Chapter 5

INTRODUCTION OF Nth DIMENSIONAL IMPRECISE NUMBERS

5.1. Introduction

N^{th} dimensional imprecise number is the study of effect of the objects, which is possible to discuss by taking n^{th} co-ordinate axis. To get more effective solution of any physical problem, we need the solution of whole dimension of the body.

Here, concept of two and three-dimensional imprecise number is extended into N^{th} dimensional imprecise numbers so that we can study the effect of impreciseness on the whole body. Identification of effect of the impreciseness characters in the specific dimension will help to solve many difficult practical problems. For examples, if a signal is set up for the safety purposes, its successful is depend on what percentage is visible from all the corner is the example of N^{th} dimensional imprecise number. And how much percentage is not visible is the complement of N^{th} dimensional imprecise numbers.

We introduce the definition of intersection and union of the N^{th} dimensional imprecise numbers with the help of maximum and minimum operators. Using these definitions all the properties of classical set occurred under the intersection and union operations are proposed for n^{th}-dimensional imprecise numbers.

5.2. N^{th} Dimensional Imprecise Numbers

N^{th} dimensional imprecise numbers is expressible in n^{th} co-ordinate geometry system comprising of n^{th} number of different faces. Here, imprecise number is defined in the n^{th}-dimensional co-ordinate system in such a way that full membership along the X_1-axis, the X_2 -axis …….Xn axes respectively is considered as a membership value one.

5.2.1. Definition: The N^{th} dimensional imprecise number

$N_{X_1X_2\ldots\ldots X_n} = [(\alpha_{x_1}, \alpha_{x_2} \ldots\ldots, \alpha_{x_n}); (\beta_{x_1}, \beta_{x_2} \ldots\ldots, \beta_{x_n}); (\gamma_{x_1}, \gamma_{x_2} \ldots\ldots\ldots\ldots, \gamma_{x_n})]$...(5.1)

is divided into sub intervals with a partial element is presence in both the intervals. Where all the points in this interval are elements of Cartesian product $X_1 \times X_2 \times \ldots \times X_n$ of n^{th} sets and X_1, X_2,……..X_n are individually imprecise numbers.

5.2.2. Definition: An element of partial presence of the N^{th}-dimensional imprecise numbers,

$N_{X_1X_2.......X_n} = [(\alpha_{x_1}, \alpha_{x_2} \dots \dots, \alpha_{x_n}); (\beta_{x_1}, \beta_{x_2} \dots .., \beta_{x_n}); (\gamma_{x_1}, \gamma_{x_2} \dots \dots \dots \dots ., \gamma_{x_n})]$ is described by the present level indicator function $p(x_1, x_2, \dots \dots, x_n)$ which is counted from the reference function, $r(x_1, x_2, \dots \dots, x_n)$ such that present level indicator for any $(x_1, x_2, \dots \dots, x_n)$, $(\alpha_{x_1}, \alpha_{x_2} \dots \dots, \alpha_{x_n}) \le (x_1, x_2, \dots \dots, x_n) \le (\gamma_{x_1}, \gamma_{x_2} \dots \dots \dots \dots ., \gamma_{x_n})$ is $(p(x_1, x_2, \dots \dots, x_n) - r(x_1, x_2, \dots \dots, x_n))$.

Where, $(0,0, \dots .., 0) \le r(x_1, x_2, \dots \dots, x_n) \le p(x_1, x_2, \dots \dots, x_n) \le (1,1, \dots \dots ., 1)$.

