Chapter 6

CONSTRUCTION OF NORMAL IMPRECISE FUNCTION

The work presented in this chapter is published in the journal "*Advances in Computational Sciences and Technology*", Vol.10 (7) (2017), 2019-2036.

6.1. Introduction

Generally, a function is most important for the study of mathematics. In the specific time-bound, if we represent graphically the path of the activities of any object, then it will frame a mathematical function. Some of them are either in controllable or uncontrollable. The activities, which frame controllable form is usable for the practical purpose. But there are numerous activities of the object, which are not controllable till now. For this, we need to investigate the behavior of objects and control it accordingly.

The diagrams, which is gliding unreservedly without meeting the ground or real line over and again is possible to control to meet the real line more than once with the assistance of augmentation of other multiplication factors. The resultant capacity function is known to imprecise function. So our point in this chapter is to control the wild capacity into controllable capacity with the assistance of the multiplication of appropriate multiplication factors, similar to sine and cosine work so it tends to be utilized in our useful purposes.

6.2. Imprecise Functions

6.2.1. Definition**:** If the function is sectional dividable into a finite number of imprecise numbers such that all the intervals have either distinct or similar maximum value, then we call it imprecise function. If the maximum value of the function of all the sectional intervals is unique, then we call it periodic and if the maximum values of all the intervals are not similar, then we call it semi-periodic. However, both the type of functions are having imprecise number properties in the many intervals within the large interval of the function. From this point of view, it can be proposed that an imprecise function has the following properties.

- (i) Function is continuous
- (ii) Function is oscillation in nature.
- (iii) Function has maximum and minimum values
- (iv) Function is semi-periodic/periodic in nature

For example, the formation of an effect of the impreciseness in the sine and cosine function form is the interval definable number. Multiplication of the sine and the cosine function with any other function is always forming the imprecise function. This function has a finite number of intervals with each interval contains maximum and minimum values within it. Here the membership value of the impreciseness of object is obtained due to the extension definition of the fuzzy number of Baruah (2011). Which we call imprecise number.

6.2.2. Imprecise function formed by the multiplication of sine function

We have mentioned in the above that any function is possible to transform into the imprecise function with the assistance of sine function. To discuss this matter, let us consider $y = x^n, n \geq 0;\ n \in N$ be a continuous function. Graph of this function is shown in the below.

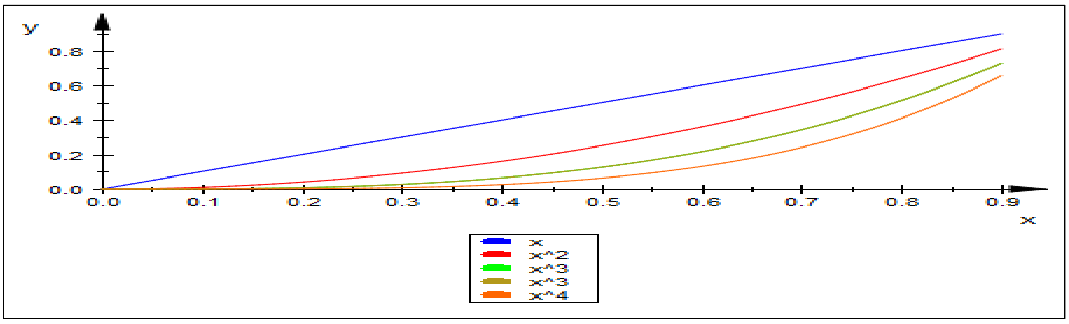

Fig.6.1. Graph of $y = x^n, n \geq 0;\ n \in N$

The given function is not imprecise function as it cuts the X-axis only at origin of the X-axis. So, the function is not controllable as it does not reach the x-axis for several times.

To convert this into imprecise function form, let us multiply it by sine function so that it becomes, $y^* = x^n \sin(x), n \geq 1;\ n \in N$

For n=1, the following data is considered to draw a graph of imprecise function, $y^* = x\, sin(x)$.

