Chapter 2

A FEW APPLICATIONS OF ONE-DIMENSIONAL IMPRECISE NUMBERS

The work presented in this Chapter is published in "International Journal of Intelligent Systems and Applications", Vol. 7(8) (2015), 9-17, ISSN: 2074-904X (Print), ISSN: 2074-9058 (Online) & "International Journal of Basic and Applied research", Vol. 8(7) (2018), 1003-1013, ISSN: 22493352.

2.1. Introduction

The imprecise number is an extended definition of fuzzy number. It is defined with respect to the membership function and reference function. Study of the effect of the object along the single axis is known as the one-dimensional imprecise number. Here, the effect of the object is studied only along the real line or the X-axis assumed that other axes are independent.

The classical set is not enough to represent all the behavior of the objects under the single set. The fuzzy set come into exist in the study of mathematics to represent all of them under the single set and was first discussed by Zadeh L A (1965). Membership value of each element is a real number lies between 0 and 1. But this set is not completely similar to the classical set. So, another author Baruah H K (2011), [6] [7] has introduced a new fuzzy set definition called imprecise set. This set satisfies the properties of the classical set. Thus imprecise set can represent all the behaviors of the elements under the single set with satisfying the universal laws of the sets.

A special type of fuzzy set which can construct a function of bell-shaped, triangular shaped and trapezoidal within some time bound is named as the fuzzy number. It has a definition similar to the fuzzy set. So it also not satisfies all properties of the classical set. A new author Baruah H K has introduced a special type of imprecise set called imprecise number with own definition and notation in the field of fuzzy mathematics. These numbers satisfy all properties of the classical set and are proved with the example in this chapter.

Definition of imprecise number helped to construct an imprecise matrix for the study of the complex link among the variables. All the properties under the operations of the classical matrix are discussible in the imprecise matrices. Among the properties of matrix, arithmetic operations are proposed to prove in the definition of the imprecise matrix with examples.

In this chapter, some of the transportation problems are represented in one-dimensional imprecise number. Transportation problem is a type of multiple link problems. Among these, some of the problems are complex in nature and are difficult to solve it. So, to represent this type of problems, the new matrix is come to exist in the field of study. It is known as imprecise matrix. Each element of an imprecise matrix is an individually imprecise number.

Classical matrix has own properties. Those properties are undertaken to prove in the imprecise matrix. For this reason, numerical examples are considered. Later, application of the imprecise matrices, complement of imprecise matrices and the properties of imprecise matrices are presented in the graph.

2.2. Properties of One-dimensional Imprecise Numbers

In the imprecise number, the membership value and membership function have different meaning. However, there is no reference function in the fuzzy number, which is available in the imprecise number. Due to this imprecise number satisfies all classical set theory properties. For convenient of writing one-dimensional imprecise numbers is represented by, $\{(\rho_1(x), \rho_2(x)): x\epsilon X\}$. Where, $\rho_1(x)$ and $\rho_2(x)$ are membership function and reference function over the X-axis respectively.

The properties of one-dimensional imprecise numbers are discussed with the assistance of union and intersection of imprecise numbers defined in the below.

Let $A(\rho_X(x)) = \{(\rho_1(x), \rho_2(x)): x\epsilon X\}$ and $B(\rho_X(x)) = \{(\rho_3(x), \rho_4(x)): x\epsilon X\}$ be two imprecise numbers. Where X is the axis of co-ordinate system. Then,

$$A(\rho_X(x)) \cap B(\rho_X(x)) = \begin{Bmatrix} (\min(\rho_1(x), \rho_3(x)), \\ max(\rho_2(x), \rho_4(x))): x\epsilon X \end{Bmatrix} \quad \ldots\ldots\ldots\ldots\ldots..(2.1)$$

And $$A(\rho_X(x)) \cup B(\rho_X(x)) = \begin{Bmatrix} (\max(\rho_1(x), \rho_3(x)), \\ min(\rho_2(x), \rho_4(x))): x\epsilon \Omega \end{Bmatrix} \quad \ldots\ldots\ldots\ldots\ldots...(2.2)$$

Where, $\rho_{X1}(x)$, $\rho_{X3}(x)$ are membership function and $\rho_{X2}(x)$, $\rho_{X4}(x)$ are the reference functions respectively.

2.2.1. Property (Universal Laws)

(i) $A(\rho_X(x)) \cap A^C(\rho_X(x)) = \emptyset(\rho_X(x))$

And (ii) $A(\rho_X(x)) \cup A^C(\rho_X(x)) = \Omega(\rho_X(x))$

Where $\emptyset(\rho_X(x)) = \{(0, \rho_{r1}(x)): x\epsilon X\}$ and $\Omega(\rho_X(x)) = \{(1, \rho_{r2}(x)): x\epsilon X\}$ are empty imprecise number and the universal imprecise number respectively. $\rho_{r1}(x)$ and $\rho_{r2}(x)$ are the reference functions of empty and the universal imprecise numbers.

To prove this law, we consider a figure of one surface of the rectangular shape of dram "A" filled with water with half portion. So the portion not filled with water is also half.

The drum filled with water and empty portions are membership value and the complement value respectively. Drum filled with water portion and the empty portions is indicated by dark and white region respectively. Here, water is filled up to half percent under the membership function, $\rho_A: X - [0,1]$. Where X is a real line. Thus the force applied to fill up this much level of water in the drum is an imprecise number.

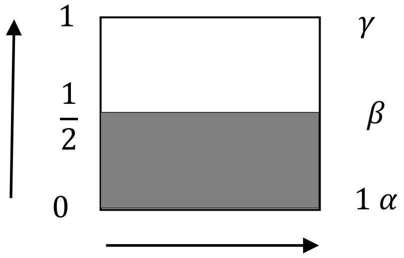

Fig. 2.1. Complement of sub-normal imprecise number

Now, in the fuzzy number definition,

Membership value of water, $\rho_A\left(\frac{1}{2}\right) = \frac{1}{2}$

and complement of membership value, $\rho_{A^C}\left(\frac{1}{2}\right) = \frac{1}{2}$

So the intersection of fuzzy number and complement of fuzzy number is

$$\rho_A\left(\frac{1}{2}\right) \cap \rho_{A^C}\left(\frac{1}{2}\right) = \min\left(\frac{1}{2}, \frac{1}{2}\right)$$
$$= \frac{1}{2}$$

It is a non-empty fuzzy number.

