frac{1}{8})$ are one seventh portion, one eighth portions of the three-dimensional object respectively.

$\emptyset((\rho_{XYZ2}(x,y,z)) = \left\{(0,0,0),(\frac{1}{8},\frac{1}{8},\frac{1}{8})\right\}$ is a null imprecise number measured from the one eighth potion of the three-dimensional object. Here membership function is zero due to null. Then,

Proof:

(i) $A(\rho_{XYZ1}(x,y,z)) \cap \emptyset(\rho_{XYZ2}(x,y,z))$

$$= \left\{\begin{matrix}\left(\min\left(\frac{1}{7},0\right),\min\left(\frac{1}{7},0\right),min\left(\frac{1}{7},0\right)\right), \\ \left(max\left(\frac{1}{8},\frac{1}{8}\right),max\left(\frac{1}{8},\frac{1}{8}\right),max\left(\frac{1}{8},\frac{1}{8}\right)\right)\end{matrix}\right\}$$

$$= \left\{(0,0,0),\left(\frac{1}{8},\frac{1}{8},\frac{1}{8}\right)\right\}$$

$$= \emptyset((\rho_{XYZ2}(x,y,z))$$

Hence proved

(ii) $A(\rho_{XYZ1}(x,y,z)) \cup \emptyset(\rho_{XYZ2}(x,y,z))$

$$= \begin{Bmatrix} \left(\max\left(\frac{1}{7},0\right),\max\left(\frac{1}{7},0\right),max\left(\frac{1}{7},0\right)\right), \\ \left(min\left(\frac{1}{8},\frac{1}{8}\right),min\left(\frac{1}{8},\frac{1}{8}\right),min\left(\frac{1}{8},\frac{1}{8}\right)\right) \end{Bmatrix}$$

$$= \left\{\left(\frac{1}{7},\frac{1}{7},\frac{1}{7}\right),\left(\frac{1}{8},\frac{1}{8},\frac{1}{8}\right)\right\}$$

$$= A((\rho_{XYZ1}(x,y,z))$$

Hence proved

To prove property 4.3.5.(iii) and 4.3.5.(iv), let $A(\rho_{XYZ1}(x,y,z)) = \left\{\left(\frac{1}{7},\frac{1}{7},\frac{1}{7}\right),(\frac{1}{8},\frac{1}{8},\frac{1}{8})\right\}$ and $X(\rho_{XYZ2}(x,y,z)) = \left\{\left(\frac{2}{7},\frac{2}{7},\frac{2}{7}\right),(\frac{1}{8},\frac{1}{8},\frac{1}{8})\right\}$ be such that membership function of three dimensional imprecise number of $A((\rho_{XYZ1}(x,y,z))$ is $\left(\frac{1}{7},\frac{1}{7},\frac{1}{7}\right)$ and measured from the reference function $(\frac{1}{8},\frac{1}{8},\frac{1}{8})$. Where $\left(\frac{1}{7},\frac{1}{7},\frac{1}{7}\right)$ and $(\frac{1}{8},\frac{1}{8},\frac{1}{8})$ are one seventh portion, one eighth portion of the three-dimensional object respectively.

$X(\rho_{XYZ2}(x,y,z)) = \left\{\left(\frac{2}{7},\frac{2}{7},\frac{2}{7}\right),(\frac{1}{8},\frac{1}{8},\frac{1}{8})\right\}$ is a universal set imprecise number measured from the one eighth potion of the three dimensional object. Here membership function is two third portion of the object and is greater than the membership value of $A((\rho_{XYZ1}(x,y,z))$. Then