X	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Y	0	$\frac{\pi}{2}$	0	$-3\frac{\pi}{2}$	0

Table 6.1. Data for imprecise function $y^* = x\, sin(x)$

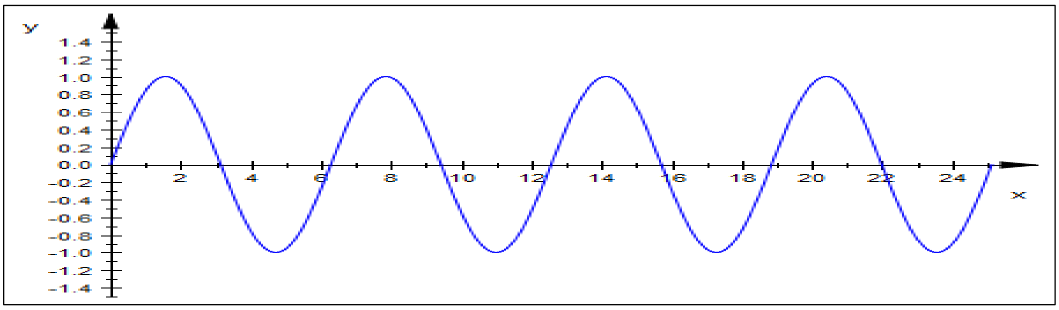

Fig.6.2. Graph of sine imprecise function $y^* = x\,sin(x)$

Here, $[0;\frac{\pi}{2};\pi]$, $[\pi;\frac{3\pi}{2};2\pi]$, $[2\pi;\frac{5\pi}{2};3\pi]$, ……$[(n-1)\pi;\frac{(2n-1)\pi}{2};\ n\pi]$ are imprecise numbers occurred towards the right part of the X-axis within the same imprecise function. In general, indicator functions of those imprecise numbers is expressible as follow.

$$\rho_{N_S}(x) = \begin{cases} \rho_1(x);\ when\ \ (n-1)\pi \le x \le \frac{(2n-1)\pi}{2} \\ \quad \rho_2(x)\ ;\ when\ \frac{(2n-1)\pi}{2} \le x \le n\pi \end{cases} ;n \in N\text{……………………..(6.1)}$$

Here, $\rho_1(x)$ is membership function and $\rho_2(x)$ is reference function of the indicator function such that

$$\rho_1(x) = 0\ at\ x = (n-1)\pi\ ,\ \rho_2(x) = 0\ \ at\ x = n\pi$$

$$\text{and}\ \ \rho_1(x) = \rho_2(x) = \frac{(2n-1)\pi}{2}\quad at\ x = \frac{(2n-1)\pi}{2}$$

………………………………..(6.2)

Here the maximum value of indicator function is found at $x = \frac{(2n-1)\pi}{2}$ and the minimum value is found at $x = 0$ and $x = n\pi$. Where the maximum value varies for the different values of n.

Thus $y^* = x\,sin(x)$ is an imprecise function.

In general $y^* = x^n \sin(x)\,, n \ge 1;\ n \in N$ is again an imprecise function having indicator function defined in (6.1) with maximum values $\left(\frac{(2n-1)\pi}{2}\right)^n$ and minimum value as 0 for all $n \in Z$. This imprecise function has a graph, which cut the x-axis for repeatedly and is shown at the below.