Thus, $\frac{1}{2}$ is the common region membership value of fuzzy number and the complement of fuzzy number. But this portion is not clearly visible in the Fig.2.1.

And the union of fuzzy number and its complement is,

$$A\left(\rho_X\left(\frac{1}{2}\right)\right) \cup A^C\left(\rho_X\left(\frac{1}{2}\right)\right) = \max\left(\frac{1}{2}, \frac{1}{2}\right)$$
$$= \frac{1}{2}$$

It is not the universal fuzzy number.

But in the imprecise number definition, membership value of the imprecise number $A\left(\rho_X\left(\frac{1}{2}, 0\right)\right)$ is

$$\{(\rho(\beta),\rho(\alpha)) : \alpha,\beta \in R\} = \{(\frac{1}{2},0)\}$$

And membership of the complement of imprecise number $A^C\left(\rho_X\left(1,\frac{1}{2}\right)\right)$ is

$$\{(\rho(\beta),\rho(\gamma)) : \beta,\gamma \in R\} = \{(1,\frac{1}{2})\} \ .$$

So the union and intersection of imprecise numbers, defined by Baruah gives us,

$$A\left(\rho_X\left(\frac{1}{2},0\right)\right) \cap A^C\left(\rho_X\left(1,\frac{1}{2}\right)\right) = \left\{\left\{\min\left(\frac{1}{2},1\right),\ \max\left(0,\frac{1}{2}\right)\right\}\right\}$$

$$= \left\{\left(\frac{1}{2},\frac{1}{2}\right)\right\}$$

Which gives us the membership value $(\frac{1}{2}-\frac{1}{2})$ =0.

So, it is an empty imprecise number.

And $A\left(\rho_X\left(\frac{1}{2},0\right)\right) \cap A^C\left(\rho_X\left(1,\frac{1}{2}\right)\right) = \left\{\left(\max\left(\frac{1}{2},\ 1\right), \min\left(0,\ \frac{1}{2}\right)\right)\right\}$

$$= \{(1,0)\}$$

Which has a membership value, 1-0=1.

So, it is the Universal imprecise number.

Another example of a dram "A" filled with $\frac{1}{4}$ portion layer of oil is considered to compare the result obtained in the fuzzy number and imprecise number. Here, the level of oil is up to $\frac{1}{2}$ (half) portion and is shown in the following Fig.2.2. Here, oil fills up to the $\frac{1}{4}$ portion due to the force applied of the membership function $\rho_A: X - [0,1]$. Where X is a set real line. Thus the force applied to fill up this much level of oil in the drum is an imprecise number.

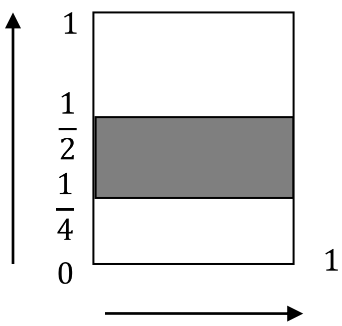

Fig. 2.2. Complement of sub-normal imprecise number

Now according to fuzzy number defined by Zadeh,

Membership value of the layer of oil,

$$\rho_A\left(\frac{1}{4}\right)=\frac{1}{2}-\frac{1}{4}$$
$$=\frac{1}{4}$$

Complementary of membership value,

$$\rho_{A^C}\left(\frac{3}{4}\right)=\left(1-\frac{1}{2}\right)+\left(1-\frac{1}{4}\right)$$
$$=\frac{3}{4}$$

By the intersection and union definition of fuzzy numbers we have,

$$\rho_A\left(\frac{1}{4}\right)\cap\rho_{A^C}\left(\frac{3}{4}\right)=\min\left(\frac{1}{4},\frac{3}{4}\right)$$
$$=\frac{1}{4}$$

and $\rho_A\left(\frac{1}{4}\right)\cup\rho_{A^C}\left(\frac{3}{4}\right)=\max\left(\frac{1}{4},\frac{3}{4}\right)$

$$=\frac{3}{4}$$

Thus the union and intersection of two fuzzy number and its complement are not become the universal fuzzy number and empty fuzzy number.

But in the definition of imprecise number,

Portion of dram filled with oil is an imprecise number, $A\left(\rho_X\ \left(\frac{1}{2},\frac{1}{4}\right)\right)=\left\{\left(\frac{1}{2},\frac{1}{4}\right)\right\}$

Portions not filled with oil are the imprecise numbers,

$$A^C\left(\rho_X\left(1,\frac{1}{2}\right)\right)=\ \left\{\left(1,\frac{1}{2}\right)\right\}\quad\text{and}\ A^C\left(\rho_X\left(\frac{1}{4},0\right)\right)=\left\{\left(\frac{1}{4},0\right)\right\}$$

Which is shown in the Fig.2.2. Combination of these two imprecise numbers is the complementary part of the imprecise number.

So the imprecise number and its complement has the intersection values as follows.

$$A\left(\rho_X\left(\frac{1}{2},\frac{1}{4}\right)\right)\cap A^C\left(\rho_X\left(1,\frac{1}{2}\right)\right)=\left\{\left(\min\left(\frac{1}{2},1\right),\max\left(\frac{1}{4},\frac{1}{2}\right)\right)\right\}$$
$$=\left\{\left(\frac{1}{2},\frac{1}{2}\right)\right\}$$

It has a membership value, $\frac{1}{2}-\frac{1}{2}=0$.