5.2.3. Definition: Indicator function of N^th^ -dimensional imprecise number $N_{X_1X_2.......X_n} = [(\alpha_{x_1}, \alpha_{x_2} \dots \dots, \alpha_{x_n}); (\beta_{x_1}, \beta_{x_2} \dots .., \beta_{x_n}); (\gamma_{x_1}, \gamma_{x_2} \dots \dots \dots \dots ., \gamma_{x_n})]$ is represented and defined by

$$\rho_{N_{X_1X_2.......X_n}}(x_1, x_2, \dots \dots, x_n)$$

$$= \begin{cases} {\rho^1}_{X_1X_2.......X_n}(x_1, x_2, \dots, x_n); (\alpha_{x_1}, \alpha_{x_2}, \dots, \alpha_{x_n}) \le (x_1, x_2 \dots, x_n) \le (\beta_{x_1}, \beta_{x_2} .., \beta_{x_n}) \\ {\rho^2}_{X_1X_2.......X_n}(x_1, x_2, \dots, x_n);\ (\beta_{x_1}, \beta_{x_2} .., \beta_{x_n}) \le (x_1, x_2, \dots, x_n) \le (\gamma_{x_1}, \gamma_{x_2} \dots ., \gamma_{x_n}) \\ 0\ ; \qquad otherwise \end{cases}$$

……………………………..(5.2)

Such that ${\rho^1}_{X_1X_2.......X_n}(\alpha_{x_1}, \alpha_{x_2} \dots \dots, \alpha_{x_n}) = {\rho^2}_{X_1X_2.......X_n}(\gamma_{x_1}, \gamma_{x_2} \dots \dots \dots \dots ., \gamma_{x_n}) = (0,0, \dots .., 0)$ and ${\rho^1}_{X_1X_2.......X_n}(\beta_{x_1}, \beta_{x_2} \dots .., \beta_{x_n}) = {\rho^2}_{X_1X_2.......X_n}(\beta_{x_1}, \beta_{x_2} \dots .., \beta_{x_n})$.

Where ${\rho^1}_{X_1X_2.......X_n}(\alpha_{x_1}, \alpha_{x_2} \dots \dots, \alpha_{x_n})$ is non-decreasing function over the interval $[(\alpha_{x_1}, \alpha_{x_2} \dots \dots, \alpha_{x_n}), (\beta_{x_1}, \beta_{x_2} \dots \dots, \beta_{x_n})]$ and ${\mu^2}_{X_1X_2.......X_n}(x_1, x_2, \dots \dots, x_n)$ is non-increasing over the interval $[(\beta_{x_1}, \beta_{x_2} \dots \dots, \beta_{x_n}), (\beta_{x_1}, \beta_{x_2} \dots . \beta_{x_n})]$ respectively. Then,

Case I: N^th^-dimensional normal imprecise number if

${\rho^1}_{X_1X_2.......X_n}(\alpha_{x_1}, \alpha_{x_2} \dots \dots, \alpha_{x_n}) = {\rho^2}_{X_1X_2.......X_n}(\gamma_{x_1}, \gamma_{x_2} \dots \dots \dots \dots ., \gamma_{x_n}) = (0,0, \dots .., 0)$ and ${\rho^1}_{X_1X_2.......X_n}(\beta_{x_1}, \beta_{x_2} \dots .., \beta_{x_n}) = {\rho^2}_{X_1X_2.......X_n}(\beta_{x_1}, \beta_{x_2} \dots .., \beta_{x_n}) = (1,1,1, \dots \dots, 1)$

………………….(5.3)

Case II: N^th^ -dimensional subnormal imprecise number if

${\rho^1}_{X_1X_2.......X_n}(\alpha_{x_1}, \alpha_{x_2} \dots \dots, \alpha_{x_n}) = {\rho^2}_{X_1X_2.......X_n}(\gamma_{x_1}, \gamma_{x_2} \dots \dots \dots \dots ., \gamma_{x_n}) = (0,0, \dots .., 0)$ and ${\rho^1}_{X_1X_2.......X_n}(\beta_{x_1}, \beta_{x_2} \dots .., \beta_{x_n}) = {\rho^2}_{X_1X_2.......X_n}(\beta_{x_1}, \beta_{x_2} \dots .., \beta_{x_n}) \ne (1,1,1, \dots \dots, 1)$

………………….(5.4)

And $\left({\rho^1}_{X_1X_2.......X_n}(x_1, x_2, \dots \dots, x_n) - {\rho^2}_{X_1X_2.......X_n}(x_1, x_2, \dots \dots, x_n)\right)$

$$= (\alpha_{x_1} - \beta_{x_1}) \times (\alpha_{x_2} - \beta_{x_2}) \times \dots\dots..\times (\alpha_{x_n} - \beta_{x_n})$$

…………………………………………………(5.5)

is called membership value of the indicator function $\rho_{N_{X_1X_2\dots\dots X_n}}(x_1, x_2, \dots\dots, x_n)$.