Table

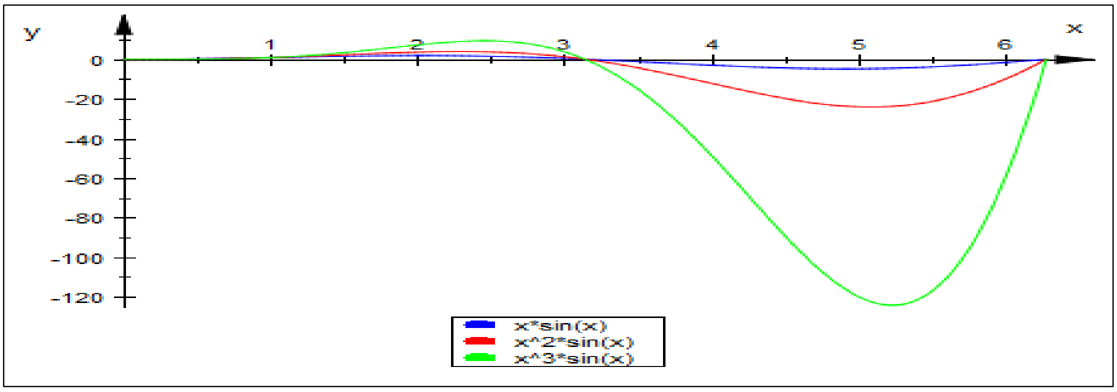

Fig.6.3. Graph of sine imprecise function $y^* = x^n\, sin(x)$

Here all the curves are semi-oscillation in nature and are forming different imprecise functions within the same interval.

6.2.3. Imprecise Functions formed by the Multiplication of Cosine Function

As, the capacity of a function is possible to transform into the imprecise function form with the assistance of cosine function. To discuss this matter, let us multiply $y = x^n, n \geq 0;\ n \in N$ by cosine function to have $y^* = x^n \cos(x), n \geq 1;\ n \in N$ an imprecise function. Here the function is semi periodic in nature and graph is shown at below.

For n=1, $y^* = x\, cos(x)$ has a graph for the following table.

X	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Y	-2π	0	π	0	0	0	$-\pi$	0	2π

Table 6.2. Data for the graph o function $y^* = x\, cos(x)$

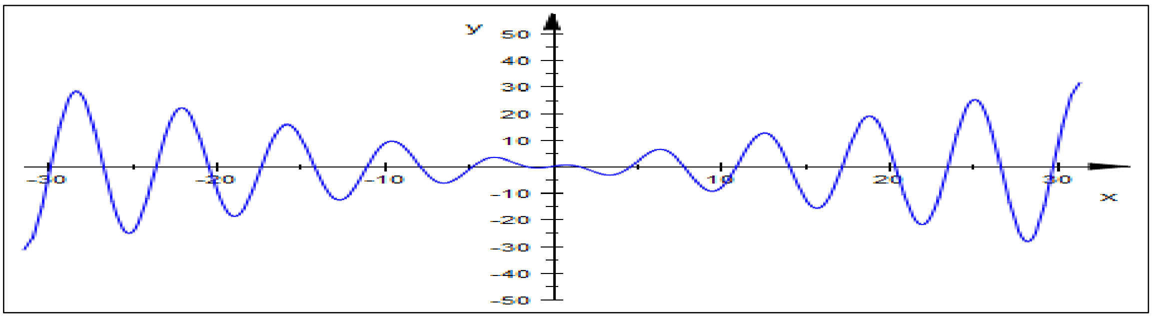

Fig.6.4. Graph of cosine imprecise function $y^* = x\,cos(x)$

Let us discuss the imprecise function for the right part of the x-axis. Thus the table 6.2. shows that $[\frac{\pi}{2};\pi;\frac{3\pi}{2}]$, $[\frac{3\pi}{2};2\pi;\frac{5\pi}{2}]$, $[\frac{5\pi}{2};3\pi;\frac{7\pi}{2}]$,…………$[\frac{(2n-1)\pi}{2};\ n\pi;\frac{(2n+1)\pi}{2}]$ are the imprecise numbers within the same imprecise function, whose indicator function is expressible in common as below.

$$\rho_{N_C}(x) = \begin{cases} \rho_1(x)\ when\ \frac{(2n-1)\pi}{2} \le x \le n\pi \\ \rho_2(x)\ \ when\ n\pi \le x \le \frac{(2n+1)\pi}{2} \end{cases} ; n \in N \text{……………………(6.3)}$$

Here, $\rho_1(x)$ and $\rho_2(x)$ are membership function and reference function of the indicator function such that

$$\rho_1(x) = 0\ at\ x = \frac{(2n-1)\pi}{2},\ \rho_2(x) = 0\ \ at\ x = \frac{(2n+1)\pi}{2}$$

$$\text{and}\ \ \rho_1(x) = \rho_2(x) = \pm n\pi\ \ \ at\ x = n\pi$$

……………………………(6.4)

Here the maximum value of indicator function (6.3) is found at $x = n\pi; \forall\, n \in N$ and the minimum value is found at $x = \frac{(2n-1)\pi}{2}$ and $= \frac{(2n+1)\pi}{2}$ for $n \in N$. Where the maximum value varies for different values of n. Thus $y^* = x\,cos(x)$ is a normal imprecise function.