And $A\left(\rho_X\left(\frac{1}{2},\frac{1}{4}\right)\right)\cap A^C\left(\rho_X\left(\frac{1}{4},0\right)\right)=\left\{\left(\min\left(\frac{1}{2},\frac{1}{4}\right),\max\left(\frac{1}{4},0\right)\right)\right\}$

$$=\left\{\left(\frac{1}{4},\frac{1}{4}\right)\right\}$$

It has a membership value, $\frac{1}{4} - \frac{1}{4} = 0$.

Here, both the intersections give us membership value 0. So it is an empty imprecise number.

By the associativity law of sets, union of imprecise numbers is given by

$$A\left(\rho_X\left(\frac{1}{2},\frac{1}{4}\right)\right) \cap \left(A^C\left(\rho_X\left(1,\frac{1}{2}\right)\right) \cup A^C\left(\rho_X\left(\frac{1}{4},0\right)\right)\right)$$

$$= \left(A\left(\rho_X\left(\tfrac{1}{2},\tfrac{1}{4}\right)\right) \cap A^C\left(\rho_X\left(1,\tfrac{1}{2}\right)\right)\right) \cup \left(A\left(\rho_X\left(\tfrac{1}{2},\tfrac{1}{4}\right)\right) \cap A^C\left(\rho_X\left(\tfrac{1}{4},0\right)\right)\right)$$

$$= \left\{\left(\max\left(\tfrac{1}{2},1\right), \min\left(\tfrac{1}{4},\tfrac{1}{2}\right)\right)\right\} \cup \left\{\left(\max\left(\tfrac{1}{2},\tfrac{1}{4}\right), \min\left(\tfrac{1}{4},0\right)\right)\right\}$$

$$= \left\{\left(1,\frac{1}{4}\right)\right\} \cup \left\{\left(\frac{1}{2},0\right)\right\}$$

$$= \left\{\left(\max\left(1,\tfrac{1}{2}\right), \min\left(\tfrac{1}{4},0\right)\right)\right\}$$

$$= \{(1,0)\}$$

Which has a membership value $(1-0) = 1$. So, it is the universal set.

Classical set has many other properties under the operations of union and intersection. These are discussed in one-dimensional imprecise number with the assistance of examples at the below.

2.2.2. Property (Idempotence Laws)

(i) $A(\rho_X(x)) \cap A(\rho_X(x)) = A(\rho_X(x))$

(ii) $A(\rho_X(x)) \cup A(\rho_X(x)) = A(\rho_X(x))$

This property is obviously true.

2.2.3. Property (Identity Laws)

(i) $A(\rho_X(x)) \cap \emptyset(\rho_X(x)) = \emptyset(\rho_X(x))$

(ii) $A(\rho_X(x)) \cup \emptyset(\rho_X(x)) = A(\rho_X(x))$

(iii) $A(\rho_X(x)) \cap \Omega(\rho_X(x)) = A(\rho_X(x))$

(iv) $A(\rho_X(x)) \cup \Omega(\rho_X(x)) = \Omega(\rho_X(x))$

Where, $\emptyset(\rho_X(x)) = \{(0, \rho_{r1}(x)) : x \epsilon X\}$ and $\Omega(\rho_X(x)) = \{(1, \rho_{r2}(x)) : x \epsilon X\}$ are empty imprecise number and the universal imprecise number respectively. And $\rho_{r1}(x)$ and $\rho_{r2}(x)$ are reference functions of the empty imprecise number and the universal imprecise numbers.

To prove the properties 2.2.3.(i) and 2.2.3.(ii), let $A(\rho_X(x)) = \left\{\left(\frac{1}{3}, \frac{1}{4}\right)\right\}$ and $\emptyset((\rho_X(x)) = \left\{\left(0, \frac{1}{4}\right)\right\}$ be one-dimensional imprecise numbers such that measurement of membership functions are done from reference function $\left(\frac{1}{4}\right)$. i.e. from the one fourth portion of one dimensional imprecise number f orm of object.

Here, $\emptyset((\rho_X(x)) = \left\{\left(0, \frac{1}{4}\right)\right\}$ is a one dimensional null imprecise number and is measured from the one fourth potion of the one dimensional object. Here the membership function has a zero value due to the null imprecise number. Now,

Proof:

(i) $$A(\rho_X(x)) \cap \emptyset(\rho_X(x)) = \left\{\left(\frac{1}{3}, \frac{1}{4}\right)\right\} \cap \left\{\left(0, \frac{1}{4}\right)\right\}$$
$$= \left\{\left(\min\left(\frac{1}{3}, 0\right), max\left(\frac{1}{4}, \frac{1}{4}\right)\right)\right\}$$
$$= \left\{\left(0, \frac{1}{4}\right)\right\}$$
$$= \emptyset((\mu_X(x))$$

Hence proved

(ii) $$A(\rho_X(x)) \cup \emptyset(\rho_X(x)) = \left\{\left(\frac{1}{3}, \frac{1}{4}\right)\right\} \cup \left\{\left(0, \frac{1}{4}\right)\right\}$$
$$= \left\{\left(\max\left(\frac{1}{3}, 0\right), min\left(\frac{1}{4}, \frac{1}{4}\right)\right)\right\}$$
$$= \left\{\left(\frac{1}{3}, \frac{1}{4}\right)\right\}$$
$$= \emptyset(\mu_X(x))$$

Hence proved

To prove 2.2.3.(iii) and 2.2.3.(iv), let , $A(\rho_X(x)) = \left\{\left(\frac{1}{5}, \frac{1}{7}\right)\right\}$ and $\Omega(\rho_X(x)) = \left\{\left(1, \frac{1}{7}\right)\right\}$ are the one-dimensional imprecise numbers such that membership functions are measured from reference function $\left(\frac{1}{7}\right)$. Here $\left(\frac{1}{5}\right)$ and $\left(\frac{1}{7}\right)$ are the membership functions of the one-dimensional objects.