(iii) $A(\rho_{XYZ1}(x,y,z)) \cap X(\rho_{XYZ2}(x,y,z))$

$$= \begin{Bmatrix} \left(\min\left(\frac{1}{7},\frac{2}{7}\right),\min\left(\frac{1}{7},\frac{2}{7}\right),min\left(\frac{1}{7},\frac{2}{7}\right)\right), \\ \left(max\left(\frac{1}{8},\frac{1}{8}\right),max\left(\frac{1}{8},\frac{1}{8}\right),max\left(\frac{1}{8},\frac{1}{8}\right)\right) \end{Bmatrix}$$

$$= \left\{\left(\frac{1}{7},\frac{1}{7},\frac{1}{7}\right),\left(\frac{1}{8},\frac{1}{8},\frac{1}{8}\right)\right\}$$

$$= A((\mu_{XYZ1}(x,y,z))$$

Hence Proved

(iv) $A((\rho_{XYZ1}(x,y,z)) \cup X((\rho_{XYZ2}(x,y,z))$

$$= \begin{Bmatrix} \left(\max\left(\frac{1}{7},\frac{2}{7}\right),\max\left(\frac{1}{7},\frac{2}{7}\right),max\left(\frac{1}{7},\frac{2}{7}\right)\right), \\ \left(min\left(\frac{1}{8},\frac{1}{8}\right),min\left(\frac{1}{8},\frac{1}{8}\right),min\left(\frac{1}{8},\frac{1}{8}\right)\right) \end{Bmatrix}$$

$$= \left\{\left(\frac{2}{7},\frac{2}{7},\frac{2}{7}\right),\left(\frac{1}{8},\frac{1}{8},\frac{1}{8}\right)\right\}$$

$$= X((\rho_{XYZ2}(x,y,z))$$

Hence proved

4.3.6. Property (Associatively Laws)

If $A(\rho_{XYZ}(x,y,z)) = \{\rho_{XYZ1}(x,y,z), \rho_{XYZ2}(x,y,z): (x,y,z) \in X \times Y \times Z\}$,
$B(\rho_{XYZ}(x,y,z)) = \{\rho_{XYZ3}(x,y,z), \rho_{XYZ4}(x,y,z): (x,y,z) \in X \times Y \times Z\}$
and $C(\rho_{XYZ}(x,y,z)) = \{\rho_{XYZ5}(x,y,z), \rho_{XYZ6}(x,y,z): (x,y,z) \in X \times Y \times Z\}$ be three-dimensional imprecise numbers. Then

(i) $$A\big(\rho_{XYZ}(x,y,z)\big) \cup \Big(B\big(\rho_{XYZ}(x,y,z)\big) \cup C\big(\rho_{XYZ}(x,y,z)\big)\Big)$$
$$= \Big(A\big(\rho_{XYZ}(x,y,z)\big) \cup B\big(\rho_{XYZ}(x,y,z)\big)\Big) \cup C\big(\rho_{XYZ}(x,y,z)\big)$$

(ii) $$A\big(\rho_{XYZ}(x,y,z)\big) \cap \Big(B\big(\rho_{XYZ}(x,y,z)\big) \cap C\big(\rho_{XYZ}(x,y,z)\big)\Big)$$
$$= \Big(A\big(\rho_{XYZ}(x,y,z)\big) \cap B\big(\rho_{XYZ}(x,y,z)\big)\Big) \cap C\big(\rho_{XYZ}(x,y,z)\big)$$

To prove this property let us consider, $A((\rho_{XYZ}(x,y,z)) = \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$,
$B(\rho_{XYZ}(x,y,z)) = \left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$ and $C((\rho_{XYZ}(x,y,z)) = \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$.
Then,

Proof:

(i) $$A\big(\rho_{XYZ}(x,y,z)\big) \cup \Big(B\big(\rho_{XYZ}(x,y,z)\big) \cup C\big(\rho_{XYZ}(x,y,z)\big)\Big)$$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cup \begin{pmatrix} \left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \\ \cup \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \end{pmatrix}$$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cup \left\{\begin{array}{c} \left(\max\left(\frac{1}{3},\frac{1}{6}\right),\max\left(\frac{1}{3},\frac{1}{6}\right),\max\left(\frac{1}{3},\frac{1}{6}\right)\right), \\ \left(min\left(\frac{1}{9},\frac{1}{9}\right),min\left(\frac{1}{9},\frac{1}{9}\right),min\left(\frac{1}{9},\frac{1}{9}\right)\right) \end{array}\right\}$$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cup \left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$= \left\{\begin{array}{c} \left(\max\left(\frac{1}{2},\frac{1}{3}\right),\max\left(\frac{1}{2},\frac{1}{3}\right),\max\left(\frac{1}{2},\frac{1}{3}\right)\right), \\ \left(min\left(\frac{1}{9},\frac{1}{9}\right),min\left(\frac{1}{9},\frac{1}{9}\right),min\left(\frac{1}{9},\frac{1}{9}\right)\right) \end{array}\right\}$$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$\Big(A\big(\rho_{XYZ}(x,y,z)\big) \cup B\big(\rho_{XYZ}(x,y,z)\big)\Big) \cup C\big(\rho_{XYZ}(x,y,z)\big)$$