In general $y^* = x^n \cos(x)\,, n \ge 1;\ n \in N$ is also an imprecise function having indicator function defined in (6.3) and the maximum value is $(n\pi\,)^n$ and minimum value is 0 for all $n \in N$.

Thus we call it an imprecise function as it cuts the x-axis repeatedly and is shown at the below.

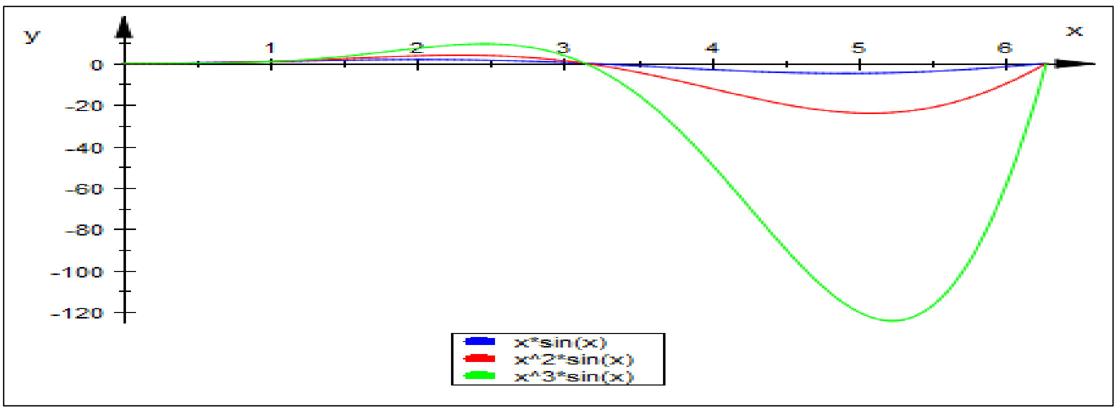

Fig.6.5. Graph of cosine imprecise function $y^* = x^n \, cos(x)\, , n \geq 1; \forall\, n \in N$

Here, the curves are oscillation in nature and are forming different imprecise functions within the same interval for different value of $n \in N$.

6.3. Conversion Points

It is a point from where non-imprecise function starts to convert into imprecise function. If we can define this point, then we can add necessary objects along with the function properly for our practical purposes.

6.3.1. Conversion points obtained by sine function

From the Fig. 6.5. It is observed that conversion point of imprecise function varies as the value of "n" varies. Thus, for $n = 1, y = x$ starts to oscillate towards the positive x-axis from the point $(\frac{\pi}{2}, \frac{\pi}{2})$ due to the multiplication of $\sin(x)$ to form an imprecise function $y^* = x \sin(x)$.

If $n = 2, y = x^2$ start to oscillate towards positive x-axis from the point $\left(\frac{\pi}{2}, \quad \left(\frac{\pi}{2}\right)^2\right)$ due to multiplication of $\sin(x)$ to form an imprecise function $y^* = x^2 \sin(x)$.

If $n = 3, y = x^3$ start to oscillate towards the positive x-axis from the point $\left(\frac{\pi}{2}, \quad \left(\frac{\pi}{2}\right)^3\right)$ due to multiplication of $\sin(x)$ to form an imprecise function $y^* = x^3 \sin(x)$ and so on.

Thus in general $\left(\frac{\pi}{2}, \quad \left(\frac{\pi}{2}\right)^n\right)$ is the conversion point of a function $y = x^n$ with respect to $\sin(x)$.