Where $\rho^1{}_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n) = (\alpha_{x_1}, \alpha_{x_2} \dots\dots, \alpha_{x_n})$and

$$\rho^2{}_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n) = (\beta_{x_1}, \beta_{x_2} \dots.\dots, \beta_{x_n})$$

5.2.4. Definition: For the nth-dimensional normal imprecise number $\rho_{N_{X_1X_2\dots\dots X_n}}$ $= \{(\rho_{N_{X_1X_2\dots\dots X_n}}(x_1, x_2, \dots\dots, x_n)), (0,0, \dots\dots,0) : (x_1, x_2, \dots\dots, x_n) \in X_1 \times X_2 \times \dots \times X_n\}$ as defined above, the complement $\rho_{N^c{}_{X_1X_2\dots\dots X_n}}$ $= \{(1,1, \dots\dots.,1), (\rho_{N_{X_1X_2\dots\dots X_n}}(x_1, x_2, \dots\dots, x_n))) : (x_1, x_2, \dots\dots, x_n) \in X_1 \times X_2 \times \dots \times X_n$, will have membership function $(1,1,1, \dots\dots,1)$ and the reference function $\rho_{N_{X_1X_2\dots\dots X_n}}(x_1, x_2, \dots\dots, x_n) < 1$ for $-\infty < (x_1, x_2, \dots\dots, x_n) < \infty$

Thus the nth-dimensional imprecise numbers is characterized by

$$\{(\rho^1{}_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n)), (\rho^2{}_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n)) : (x_1, x_2, \dots\dots, x_n) \in X_1 \times X_2 \times \dots \times X_n\}.$$

Where, $\rho^1{}_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n)$ and $\rho^2{}_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n)$ are called membership function and the reference function of the indicator function $\rho_{N_{X_1X_2\dots\dots X_n}}(x_1, x_2, \dots\dots, x_n)$. And

$$(\rho^1{}_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n) - \rho^2{}_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n))$$
$$= (\alpha_{x_1} - \beta_{x_1}) \times (\alpha_{x_2} - \beta_{x_2}) \times \dots\dots..\times (\alpha_{x_n} - \beta_{x_n})$$

…………………………..(5.6)

is called the membership value of the indicator function.

Where $\rho^1{}_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n) = (\alpha_{x_1}, \alpha_{x_2} \dots\dots, \alpha_{x_n})$

and $\rho^2{}_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n) = (\beta_{x_1}, \beta_{x_2} \dots.\dots, \beta_{x_n})$ respectively.

If the membership value is equal to 1, then the imprecise number is called the nth dimensional normal imprecise number otherwise subnormal.

5.2.5. Definition: Intersection and union of Nth-dimensional imprecise numbers is defined as follows.

If $A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$=\left\{\left(\rho^1{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right),\left(\rho^2{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right):(x_1,x_2,\ldots\ldots,x_n)\in X_1\times X_2\times\ldots\times X_n\right\},$$

And $B\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$=\left\{(\rho^3{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)),(\rho^4{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)):(x_1,x_2,\ldots\ldots,x_n)\in X_1\times X_2\times\ldots\times X_n\right\},$$

Then, intersection and the union of imprecise numbers of Nth-dimensional is defined by

$$A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cap B\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$$
$$=\left\{\begin{array}{c}min\left(\rho^1{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n),\rho^3{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right),\\ max\left(\rho^2{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n),\rho^4{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right);\\ (x_1,x_2,\ldots\ldots,x_n)\in X_1\times X_2\times\ldots\times X_n\end{array}\right\}$$