$$= \left(\left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cup \left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}\right) \cup \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$= \left\{\begin{matrix}\left(\max\left(\frac{1}{2},\frac{1}{3}\right),\max\left(\frac{1}{2},\frac{1}{3}\right),\max\left(\frac{1}{2},\frac{1}{3}\right)\right), \\ \left(min\left(\frac{1}{9},\frac{1}{9}\right),min\left(\frac{1}{9},\frac{1}{9}\right),min\left(\frac{1}{9},\frac{1}{9}\right)\right)\end{matrix}\right\} \cup \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cup \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$= \left\{\begin{matrix}\left(\max\left(\frac{1}{2},\frac{1}{6}\right),\max\left(\frac{1}{2},\frac{1}{6}\right),\max\left(\frac{1}{2},\frac{1}{6}\right)\right), \\ \left(min\left(\frac{1}{9},\frac{1}{9}\right),min\left(\frac{1}{9},\frac{1}{9}\right)min\left(\frac{1}{9},\frac{1}{9}\right),\right)\end{matrix}\right\}$$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

Hence proved

(ii) $A(\rho_{XYZ}(x,y,z)) \cap \left(B(\rho_{XYZ}(x,y,z)) \cap C(\rho_{XYZ}(x,y,z))\right)$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cap \left(\begin{matrix}\left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \\ \cap\left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}\end{matrix}\right)$$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cap \left\{\begin{matrix}\left(\min\left(\frac{1}{3},\frac{1}{6}\right),\min\left(\frac{1}{3},\frac{1}{6}\right),\min\left(\frac{1}{3},\frac{1}{6}\right)\right), \\ \left(max\left(\frac{1}{9},\frac{1}{9}\right),max\left(\frac{1}{9},\frac{1}{9}\right),max\left(\frac{1}{9},\frac{1}{9}\right)\right)\end{matrix}\right\}$$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cap \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$= \left\{\begin{matrix}\left(\min\left(\frac{1}{2},\frac{1}{6}\right),\min\left(\frac{1}{2},\frac{1}{6}\right),\min\left(\frac{1}{2},\frac{1}{6}\right)\right), \\ \left(max\left(\frac{1}{9},\frac{1}{9}\right),max\left(\frac{1}{9},\frac{1}{9}\right),max\left(\frac{1}{9},\frac{1}{9}\right)\right)\end{matrix}\right\}$$

$$= \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

$\left(A(\rho_{XYZ}(x,y,z)) \cup B(\rho_{XYZ}(x,y,z))\right) \cup C(\rho_{XYZ}(x,y,z))$

$$= \left(\left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cap \left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}\right) \cap \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$= \left\{\begin{matrix}\left(\min\left(\frac{1}{2},\frac{1}{3}\right),\min\left(\frac{1}{2},\frac{1}{3}\right),\min\left(\frac{1}{2},\frac{1}{3}\right)\right), \\ \left(max\left(\frac{1}{9},\frac{1}{9}\right),max\left(\frac{1}{9},\frac{1}{9}\right),max\left(\frac{1}{9},\frac{1}{9}\right)\right)\end{matrix}\right\} \cap \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$= \left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cap \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$= \left\{\begin{matrix} \left(\min\left(\frac{1}{3},\frac{1}{6}\right),\min\left(\frac{1}{3},\frac{1}{6}\right),\min\left(\frac{1}{3},\frac{1}{6}\right)\right), \\ \left(max\left(\frac{1}{9},\frac{1}{9}\right),max\left(\frac{1}{9},\frac{1}{9}\right)max\left(\frac{1}{9},\frac{1}{9}\right),\right) \end{matrix}\right\}$$