6.3.2. Conversion points obtained by cosine function

If $n = 1,\ then\, y = x$ tends to oscillate from the point $\left(\frac{\pi}{4}, \frac{\pi}{4} \cos\left(\frac{\pi}{4}\right)\right)$ towards positive x-axis due to multiplication of $\cos(x)$ to have an imprecise function $y^* = x \cos(x)$.

If $n = 2, then\, y = x^2$ tens to oscillate from the point $\left(\frac{\pi}{4}, \quad \left(\frac{\pi}{4}\right)^2 \cos\left(\left(\frac{\pi}{4}\right)^2\right)\right)$ towards the positive x-axis due to multiplication of $\cos(x)$ to have an imprecise function $y^* = x^2 \cos(x)$.

If $n = 3, y = x^3$ tens to oscillate from the point $\left(\frac{\pi}{4}, \left(\frac{\pi}{4}\right)^3 \cos\left(\left(\frac{\pi}{4}\right)^3\right)\right)$ towards positive x-axis due to multiplication of $\cos(x)$ to have an imprecise function $y^* = x^3 \cos(x)$ and so on.

Thus, in general, $\left(\frac{\pi}{4}, \left(\frac{\pi}{4}\right)^n \cos\left(\left(\frac{\pi}{4}\right)^n\right)\right)$ is a conversion point of a function $y = x^n; \forall\, n \in N$ which lead to transform function into imprecise form with respect to $\cos(x)$ towards the positive x-axis.

6.4. Rate effect of Impreciseness of the Imprecise Functions

Rate effect of impreciseness of the function is the ratio of the area/volume occupied by the objects and the area/volume of the place where the object is used. This ratio gives us information to know whether the designed imprecise function is useful for some particular place or not. When it is not properly designed then we need further modification. Normally 1(One) is the value of the rate effect of the impreciseness of object for which a function is well designed for a particular place. So to know the use of the effectiveness of the object for a particular medium it is essential to know what is the capacity of an object that can cover the area.

In the defining area of imprecise function some of the intervals is not allowed to use as the limit of integration. For the example,

$$\int_0^{2\pi} \sin(x)\, dx = 0, \int_0^{\pi} \sin(2x)\, dx = 0$$

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos(x)\, dx = 0, \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \cos(2x)\, dx = 0$$

etc. For this matter, area of an imprecise function is obtained with the assistance of imprecise number. Maximum and the minimum value point is used as the limit of integration. These limits help us to get the area of imprecise function for any interval in the summation form. In the every half period of sine imprecise function, we get an imprecise number.

Thus, right part area of x-axis of the imprecise function $y^* = x \sin(x)$ is obtained with the assistance of imprecise numbers,

$\left[0;\frac{\pi}{2};\pi\right],\left[\pi;\frac{3\pi}{2};2\pi\right],[2\pi;\frac{5\pi}{2};3\pi]........[(n-1)\pi;\frac{(2n-1)\pi}{2};n\pi]\ ;\forall\ n\epsilon N.$

Here, the sine function has an angle with integral multiple of 1(one). So to have an area of the half period of this imprecise function, we take integral multiple 1(one) with π is the limit of integration and is shown in below.

$$\int_0^{\pi} x\sin(x)\,dx = \pi \text{ square unit}$$

$$\int_{\pi}^{2\pi} x\sin(x)\,dx = -3\,\pi \text{ square unit}$$

$$\int_{2\pi}^{3\pi} x\sin(x)\,dx = 5\,\pi \text{ square unit}$$

So on.

In general, area of the imprecise function $y^* = x\sin(x)$ in the interval $[(n-1)\pi, n\pi]$ is obtained with assistance of integration and is shown in below.

$$\int_{(n-1)\pi}^{n\pi} x\sin(x)\,dx = (-xcosx)_{(n-1)\pi}^{n\pi} + \int_{(n-1)\pi}^{n\pi} cosx\,dx = (-1)^{n+1}(2n-1)\pi + 0$$

$$= (-1)^{n+1}(2n-1)\pi;\forall\ n\in N$$

Where, negative and positive signs are the below and above area signs of the axis.