…………………..(5.7)

$$A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cup B\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$$
$$=\left\{\begin{array}{c}max\left(\rho^1{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n),\rho^3{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right),\\ min\left(\rho^2{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n),\rho^4{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right):\\ (x_1,x_2,\ldots\ldots,x_n)\in X_1\times X_2\times\ldots\times X_n\end{array}\right\}$$

……………………(5.8)

5.3. Properties of Nth-dimensional Imprecise Numbers

Based on classical set theory properties under the operations of intersection and union we can obtain the nth dimensional imprecise numbers.

5.3.1. Property(Universal Laws)

(i) $A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cap A^C\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$=\emptyset\left(\mu_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$$

and (ii) $A\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)\cup A^C\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)$

$$=\Omega\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)$$

Where, $A^C\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right),\emptyset\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)$ and $\Omega\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)$ complement, null and the universal of N[th]-dimensional imprecise numbers respectively.

Proof:

Let $A\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)$

$$=\left\{\left({\rho^1}_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots,x_n)\right),(0,0,0\dots\dots0):(x_1,x_2,\dots\dots,x_n)\in X_1\times X_2\times\dots\times X_n\right\}$$

$$=\left\{\left({\rho^1}_{X_1}(x_1),{\rho^1}_{X_2}(x_2)\dots\dots,{\rho^1}_{X_n}(x_n)\right),\left((0,0,0\dots\dots0)\right):x_i\in X_i;1\le i\le n\in N\right\}$$

$$=\left\{\left({\rho^1}_{X_i}(x_i)\right),(0):x_i\in X_i;1\le i\le n\in N\right\}$$

$A^C\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)$

$$=\left\{(1,1,1,\dots.,1),\left({\rho^1}_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots,x_n)\right):(x_1,x_2,\dots\dots,x_n)\in X_1\times X_2\times\dots\times X_n\right\}$$

$$=\left\{(1,1,1,\dots.,1),\left({\rho^1}_{X_1}(x_1),{\rho^1}_{X_2}(x_2)\dots\dots,{\rho^1}_{X_n}(x_n)\right):x_i\in X_i;1\le i\le n\in N\right\}$$

$$=\left\{(1),\left({\rho^1}_{X_i}(x_i)\right):x_i\in X_i;1\le i\le n\in N\right\},$$

Where, $0<{\rho^1}_{X_i}(x_i)<1;\ i=1,2,3\dots\dots\dots.n$ are individually imprecise numbers for the respective dimension.

Now, $A\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)\cap A^C\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)$

$$=\left\{\left(\min\left({\rho^1}_{X_i}(x_i),1\right)\right),\left(\max(0,{\rho^1}_{X_i}(x_i)\right):x_i\in X_i;1\le i\le n\in N\right\}$$

$$=\left\{\left({\rho^1}_{X_1}(x_1),{\rho^1}_{X_2}(x_2)\dots\dots,{\rho^1}_{X_n}(x_n)\right),\left({\rho^1}_{X_1}(x_1),{\rho^1}_{X_2}(x_2)\dots\dots,{\rho^1}_{X_n}(x_n)\right)\right\}$$

Its membership value is, $\left({\rho^1}_{X_1}(x_1)-{\rho^1}_{X_1}(x_1)\right)\times\left({\rho^1}_{X_1}(x_1)-{\rho^1}_{X_1}(x_1)\right)\times\dots\dots..\times$

$$\left({\rho^1}_{X_1}(x_1)-{\rho^1}_{X_1}(x_1)\right)=0.$$

So the intersection of N^th-dimensional imprecise number and its complement is a null set.