$$= \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

Hence proved

4.3.7. Property (De-Morgan's Laws)

If $A(\rho_{XYZ}(x,y,z)) = \{\rho_{XYZ1}(x,y,z), \rho_{XYZ2}(x,y,z) : (x,y,z) \in X \times Y \times Z\}$ and $B(\rho_{XYZ}(x,y,z)) = \{\rho_{XYZ3}(x,y,z), \rho_{XYZ4}(x,y,z) : (x,y,z) \in X \times Y \times Z\}$ be three-dimensional imprecise numbers. Then,

(i) $\left(A\left(\rho_{XYZ}(x,y,z)\right) \cup B\left(\rho_{XYZ}(x,y,z)\right)\right)^C$

$$= A^C\left(\rho_{XYZ}(x,y,z)\right) \cap B^C\left(\rho_{XYZ}(x,y,z)\right)$$

(ii) $\left(A\left(\rho_{XYZ}(x,y,z)\right) \cap B\left(\rho_{XYZ}(x,y,z)\right)\right)^C$

$$= A^C\left(\rho_{XYZ}(x,y,z)\right) \cup B^C\left(\rho_{XYZ}(x,y,z)\right)$$

To prove this property, let us take the above three-dimensional imprecise numbers $A(\rho_{XYZ}(x,y,z)) = \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$, $B(\rho_{XYZ}(x,y,z)) = \left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$. Then,

Proof:

(i) $\left(A\left(\rho_{XYZ}(x,y,z)\right) \cup B\left(\rho_{XYZ}(x,y,z)\right)\right)^C$

$$= \left(\left\{\begin{matrix} \left(\max\left(\frac{1}{2},\frac{1}{3}\right),\max\left(\frac{1}{2},\frac{1}{3}\right),\max\left(\frac{1}{2},\frac{1}{3}\right)\right), \\ \left(min\left(\frac{1}{9},\frac{1}{9}\right),min\left(\frac{1}{9},\frac{1}{9}\right)min\left(\frac{1}{9},\frac{1}{9}\right),\right) \end{matrix}\right\}\right)^C$$

$$= \left(\left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}\right)^C$$

$$= \left\{(1,1,1),\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)\right\}$$

$A^C\left(\rho_{XYZ}(x,y,z)\right) \cap B^C\left(\rho_{XYZ}(x,y,z)\right)$

$$= \left(\left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}\right)^C \cap \left(\left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}\right)^C$$

$$= \left\{(1,1,1),\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)\right\} \cap \left\{(1,1,1),\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)\right\}$$

$$= \left\{\begin{matrix} (\min(1,1),\min(1,1),\min(1,1)), \\ \left(max\left(\frac{1}{2},\frac{1}{3}\right),max\left(\frac{1}{2},\frac{1}{3}\right),max\left(\frac{1}{2},\frac{1}{3}\right)\right) \end{matrix}\right\}$$

$$= \left\{(1,1,1), \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)\right\}$$

Hence proved

(ii) $\left(A\left(\rho_{XYZ}(x,y,z)\right) \cap B\left(\rho_{XYZ}(x,y,z)\right)\right)^C$

$$= \left(\left\{\begin{array}{c}\left(\min\left(\frac{1}{2},\frac{1}{3}\right), \min\left(\frac{1}{2},\frac{1}{3}\right), \min\left(\frac{1}{2},\frac{1}{3}\right)\right), \\ \left(max\left(\frac{1}{9},\frac{1}{9}\right), max\left(\frac{1}{9},\frac{1}{9}\right) max\left(\frac{1}{9},\frac{1}{9}\right),\right)\end{array}\right\}\right)^C$$

$$= \left(\left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right), \left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}\right)^C$$

$$= \left\{(1,1,1), \left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)\right\}$$