Here, $\Omega(\rho_X(x)) = \left\{\left(1, \frac{1}{7}\right)\right\}$ is the Universal imprecise number counted from the one-seventh portion of one-dimensional object. Here membership function of the universal imprecise number must be greater than the membership function of other imprecise number. Then,

Proof:

(iii) $A(\rho_X(x)) \cap \Omega(\rho_X(x)) = \left\{\left(\frac{1}{5}, \frac{1}{7}\right)\right\} \cap \left\{\left(1, \frac{1}{7}\right)\right\}$

$$= \left\{\left(\min\left(\frac{1}{5}, 1\right), max\left(\frac{1}{7}, \frac{1}{7}\right)\right)\right\}$$

$$= \left\{\left(\frac{1}{5}, \frac{1}{7}\right)\right\}$$

$$= A(\mu_X(x))$$

Hence proved

(iv) $A(\rho_X(x)) \cup \Omega(\rho_X(x)) = \left\{\left(\frac{1}{5}, \frac{1}{7}\right)\right\} \cup \left\{\left(1, \frac{1}{7}\right)\right\}$

$$= \left\{\left(\max\left(\frac{1}{5}, 1\right), min\left(\frac{1}{7}, \frac{1}{7}\right)\right)\right\}$$

$$= \left\{\left(1, \frac{1}{7}\right)\right\}$$

$$= X(\mu_X(x))$$

Hence proved

2.2.4. Property (Commutative laws)

Let $A(\rho_X(x)) = \{(\rho_1(x), \rho_2(x)): x \in X\}$ and $B(\rho_X(x)) = \{(\rho_3(x), \rho_4(x)): x \in X\}$ be two one-dimensional imprecise numbers. Here X is the axis of the co-ordinate system. $\rho_1(x)$, $\rho_3(x)$ are membership functions and $\rho_2(x)$, $\rho_4(x)$ are reference functions of the imprecise numbers respectively. Then commutative property is obtained in the form.

(i) $A(\rho_X(x)) \cup B(\rho_X(x)) = B(\rho_X(x)) \cup A(\rho_X(x))$

(ii) $A(\rho_X(x)) \cap B(\rho_X(x)) = B(\rho_X(x)) \cap A(\rho_X(x))$

To prove these property, let $A(\rho_X(x)) = \left\{\left(\frac{1}{5}, \frac{1}{6}\right)\right\}$ and $B(\rho_X(x)) = \left\{\left(\frac{1}{4}, \frac{1}{6}\right)\right\}$ are the one dimensional imprecise numbers counted from the reference function $\frac{1}{6}$. i.e. one sixth portion of one-dimensional imprecise number object. Then,

Proof:

(i) $A(\rho_X(x)) \cup B(\rho_X(x)) = \left\{\left(\frac{1}{5}, \frac{1}{6}\right)\right\} \cup \left\{\left(\frac{1}{4}, \frac{1}{6}\right)\right\}$

$$= \left\{\left(\max\left(\frac{1}{5}, \frac{1}{4}\right), min\left(\frac{1}{6}, \frac{1}{6}\right)\right)\right\}$$

$$= \left\{\left(\frac{1}{4}, \frac{1}{6}\right)\right\}$$

$B(\rho_X(x)) \cup A(\rho_X(x))=\left\{\left(\frac{1}{4}, \frac{1}{6}\right)\right\} \cup \left\{\left(\frac{1}{5}, \frac{1}{6}\right)\right\}$

$$= \left\{\left(\max\left(\frac{1}{4},\frac{1}{5}\right), min\left(\frac{1}{6},\frac{1}{6}\right)\right)\right\}$$
$$= \left\{\left(\frac{1}{4},\frac{1}{6}\right)\right\}$$

Hence proved

(ii) $A(\rho_X(x)) \cap B(\rho_X(x)) = \left\{\left(\frac{1}{5},\frac{1}{6}\right)\right\} \cap \left\{\left(\frac{1}{4},\frac{1}{6}\right)\right\}$

$$= \left\{\left(\min\left(\frac{1}{5},\frac{1}{4}\right), max\left(\frac{1}{6},\frac{1}{6}\right)\right)\right\}$$
$$= \left\{\left(\frac{1}{5},\frac{1}{6}\right)\right\}$$

$B(\rho_{XY2}(x,y)) \cap A(\rho_{XY1}(x,y))=\left\{\left(\frac{1}{4},\frac{1}{6}\right)\right\} \cap \left\{\left(\frac{1}{5},\frac{1}{6}\right)\right\}$

$$= \left\{\left(\min\left(\frac{1}{4},\frac{1}{5}\right), max\left(\frac{1}{6},\frac{1}{6}\right)\right)\right\}$$
$$= \left\{\left(\frac{1}{5},\frac{1}{6}\right)\right\}$$

Hence Proved

2.2.5. Property (Distributive Laws)

Let, $A(\rho_X(x)) = \{(\rho_1(x), \rho_2(x)) : x \in X\}$, $B(\rho_X(x)) = \{(\rho_3(x), \rho_4(x)) : x \in X\}$ and $C(\rho_X(x)) = \{(\rho_5(x), \rho_6(x)) : x \in X\}$ be one-dimensional imprecise numbers. Here X is the axis of the co-ordinate system. $\rho_1(x)$, $\rho_3(x)$ are membership functions and $\rho_2(x)$, $\rho_4(x)$ are reference functions of imprecise numbers respectively. Then,

(i) $A(\rho_X(x)) \cap \left(B(\rho_X(x)) \cup C(\rho_X(x))\right)$

$$= \left(A\left(\rho_X(x)\right) \cap B\left(\rho_X(x)\right)\right) \cup \left(A\left(\rho_X(x)\right) \cap C\left(\rho_X(x)\right)\right)$$

(ii) $A(\rho_X(x)) \cup \left(B(\rho_X(x)) \cap C(\rho_X(x))\right)$

$$= \left(A(\rho_X(x)) \cup B(\rho_X(x))\right) \cap \left(A(\rho_X(x)) \cup C(\rho_X(x))\right)$$