And $A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cup A^C\ \left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$=\left\{\left(\max\left(\rho^1{}_{X_i}(x_i),1\right)\right),\left(\min(0,\rho^1{}_{X_i}(x_i)\right):x_i\in X_i;1\le i\le n\ \in N\right\}$$

$$=\{(1)(0):x_i\in X_i;1\le i\le n\in N\}$$

$$=\{(1,1\ldots\ldots,1)(0,0\ldots\ldots,0)\}$$

Its membership value is, $(1-0)\times(1-0)\times\ldots\ldots..\times(1-0)=1$.

So, union of N^th-dimensional imprecise number and its complement is the universal set.

Remaining properties are discussed at the below.

5.3.2. Property (Commutative laws)

If $A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$=\left\{\left(\rho^1{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right),\left(\rho^2{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right):(x_1,x_2,\ldots\ldots,x_n)\in X_1\times X_2\times\ldots\times X_n\right\},$$

And $B\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$=\left\{\begin{matrix}\left(\rho^3{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right),\left(\rho^4{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\\ :(x_1,x_2,\ldots\ldots,x_n)\in X_1\times X_2\times\ldots\times X_n\end{matrix}\right\},$$

be the N^th dimensional imprecise numbers. Then,

(i) $A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cup B\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$=B\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cup A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$$

(ii) $A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cap B\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$=B\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cap A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$$

These properties are obvious true.

5.3.3. Property (Distributive Laws)

If $A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$=\left\{\begin{matrix}\left(\rho^1{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right),\left(\rho^2{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\\ :(x_1,x_2,\ldots\ldots,x_n)\in X_1\times X_2\times\ldots\times X_n\end{matrix}\right\}$$

$$B\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)$$

$$=\left\{\begin{matrix}\left({\rho^3}_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right),\left({\rho^4}_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)\\ :(x_1,x_2,\dots\dots,x_n)\in X_1\times X_2\times\dots\times X_n\end{matrix}\right\} \text{ and}$$

$$C\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)$$

$$=\left\{\begin{matrix}\left({\rho^3}_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right),\left({\rho^4}_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)\\ :(x_1,x_2,\dots\dots,x_n)\in X_1\times X_2\times\dots\times X_n\end{matrix}\right\}$$

be three N^th dimensional imprecise numbers. Then,

(i) $$A\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)\cap\left(B\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)\cup C\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)\right)$$

$$=A\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)\cap B\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)$$
$$\cup A\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)\cap C\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)$$

(ii) $$A\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)\cup\left(B\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)\cap C\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)\right)$$

$$=A\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)\cup B\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)$$
$$\cap A\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)\cup C\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)$$

Proof:

Let, $A\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)$

$$=\left\{\left({\rho^1}_{X_1}(x_1),{\rho^1}_{X_2}(x_2)\dots\dots,{\rho^1}_{X_n}(x_n)\right),(0,0,\dots,0):x_i\in X_i;1\le i\le n\in N\right\}$$

$B\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)$

$$=\left\{\left({\rho^2}_{X_1}(x_1),{\rho^2}_{X_2}(x_2)\dots\dots,{\rho^2}_{X_n}(x_n)\right),(0,0,\dots,0):x_i\in X_i;1\le i\le n\in N\right\}$$

$C\left(\rho_{X_1X_2\dots\dots X_n}(x_1,x_2,\dots\dots,x_n)\right)$

$$=\left\{\left({\rho^3}_{X_1}(x_1),{\rho^3}_{X_2}(x_2)\dots\dots,{\rho^3}_{X_n}(x_n)\right),(0,0,\dots,0):x_i\in X_i;1\le i\le n\in N\right\}$$

Where, $0 < \rho^1{}_{X_i}(x_i) < \rho^2{}_{X_i}(x_i) < \rho^3{}_{X_i}(x_i) < 1;\ i = 1,2,3 \dots \dots \dots . n$ are individually imprecise numbers for the respective dimension. Now,