$A^C\left(\rho_{XYZ}(x,y,z)\right) \cup B^C\left(\rho_{XYZ}(x,y,z)\right)$

$$= \left(\left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right), \left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}\right)^C \cup \left(\left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right), \left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}\right)^C$$

$$= \left\{(1,1,1), \left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)\right\} \cup \left\{(1,1,1), \left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)\right\}$$

$$= \left\{\begin{array}{c}(\max(1,1), \text{miax}(1,1), \max(1,1)), \\ \left(min\left(\frac{1}{2},\frac{1}{3}\right), min\left(\frac{1}{2},\frac{1}{3}\right), min\left(\frac{1}{2},\frac{1}{3}\right)\right)\end{array}\right\}$$

$$= \left\{(1,1,1), \left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)\right\}$$

Hence proved

Here, membership function of $A((\rho_{XYZ}(x,y,z))$ and $B((\rho_{XYZ}(x,y,z))$ are $\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ and $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$ and are measured from the reference function, $\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)$. So the complement of $A((\rho_{XYZ}(x,y,z))$ and $B((\rho_{XYZ}(x,y,z))$ are measured from reference function $\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ and $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$ till to the highest membership function of the three dimensional imprecise function (1,1,1).

4.4. Application of Three-dimensional Imprecise Numbers

Three-dimensional imprecise number can be obtained as an application in the field of economics. Assume that production, demand and service of our country is in such a way that at present 60% effort of production of different crops is done in every year to fulfill 75% needs or demand of our peoples with service capacity of 65%. So, the production of crops should be increased up to 85% to fulfill the demand up to hundred percent with service effort 90%. In this case demand, production and service situation can be expressed into three-dimensional imprecise number. Thus the imprecise number is obtained in the following form.

$$A\big(\rho_{XYZ}(x,y,z)\big) = \{(85\% ,100\%, 90\%), (60\%, 75\%, 65\%)\}$$
$$= \left\{\left(\frac{17}{20}, 1, \frac{9}{10}\right), \left(\frac{3}{5}, \frac{3}{4}, \frac{13}{20}\right)\right\}$$

Here, membership value can be modelled in the following form.

$$A\{\rho_{XYZ1}(x,y,z), \rho_{XYZ2}(x,y,z) : (x,y,z) \in X \times Y \times Z\}$$
$$= |x_1 - x_2|, |y_1 - y_2|, |z_1 - z_2|;$$
$$\rho_{XYZ1}(x,y,z) = (x_1, y_1, z_1), \rho_{XYZ2}(x,y,z) = (x_2, y_2, z_2)$$

Where $|x_1 - x_2|$, $|y_1 - y_2|$ and $|z_1 - z_2|$ are distinct behaviors. In case the behaviors of variables are similar, then their product will be a membership function of the above imprecise number. Otherwise membership function will be counted separately.

Thus, membership value of production, demand and service of this problem is,

$$\left|\frac{17}{20} - \frac{3}{5}\right|, \left|1 - \frac{3}{4}\right|, \left|\frac{9}{10} - \frac{13}{20}\right| = \left|\frac{-5}{20}\right|, \left|\frac{1}{4}\right|, \left|\frac{5}{20}\right| = \left|\frac{1}{4}\right|, \left|\frac{1}{4}\right|, \left|\frac{1}{4}\right|.$$

Which shows that 25% more effort of production, demand and service has to increase to fulfill the people's need of our country.

4.5. Conclusions

When the effect of the object is enough to study with the information of three surface of the body it is suggested to use three-dimensional imprecise numbers definition. Resultant of interaction among the body in common and whole effect of the system is complex to solve and represent in mathematically. But using the definition of intersection and the union of three-dimensional imprecise numbers, we can solve those problems. Multiple objects containing three variables may decompose at the same time. These situations can be studied with the help of properties of three-dimensional imprecise numbers. So, the properties of classical set theory are discussed in three-dimensional imprecise number in the various sections of this chapter.