To prove these properties, let $A(\rho_X(x)) = \left\{\left(\frac{1}{2},\frac{1}{9}\right)\right\}$, $B(\rho_X(x)) = \left\{\left(\frac{1}{3},\frac{1}{9}\right)\right\}$ and $C(\rho_X(x)) = \left\{\left(\frac{1}{6},\frac{1}{9}\right)\right\}$ be the two one-dimensional imprecise numbers and are counted from reference function $\frac{1}{9}$. Thus,

Proof:

(i) $A(\rho_X(x)) \cap \left(B(\rho_X(x)) \cup C(\rho_X(x))\right)$

$$= \left\{\left(\frac{1}{2},\frac{1}{9}\right)\right\} \cap \left\{\left(\max\left(\frac{1}{3},\frac{1}{6}\right), min\left(\frac{1}{9},\frac{1}{9}\right)\right)\right\}$$

$$=\left\{\left(\frac{1}{2},\frac{1}{9}\right)\right\}\cap\left\{\left(\frac{1}{3},\frac{1}{9}\right)\right\}$$

$$=\left\{\left(\min\left(\frac{1}{2},\frac{1}{3}\right),max\left(\frac{1}{9},\frac{1}{9}\right)\right)\right\}$$

$$=\left\{\left(\frac{1}{3},\frac{1}{9}\right)\right\}$$

$\left(A(\rho_X(x))\cap B(\rho_X(x))\right)\cup\left(A(\rho_X(x))\cap C(\rho_X(x))\right)$

$$=\left\{\left\{\left(\min\left(\frac{1}{2},\frac{1}{3}\right),max\left(\frac{1}{9},\frac{1}{9}\right)\right)\right\}\cup\left\{\left(\min\left(\frac{1}{2},\frac{1}{6}\right),max\left(\frac{1}{9},\frac{1}{9}\right)\right)\right\}\right\}$$

$$=\left\{\left(\frac{1}{3},\frac{1}{9}\right)\right\}\cup\left\{\left(\frac{1}{6},\frac{1}{9}\right)\right\}$$

$$=\left\{\left(\max\left(\frac{1}{3},\frac{1}{6}\right),min\left(\frac{1}{9},\frac{1}{9}\right)\right)\right\}$$

$$=\left\{\left(\frac{1}{3},\frac{1}{9}\right)\right\}$$

Hence proved

(ii) $A(\rho_X(x))\cup\left(B(\rho_X(x))\cap C(\rho_X(x))\right)$

$$=\left\{\left(\frac{1}{2},\frac{1}{9}\right)\right\}\cup\left\{\left(\min\left(\frac{1}{3},\frac{1}{6}\right),max\left(\frac{1}{9},\frac{1}{9}\right)\right)\right\}$$

$$=\left\{\left(\frac{1}{2},\frac{1}{9}\right)\right\}\cup\left\{\left(\frac{1}{6},\frac{1}{9}\right)\right\}$$

$$=\left\{\left(\max\left(\frac{1}{2},\frac{1}{3}\right),min\left(\frac{1}{9},\frac{1}{9}\right)\right)\right\}$$

$$=\left\{\left(\frac{1}{2},\frac{1}{9}\right)\right\}$$

$\left(A(\rho_X(x))\cup B(\rho_X(x))\right)\cap\left(A(\rho_X(x))\cup C(\rho_X(x))\right)$

$$=\left\{\left\{\left(\max\left(\frac{1}{2},\frac{1}{3}\right),min\left(\frac{1}{9},\frac{1}{9}\right)\right)\right\}\cap\left\{\left(\max\left(\frac{1}{2},\frac{1}{6}\right),min\left(\frac{1}{9},\frac{1}{9}\right)\right)\right\}\right\}$$

$$=\left\{\left(\frac{1}{2},\frac{1}{9}\right)\right\}\cap\left\{\left(\frac{1}{2},\frac{1}{9}\right)\right\}$$

$$=\left\{\left(\min\left(\frac{1}{2},\frac{1}{2}\right),max\left(\frac{1}{9},\frac{1}{9}\right)\right)\right\}$$

$$=\left\{\left(\frac{1}{2},\frac{1}{9}\right)\right\}$$

Hence Proved

2.2.6. Property (Associatively Laws)

If $A(\rho_X(x))=\{(\rho_1(x),\rho_2(x)):x\in X\}$, $B(\rho_X(x))=\{(\rho_3(x),\rho_4(x)):x\in X\}$

and $C((\rho_X(x)) = \{(\rho_5(x), \rho_6(x)) : x \in X\}$ be the three one-dimensional imprecise numbers. Then,

(i) $A(\rho_X(x)) \cup \left(B(\rho_X(x)) \cup C(\rho_X(x))\right) = \left(A(\rho_X(x)) \cup B(\rho_X(x))\right) \cup C(\rho_X(x))$

(ii) $A(\rho_X(x)) \cap \left(B(\rho_X(x)) \cap C(\rho_X(x))\right) = \left(A(\rho_X(x)) \cap B(\rho_X(x))\right) \cap C(\rho_X(x))$

To prove these properties, let $A(\rho_X(x)) = \left\{\left(\frac{1}{2}, \frac{1}{9}\right)\right\}$, $B(\rho_X(x)) = \left\{\left(\frac{1}{3}, \frac{1}{9}\right)\right\}$ and $C(\rho_X(x)) = \left\{\left(\frac{1}{6}, \frac{1}{9}\right)\right\}$ be the three one-dimensional imprecise numbers counted from reference function $\frac{1}{9}$. Thus,

Proof:

(i) $A(\rho_X(x)) \cup \left(B(\rho_X(x)) \cup C(\rho_X(x))\right)$

$$= \left\{\left(\frac{1}{2}, \frac{1}{9}\right)\right\} \cup \left\{\left(\max\left(\frac{1}{3}, \frac{1}{6}\right), min\left(\frac{1}{9}, \frac{1}{9}\right)\right)\right\}$$

$$= \left\{\left(\frac{1}{2}, \frac{1}{9}\right)\right\} \cup \left\{\left(\frac{1}{3}, \frac{1}{9}\right)\right\}$$

$$= \left\{\left(\max\left(\frac{1}{2}, \frac{1}{3}\right), min\left(\frac{1}{9}, \frac{1}{9}\right)\right)\right\}$$

$$= \left\{\left(\frac{1}{2}, \frac{1}{9}\right)\right\}$$

$$\left(A(\rho_X(x)) \cup B(\rho_X(x))\right) \cup C(\rho_X(x)) = \left\{\left(\max\left(\frac{1}{2}, \frac{1}{3}\right), min\left(\frac{1}{9}, \frac{1}{9}\right)\right)\right\} \cup \left\{\left(\frac{1}{6}, \frac{1}{9}\right)\right\}$$

$$= \left\{\left(\frac{1}{2}, \frac{1}{9}\right)\right\} \cup \left\{\left(\frac{1}{6}, \frac{1}{9}\right)\right\}$$

$$= \left\{\left(\max\left(\frac{1}{2}, \frac{1}{6}\right), min\left(\frac{1}{9}, \frac{1}{9}\right)\right)\right\}$$

$$= \left\{\left(\frac{1}{2}, \frac{1}{9}\right)\right\}$$

Hence proved

(ii) $A(\rho_X(x)) \cap \left(B(\rho_X(x)) \cap C(\rho_X(x))\right)$

$$= \left\{\left(\frac{1}{2}, \frac{1}{9}\right)\right\} \cup \left\{\left(\min\left(\frac{1}{3}, \frac{1}{6}\right), max\left(\frac{1}{9}, \frac{1}{9}\right)\right)\right\}$$

$$= \left\{\left(\frac{1}{6}, \frac{1}{9}\right)\right\} \cap \left\{\left(\frac{1}{3}, \frac{1}{9}\right)\right\}$$

$$= \left\{\left(\min\left(\frac{1}{6},\frac{1}{3}\right), max\left(\frac{1}{9},\frac{1}{9}\right)\right)\right\}$$

$$= \left\{\left(\frac{1}{6},\frac{1}{9}\right)\right\}$$

$$\left(A\left(\rho_X(x)\right) \cap B\left(\rho_X(x)\right)\right) \cap C\left(\rho_X(x)\right) = \left\{\left(\min\left(\frac{1}{2},\frac{1}{3}\right), max\left(\frac{1}{9},\frac{1}{9}\right)\right)\right\} \cap \left\{\left(\frac{1}{6},\frac{1}{9}\right)\right\}$$

$$= \left\{\left(\frac{1}{3},\frac{1}{9}\right)\right\} \cap \left\{\left(\frac{1}{6},\frac{1}{9}\right)\right\}$$

$$= \left\{\left(\min\left(\frac{1}{3},\frac{1}{6}\right), max\left(\frac{1}{9},\frac{1}{9}\right)\right)\right\}$$

$$= \left\{\left(\frac{1}{6},\frac{1}{9}\right)\right\}$$

Hence proved

2.2.7. Property(De-Morgan's Laws)

Let $A(\rho_X(x)) = \{(\rho_1(x), \rho_2(x)): x \in X\}$ and $B(\rho_X(x)) = \{(\rho_3(x), \rho_4(x)): x \in X\}$ and $C(\rho_X(x)) = \{(\rho_5(x), \rho_6(x)): x \in X\}$ be three one-dimensional imprecise numbers. Then, De-Morgan's law is obtained in the following form.

(i) $\left(A\left(\rho_X(x)\right) \cup B\left(\rho_X(x)\right)\right)^C = A^C(\rho_X(x)) \cap B^C(\rho_X(x))$

(ii) $\left(A\left(\rho_X(x)\right) \cap B\left(\rho_X(x)\right)\right)^C = A^C(\rho_X(x)) \cup B^C(\rho_X(x))$

To prove these properties, let us consider $A(\rho_X(x)) = \left\{\left(\frac{1}{2},\frac{1}{9}\right)\right\}$ and $B(\rho_X(x)) = \left\{\left(\frac{1}{3},\frac{1}{9}\right)\right\}$.

Here membership function of $A(\rho_X(x))$ and $B(\rho_X(x))$ is counted from $\frac{1}{9}$ th portion of the one dimensional object. So, the complement of the imprecise numbers are counted from reference functions $\frac{1}{2}$ and $\frac{1}{3}$ respectively.

Proof:

(i) $\left(A\left(\rho_X(x)\right) \cup B\left(\rho_X(x)\right)\right)^C = \left(\left\{\left(\max\left(\frac{1}{2},\frac{1}{3}\right), min\left(\frac{1}{9},\frac{1}{9}\right)\right)\right\}\right)^C$

$$= \left\{\left(\frac{1}{2},\frac{1}{9}\right)\right\}^C$$

$$= \left\{\left(1,\frac{1}{2}\right)\right\}$$

$$A^C(\rho_X(x)) \cap B^C(\rho_X(x)) = \left\{\left(1,\frac{1}{2}\right)\right\} \cap \left\{\left(1,\frac{1}{3}\right)\right\}$$

$$= \left\{\left(\min(1,1), max\left(\frac{1}{2},\frac{1}{3}\right)\right)\right\}$$

$$= \left\{\left(1, \frac{1}{2}\right)\right\}$$

Hence proved

(ii) $$\left(A\left(\rho_X(x)\right) \cap B\left(\rho_X(x)\right)\right)^C = \left(\left\{\left(\min\left(\frac{1}{2}, \frac{1}{3}\right), max\left(\frac{1}{9}, \frac{1}{9}\right)\right)\right\}\right)^C$$

$$= \left\{\left(\frac{1}{3}, \frac{1}{9}\right)\right\}^C$$

$$= \left\{\left(1, \frac{1}{3}\right)\right\}$$

$$A^C(\rho_X(x)) \cup B^C(\rho_X(x)) = \left\{\left(1, \frac{1}{2}\right)\right\} \cup \left\{\left(1, \frac{1}{3}\right)\right\}$$

$$= \left\{\left(\max(1,1), min\left(\frac{1}{2}, \frac{1}{3}\right)\right)\right\}$$

$$= \left\{\left(1, \frac{1}{3}\right)\right\}$$

Hence proved

2.3. Imprecise Matrices

Multilink problems are expressible in the matrix form that has been discussed in the definition of the classical matrix. But the multilink problem with complex in nature is not possible to discuss in the definition of the classical matrix. It is very much related to the fuzzy concept. Fuzzy matrix was first time introduced in the field of mathematics by Thomson (1977). Here, each element of the fuzzy matrix is a fuzzy number and are only expressed in the membership value form. To discuss those multilink problems in imprecise matrices, each element of the imprecise matrix is expressed in terms of membership function and reference function. Thus the elements of a matrix with real value entries lies between 0 and 1 having imprecise number properties is known as imprecise matrix. The similarity to this definition, Dhar M (2013) has contributed an article of fuzzy matrix based on the reference function in the field of fuzzy study.