(i) $$A\left(\rho_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots \dots, x_n)\right) \cap \left(B\left(\rho_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots \dots, x_n)\right) \cup C\left(\rho_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots \dots, x_n)\right)\right)$$

$$= \left((\rho^1{}_{X_i}), (0)\right) \cap \left(\left(max\left(\rho^2{}_{X_i}(x_i), \rho^3{}_{X_i}(x_i)\right)\right), \left(min(0,0)\right)\right); i = 1,2, . . , n$$

$$= \left((\rho^1{}_{X_i}), (0)\right) \cap \left((\rho^3{}_{X_i}), (0)\right); i = 1,2, \dots, n$$

$$= \left(\left(min\left(\rho^2{}_{X_i}(x_i), \rho^3{}_{X_i}(x_i)\right)\right), \left(max(0,0)\right)\right); i = 1,2, \dots, n$$

$$= \left((\rho^1{}_{X_i}), (0)\right); = 1,2, \dots, n$$

$$A\left(\rho_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots \dots, x_n)\right) \cap B\left(\rho_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots \dots, x_n)\right) \cup A\left(\rho_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots \dots, x_n)\right) \cap C\left(\rho_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots \dots, x_n)\right)$$

$$= \left(\left(min\left(\rho^1{}_{X_i}(x_i), \rho^2{}_{X_i}(x_i)\right)\right), \left(max(0,0)\right)\right) \cup \left(\left(min\left(\rho^1{}_{X_i}(x_i), \rho^3{}_{X_i}(x_i)\right)\right), \left(max(0,0)\right)\right); i = 1,2, \dots, n$$

$$= \left((\rho^1{}_{X_i}), (0)\right) \cup \left((\rho^1{}_{X_i}), (0)\right); = 1,2, \dots, n$$

$$= \left(\left(max\left(\rho^1{}_{X_i}(x_i), \rho^1{}_{X_i}(x_i)\right)\right), \left(min(0,0)\right)\right); i = 1,2, \dots, n$$

$$= \left((\rho^1{}_{X_i}), (0)\right); = 1,2, \dots, n$$

Hence Proved

Similarly, proof of the property 5.4.3. (ii) can be done.

5.3.4. Property(Idempotence Laws)

(i) $$A\left(\rho_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots \dots, x_n)\right) \cap A\left(\rho_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots \dots, x_n)\right) = A\left(\rho_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots \dots, x_n)\right)$$

and (ii) $A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right) \cup A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$= A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$$

where, $A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$= \left\{\begin{matrix}\left({\rho^1}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right),\left({\rho^2}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right) \\ :(x_1,x_2,\ldots\ldots,x_n)\in X_1\times X_2\times\ldots\times X_n\end{matrix}\right\}.$$

5.3.5. Property (Identity Laws)

(i) $A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right) \cap \emptyset\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$= \emptyset\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$$

(ii) $A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right) \cup \emptyset\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$= A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$$

(ii) $A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right) \cap X\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$= A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$$

(iv) $A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right) \cup X\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$= X\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$$

Where, $X\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$ is the universal set and $\emptyset\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$ is the null set respectively.

The property is obviously true.

5.3.6. Property (Associatively Laws)

If $A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$= \left\{\begin{matrix}\left({\rho^1}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right),\left({\rho^2}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right) \\ :(x_1,x_2,\ldots\ldots,x_n)\in X_1\times X_2\times\ldots\times X_n\end{matrix}\right\},$$

$B\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$= \left\{\begin{matrix}\left({\rho^3}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right),\left({\rho^4}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right) \\ :(x_1,x_2,\ldots\ldots,x_n)\in X_1\times X_2\times\ldots\times X_n\end{matrix}\right\}$$

and $C\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$=\left\{\begin{matrix}\left(\rho^5{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right),\left(\rho^6{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\\ :(x_1,x_2,\ldots\ldots,x_n)\in X_1\times X_2\times\ldots\times X_n\end{matrix}\right\}$$ be three n^th dimensional imprecise number, then

(i) $A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cup\left(B\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cup C\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\right)=\left(A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cup B\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\right)\cup C\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