Thus the total area of imprecise function $y^* = x\ sin\ (x)$ of the positive axis is

$$\pi + 3\pi + 5\pi + 7\pi + \cdots \ldots\ldots\ldots\ldots..to\ \infty = \pi(1+3+5+7+\cdots\ldots\ldots\ldots\ldots\ldots.to\ \infty)$$

$$= \sum(2n-1)\pi$$

Thus the rate effect of the impreciseness of imprecise function given by

$$\frac{(2n-1)\pi}{Total\ area\ of\ the\ experiment}\ ;\forall\ n\in I^+$$

And the total rate effect of the impreciseness over the axis of any object is given by

$$\frac{2\sum(2n-1)\pi}{Total\ area\ of\ the\ experiment}\ ;\forall\ n\in Z^+$$

Area of imprecise function $y^* = x\cos(x)$ along the right part of the x-axis is obtained with the assistance of interval $\left[0,\frac{\pi}{2}\right]$and imprecise numbers,

$\left[\frac{\pi}{2};\pi;\frac{3\pi}{2}\right],,\left[\frac{3\pi}{2};2\pi;\frac{5\pi}{2}\right],\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\left[\frac{(2n-1)\pi}{2};n\pi;\frac{(2n+1)\pi}{2}\right];\forall\ n\epsilon N$

Here, angle of cosine is the integral multiple of 1(one), so, we take integral multiple of 1(one) with $\frac{\pi}{2}$ as the limit of the integration, which is shown in the below.

$$\int_0^{\frac{\pi}{2}} x\cos(x)\,dx = \frac{\pi}{2} - 1 \text{ square unit}$$

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} x\cos(x)\,dx = -2\,\pi \text{ square unit}$$

$$\int_{\frac{3\pi}{2}}^{\frac{5\pi}{2}} x\cos(x)\,dx = 4\pi \text{ square unit}$$

So on.

In general, area of imprecise function $y^* = x\cos(x)$ in the interval $\left[\frac{(2n-1)\pi}{2}, \frac{(2n+1)\pi}{2}\right]$ is the area of indicator function of the imprecise number $\left[\frac{(2n-1)\pi}{2}, n\pi, \frac{(2n+1)\pi}{2}\right]$. Thus,

$$\int_{\frac{(2n-1)\pi}{2}}^{\frac{(2n+1)\pi}{2}} x\cos(x)\,dx = (xsin(x)_{\frac{(2n-1)\pi}{2}}^{\frac{(2n+1)\pi}{2}} + \int_{\frac{(2n-1)\pi}{2}}^{\frac{(2n+1)\pi}{2}} \sin(x)\,dx = (-1)^n 2n\pi + 0$$

$$= (-1)^n 2n\pi; \forall\, n \in N$$

Where, negative and positive sign are the above and below area signs of the x-axis.

So the total area of imprecise function $y^* = xcos(x)$ of the positive X-axis is

$$1 - \frac{\pi}{2} + 2\pi + 4\pi + 6\pi + \cdots \ldots..to\ \infty = 1 - \frac{\pi}{2} + \pi(1 + 3 + 5 + 7 + \cdots \ldots \ldots.to\ \infty)$$

$$= [1 - \frac{\pi}{2} + \sum (2n-1)\pi]$$

Thus the rate effect of impreciseness over the axis of the object is given by

$$\frac{1 - \frac{\pi}{2} + (2n-1)\pi}{Total\ area\ of\ the\ experiment}\ ; \forall\, n \in I^+$$

And the total rate effect of impreciseness over the axis of the object is given by

$$\frac{2[1 - \frac{\pi}{2} + \sum(2n-1)\pi]}{Total\ area\ of\ the\ experiment}\ ; \forall\, n \in Z^+$$

6.5. Propositions

6.5.1. Proposition: If the total rate effect of the impreciseness of a sine imprecise function is $\left\{\frac{2\sum(2n-1)\pi}{Total\ area\ of\ the\ experiment}\ ; \forall\, n \in Z^+\right\} \leq 1$, then function $y^* = x\sin(x)$ is known as the convergence for the place.