Though the imprecise number with reference function 0 (zero) is characterized by $\{(\rho_X(x), 0): x \in X\}$, for the convenience of writing, elements of imprecise matrix is represented in the order pair form, $\left(\rho_{ij}(x), 0\right); i, j \in N$ and $x \in R$ such that $\sum_{i=1}^{n} \rho_{ij} \leq 1$. Where, $\rho_{ij}(x)$'s are the membership functions of the elements of imprecise matrix and are individually imprecise numbers, i and j are row and column of the imprecise matrix. Here, many of the definition and the properties of imprecise numbers are found as an important application in the field of transportation problems and are discussed in this section.

2.3.1. Definition: Order two imprecise matrices measured from its reference function $0 \le r_{ij} \le 1$ is defined by

$$M(V) = \begin{bmatrix} (v_{11}, r_{11}) & (v_{12}, r_{12}) \\ (v_{21}, r_{21}) & (v_{22}, r_{22}) \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.3)$$

Where, each element of imprecise matrix is an imprecise number. So, imprecise matrix of order two is denoted by

$$M(V) = [(v_{ij}, r_{ij})]_{2\times 2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.4)$$

In general $M(V) = \left[(v_{ij}, r_{ij})\right]_{m\times n}$ is the imprecise matrix of order $m \times n$. Here m is the row and n is the column of the imprecise matrix. v_{ij} and r_{ij} are membership function and reference function of the element of imprecise matrix respectively.

This definition represents two types of links between the two objects in various situations.

2.3.2. Definition: The complement of an imprecise matrix $M(V) = \left[(v_{ij}, r_{ij})\right]_{m\times n}$ of order $m \times n$ is represented by

$$M^C(V) = \left[(1, v_{ij})\right]_{m\times n} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.5)$$

Where the elements of complement matrix are individually imprecise numbers and are measured from reference function v_{ij}, which is a membership function of the imprecise matrix

2.3.2.1. Proposition: Complement of imprecise matrix is always an imprecise matrix.

Let $M(A) = \begin{bmatrix} (0.2,0) & (0.8,0) \\ (0.9,0) & (0.1,0) \end{bmatrix}$ be an order two square imprecise matrix, whose elements are counted from reference function 0. Then its complement becomes,

$$M^C(A) = \begin{bmatrix} (1,0.2) & (1,0.8) \\ (1,0.9) & (1,0.1) \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.6)$$

Which is an order two imprecise matrix having different reference function for the different elements of imprecise matrix. Here the membership function of the element of imprecise matrix, $a_{11} = 0.2, a_{12} = 0.8, a_{21} = 0.9, a_{22} = 0.1$ become the reference function of the complement of imprecise matrix as per the definition of imprecise number.

2.3.3. Definition (Identity Imprecise Matrices): Multiplicative identity of imprecise matrix can be represented and defined by,

$$M(I) = \left[(a_{ij}, r_{ij})\right]_{m\times n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2.7)}$$

Where $if\ i = j, a_{ij} = 1\ and\ if\ i \neq j,\ a_{ij} = 0$

Example- Matrix, $M(I) = \begin{bmatrix} (1,0) & (0,0) \\ (0,0) & (1,0) \end{bmatrix}$ is an order 2 identity square imprecise matrix measured from reference function 0.

2.3.4. Definition (Null Imprecise Matrix): Null matrix measured from the reference function $0 \leq r_{ij} \leq 1$ is represented by

$$M(0) = \left[(0\ , r_{ij})\right]_{m\times n} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2.8)}$$

Example- $M(0) = \begin{bmatrix} (0,1) & (0,1) \\ (0,1) & (0,1) \end{bmatrix}$ is an order 2 null imprecise matrix. Here, elements of the matrix are measured from the reference function 1.

2.3.5. Definition (Transpose of Imprecise Matrices): Transpose of order $m \times n$ imprecise matrix, $M(V) = [(v_{ij}, r_{ij})]_{m\times n}$ is represented and defined by

$$(M(v')) = [(v_{ji}, r_{ji})]_{m\times n} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2.9)}$$

Graphically,

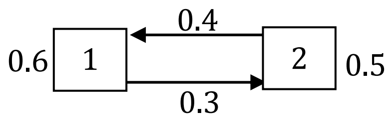

Fig.2.3. Imprecise matrix of order two

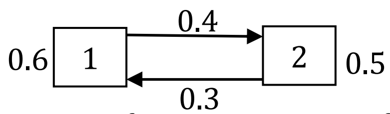

Fig. 2.4. Transpose of imprecise matrix of order two

In the figure, we see that a transpose of the matrix is forming after the direction of the activity of Fig. 2.3. is changed to Fig.2.4. Where the conditions for allocation of the elements at center 1 and 2 are the imprecise number form and are counted from the reference function 0. Each value of the path of the figure is the membership value of the element of the imprecise matrix. Thus the logic of the fig.2.3. and fig.2.4. are obtained in the form of imprecise matrix of order two, $M(V) = \begin{bmatrix} (0.6,0) & (0,3.0) \\ (0.4,0) & (0.5,0) \end{bmatrix}$ and the transpose of imprecise matrix of order two $M(V') = \begin{bmatrix} (0.6,0) & (0.4,0) \\ (0.3,0) & (0.5,0) \end{bmatrix}$

Thus the Fig.2.3. and Fig.2.4. are the transposes of the imprecise matrices to each other.