(ii) $A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cap\left(B\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cap C\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\right)=\left(A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cap B\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\right)\cap C\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

Proof:

Let, $A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$=\left\{\left(\rho^1{}_{X_1}(x_1),\rho^1{}_{X_2}(x_2)\ldots\ldots,\rho^1{}_{X_n}(x_n)\right),(0,0,\ldots,0):x_i\in X_i;1\le i\le n\right\}$$

$B\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$=\left\{\left(\rho^2{}_{X_1}(x_1),\rho^2{}_{X_2}(x_2)\ldots\ldots,\rho^2{}_{X_n}(x_n)\right),(0,0,\ldots,0):x_i\in X_i;1\le i\le n\right\}$$

$C\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$=\left\{\left(\rho^3{}_{X_1}(x_1),\rho^3{}_{X_2}(x_2)\ldots\ldots,\rho^3{}_{X_n}(x_n)\right),(0,0,\ldots,0):x_i\in X_i;1\le i\le n\right\}$$

Where, $0<\rho^1{}_{X_i}(x_i)<\rho^2{}_{X_i}(x_i)<\rho^3{}_{X_i}(x_i)<1;\ i=1,2,3\ldots\ldots\ldots.n$ are individually imprecise numbers for the respective dimension. Now,

(i) $A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cup\left(B\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cup C\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\right)$

$$=\left((\rho^1{}_{X_i}),(0)\right)\cup\left(\left(max\left(\rho^2{}_{X_i}(x_i),\rho^3{}_{X_i}(x_i)\right)\right),\left(min(0,0)\right)\right);i=1,2,..,n$$

$$= \left((\rho^1{}_{X_i}), (0)\right) \cup \left((\rho^3{}_{X_i}), (0)\right); i = 1,2, \dots, n$$

$$= \left(\left(max\left(\rho^1{}_{X_i}(x_i), \rho^3{}_{X_i}(x_i)\right)\right), \left(min(0,0)\right)\right); i = 1,2, \dots, n$$

$$= \left((\rho^3{}_{X_i}), (0)\right); = 1,2, \dots, n$$

$$\left(A\left(\rho_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n)\right) \cup B\left(\rho_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n)\right)\right) \cup C\left(\rho_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n)\right)$$

$$= \left(\left(max\left(\rho^1{}_{X_i}(x_i), \rho^2{}_{X_i}(x_i)\right)\right), \left(min(0,0)\right)\right) \cup \left((\rho^3{}_{X_i}), (0)\right); i = 1,2, \dots, n$$

$$= \left((\rho^2{}_{X_i}), (0)\right) \cup \left((\rho^1{}_{X_i}), (0)\right); = 1,2, \dots, n$$

$$= \left(\left(max\left(\rho^2{}_{X_i}(x_i), \rho^3{}_{X_i}(x_i)\right)\right), \left(min(0,0)\right)\right); i = 1,2, \dots, n$$

$$= \left((\rho^3{}_{X_i}), (0)\right); = 1,2, \dots, n$$

Hence Proved

Similarly, proof of the property 5.4.6. (ii) can be done.

5.3.7. Property (De-Morgan's Laws)

If $A\left(\rho_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n)\right)$

$$= \left\{\begin{matrix}\left(\rho^1{}_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n)\right), \left(\rho^2{}_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n)\right) \\ : (x_1, x_2, \dots\dots, x_n) \in X_1 \times X_2 \times \dots \times X_n\end{matrix}\right\},$$

$$B\left(\mu_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n)\right)$$

$$= \left\{\begin{matrix}\left(\rho^3{}_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n)\right), \left(\rho^4{}_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n)\right) \\ : (x_1, x_2, \dots\dots, x_n) \in X_1 \times X_2 \times \dots \times X_n\end{matrix}\right\}$$ be the nth dimensional imprecise number. Then,