6.5.2. Proposition: If the total rate effect of the impreciseness of a sine imprecise function is $\left\{\frac{2\sum(2n-1)\pi}{Total\ area\ of\ the\ experiment}\ ; \forall\, n \in Z^+\right\} > 1$, then function $y^* = x\sin(x)$ is known as not convergence for the place.

Thus the sound travelled by the object in a particular place will be audible or not is depending on the condition of proposition 6.5.1.(i) and 6.5.1.(ii). If the condition is not convergence, then either decreasing angle of sine function helps us to keep them in convergence mode.

Variance of effect of the impreciseness of a sine imprecise function $y^* = x\sin(x)$ is obtained as $\sigma^2 = \frac{\mathbf{4}\sum\{(2n-1)\pi\}^{\mathbf{2}}}{\boldsymbol{n}}$

Standard deviation of the effect of sine imprecise function $y^* = x\sin(x)$ is $\sqrt{\frac{\mathbf{4}\sum\{(2n-1)\pi\}^{\mathbf{2}}}{\boldsymbol{n}}}$

6.5.3. Proposition: If the total rate effect of impreciseness caused by cosine having function is $\left\{\frac{2\left[1-\frac{\pi}{2}+\sum(2n-1)\pi\right]}{Total\ area\ of\ the\ experiment}\ ; n \in Z^+\right\} \leq 1$, then function $y^* = x\cos(x)$ is known as convergence for the place.

6.5.4 Proposition: If the total rate effect of impreciseness caused by a cosine imprecise function is $\left\{\frac{2\left[1-\frac{\pi}{2}+\sum(2n-1)\pi\right]}{Total\ area\ of\ the\ experiment}\ ; n \in Z^+\right\} > 1$, then function $y^* = x\cos(x)$ is known as not convergence for the place.

Thus signal covered by the object in a particular place will be completely visible or not is depending on the condition of proposition 6.5.3.(i) and 6.5.4.(ii). If the condition is not convergence, then either decreasing the angle of cosine function helps us to keep them in convergence mode.

Variance of the effect of impreciseness of the cosine imprecise function $y^* = x\cos(x)$ is obtained as $\sigma^2 = \frac{\mathbf{4}\left[\left(\mathbf{1}-\frac{\pi}{2}\right)^{\mathbf{2}}+\sum\{(2n-1)\pi\}^{\mathbf{2}}\right]}{\boldsymbol{n}}$

Standard deviation of the effect of cosine imprecise function $y^* = x\cos(x)$ is $\sqrt{\frac{\mathbf{4}\left[\left(\mathbf{1}-\frac{\pi}{2}\right)^{\mathbf{2}}+\sum\{(2n-1)\pi\}^{\mathbf{2}}\right]}{\boldsymbol{n}}}$

6.6. Diversion Points

It is a point from where a controllable function or imprecise functions return back to the original function form. Thus the diversion point is the main reason for the point for why function recovered from an imprecise function, which may recreate problems in our system. For example, the eye problem of a person may recreate more problems in eye sight when he gives up the habit of using optical glass. Whenever we remove the conversion point from the graph it can be obtained the original function of the graph. In reverse sense, conversion point is also known as the diversion point of the original function.

6.7. Conclusions

Uses of imprecise number for the construction of imprecise function have been discussed in details in the various sections of this chapter. Activities of the different objects are found as a graph of the function. When the function is not the imprecise form we cannot control their behavior easily and are not easy to apply in our practical purposes. So to control those functions, sine and the cosine functions are used as the multiplication factors. We have obtained that multiplication factors are always transforming the function into oscillation in nature known as the imprecise function. This new function starts to form an imprecise function from the conversion point and recover from the imprecise function to the original function from the diversion point. Searching for other multiplication factors that can convert the uncontrollable function into a controllable imprecise function is the future prospect of the research, which will be benefitted for the researchers.