(i) $\left(A\left(\rho_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n)\right) \cup B\left(\rho_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n)\right)\right)^C$

$$= A^C\left(\rho_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n)\right) \cap B^C\left(\rho_{X_1X_2\dots\dots X_n}(x_1, x_2, \dots\dots, x_n)\right)$$

(ii) $\left(A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cap B\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\right)^C$

$$= A^C\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cup B^C\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$$

Proof:

Let, $A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$=\left\{\left({\rho^1}_{X_1}(x_1),{\rho^1}_{X_2}(x_2)\ldots\ldots,{\rho^1}_{X_n}(x_n)\right),(0,0,\ldots,0): x_i\in X_i; 1\le i\le n\right\}$$

$B\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$=\left\{\left({\rho^2}_{X_1}(x_1),{\rho^2}_{X_2}(x_2)\ldots\ldots,{\rho^2}_{X_n}(x_n)\right),(0,0,\ldots,0): x_i\in X_i; 1\le i\le n\right\}$$

Where, $0<{\rho^1}_{X_i}(x_i)<{\rho^2}_{X_i}(x_i)<1;\ i=1,2,3\ldots\ldots\ldots.n$ are individually imprecise numbers for the respective dimension. Then,

Then,

$A^C\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$=\left\{(1,1,1\ldots\ldots,1),\left({\rho^1}_{X_1}(x_1),{\rho^1}_{X_2}(x_2)\ldots\ldots,{\rho^1}_{X_n}(x_n)\right): x_i\in X_i; 1\le i\le n\right\}$$

$B^C\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$=\left\{(1,1,1,\ldots\ldots,1),\left({\rho^2}_{X_1}(x_1),{\rho^2}_{X_2}(x_2)\ldots\ldots,{\rho^2}_{X_n}(x_n)\right): x_i\in X_i; 1\le i\le n\right\}$$

Now,

(i) $\left(A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cup B\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\right)^C$

$$=\left(\left(max\left({\rho^1}_{X_i}(x_i),{\rho^2}_{X_i}(x_i)\right)\right),\left(min(0,0)\right)\right)^C; i=1,2,\ldots,n$$

$$=\left(\left(({\rho^2}_{X_i}),(0)\right)\right)^C; 1,2,\ldots,n$$

$$=\left((1),({\rho^2}_{X_i})\right); 1,2,\ldots,n$$

$A^C\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cap B^C\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$

$$=\left(\left(min(1,1)\right),\left(max\left({\rho^1}_{X_i}(x_i),{\rho^2}_{X_i}(x_i)\right)\right)\right); i=1,2,\ldots,n$$

$$=\left((1),({\rho^2}_{X_i})\right); =1,2,\ldots,n$$

Hence Proved

(ii) $\left(A\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cap B\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\right)^C$

$$=\left(\left(min\left({\rho^1}_{X_i}(x_i),{\rho^2}_{X_i}(x_i)\right)\right),\left(max(0,0)\right)\right)^C;\ i=1,2,\ldots,n$$

$$=\left(\left(({\rho^1}_{X_i}),(0)\right)\right)^C;1,2,\ldots,n$$

$$=\left((1),({\rho^1}_{X_i})\right);1,2,\ldots,n$$

$$A^C\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)\cup B^C\left(\rho_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right)$$

$$=\left(\left(max(1,1)\right),\left(min\left({\rho^1}_{X_i}(x_i),{\rho^2}_{X_i}(x_i)\right)\right)\right);i=1,2,\ldots,n$$

$$=\left((1),({\rho^1}_{X_i})\right);=1,2,\ldots,n$$

Hence proved

5.4. Conclusions

The solution of the complex problem is depending on the study of the whole dimension of the body. So the effect of impreciseness of those type objects is suggested to study along with all the axes. Identification of the common and whole effeteness in the system is one of the very important for the study. So, intersection and union of N^th-dimensional numbers are defined with the help of maximum and minimum operators. The existence of impreciseness of the Nth-dimensional object is studied in various sections of this chapter for different properties of the classical set.