2.3.6. Applications

Example-1: Suppose a bus deliver daily necessary foods to the three different locations namely at center 1, 2 and 3 respectively. Bus has a capacity to provide services in such a way that total 100% and when at the center 1 is 40%, then its capacity of services at the center 2 and 3 are 30% and 30%, when at center 2 is 30%, then its capacity at the center 1 and 3 are 35% and 35%, when at center 3 is 20% ,then its capacity at the center 1 and 2 are 40% and 40% respectively. This is a transportation problem and the conditions for allocation of services at the center 1, 2 and 3 is an imprecise matrix of order 3×3. Here,

$$
\begin{aligned}
v_{11} &= 40\% = 0.4 \\
v_{12} &= 30\% = 0.3 \\
v_{13} &= 30\% = 0.3 \\
v_{21} &= 35\% = 0.35 \\
v_{22} &= 30\% = 0.3 \\
v_{23} &= 35\% = 0.35 \\
v_{31} &= 40\% = 0.4 \\
v_{32} &= 40\% = 0.4 \\
v_{33} &= 20\% = 0.2
\end{aligned} \qquad (2.10)
$$

which are membership function of the indicator function of center 1, 2 and 3 respectively and are measured from the reference functions 0. Indices of v are location of the elements of matrix. Thus the problem form an imprecise matrix.

$$
= \begin{bmatrix} (v_{11},0) & (v_{12},0) & (v_{13},0) \\ (v_{21},0) & (v_{22},0) & (v_{23},0) \\ (v_{31},0) & (v_{32},0) & (v_{33},0) \end{bmatrix}_{3\times 3}
$$

$$
= \begin{bmatrix} (0.4,0) & (0.3,0) & (0.3,0) \\ (0.35,0) & (0.3,0) & (0.35,0) \\ (0.4,0) & (0.4,0) & (0.2,0) \end{bmatrix}_{3\times 3} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2.11)
$$

The logic of imprecise matrix has formed a network diagram. Each path of the diagram is represented by membership functions and are measured from the reference functions 0 for all the centers.

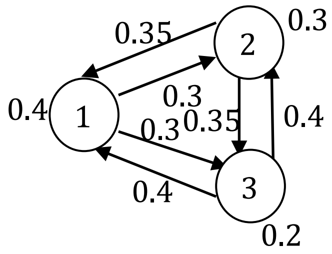

Fig. 2.5. Network diagram of order three imprecise matrix.

Eexample-2: If $M(V) = \begin{bmatrix} (0.7,0) & (0.3,0) \\ (0.3,0) & (0.4,0) \end{bmatrix}$ is an imprecise matrix of order two with each element is measured from the reference function 0, then its complement becomes,

$$M^C(W) = \begin{bmatrix} (1,0.7) & (1,0.3) \\ (1,0.3) & (1,0.4) \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.12)$$

This represents a problem in such a way that when a bus has not capable to provide service is 70% at center 1, there are 30% chances at the center 2. Similarly, if the bus cannot provide service at center 1 is 30%, their chances at center 2 are 40%. Here the conditions and the allocation of the center 1 and 2 are the imprecise matrices of order 2. The elements of the imprecise matrix are the membership values of the imprecise numbers measured from the reference function 0 and the membership values of the complement of imprecise matrix are measured from the reference functions, $v_{11} = 0.7, v_{12} = 0.3, v_{21} = 0.3, v_{22} = 0.4$. Diagram is obtained with the help of membership value.

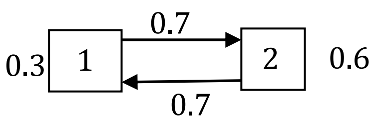

Fig.2.6. Network diagram of the complement of order two imprecise matrix

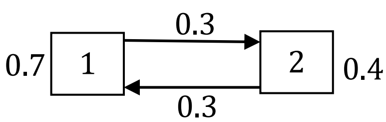

Fig. 2.7. Network diagram of the order two imprecise matrix

2.4. Arithmetic Operations of Imprecise Matrices

Here, operations of subtraction, addition, multiplication and division are the arithmetic operations of imprecise matrices. The imprecise matrices formulae of those operations are obtained in terms of minimum and maximum operators. These operators are helping to obtain the properties of imprecise matrix and are proved with examples. Applications of imprecise matrix properties are discussed in the diagram.

2.4.1. Addition: Addition of the two imprecise matrices $M(V)$ and $M(W)$ can be represented and defined by

$$M(V) + M(W) = \left(\max\left(v_{ij}, w_{ij}\right), \min\left(r_{ij}, r'_{ij}\right)\right)\ldots\ldots\ldots\ldots\ldots(2.13)$$

provided $M(V)$ and $M(W)$ are imprecise matrices of same type. Where v_{ij} , w_{ij} *and* r_{ij} , r'_{ij} are membership functions and reference functions of the imprecise matrices $M(V)$ and $M(W)$ respectively.

This definition helps us to know what is the maximum service providing between two objects at the different locations.

2.4.1.1. Proposition: Addition of any two imprecise matrices of same order is always commutative.

Example- Let $M(K) = \begin{bmatrix} (0.1,0) & (0.2,0) \\ (0.3,0) & (0.4,0) \end{bmatrix}$ and $M(L) = \begin{bmatrix} (0.2,0) & (0.4,0) \\ (0.3,0) & (0.1,0) \end{bmatrix}$ be two imprecise matrices with each elements is counted from the reference function 0. Then

$$M(K) + M(L) = \begin{bmatrix} (0.2,0) & (0.4,0) \\ (0.3,0) & (0.4,0) \end{bmatrix}$$

$$M(K) + M(L) = \begin{bmatrix} (0.2,0) & (0.4,0) \\ (0.3,0) & (0.4,0) \end{bmatrix}$$

So we have,

$$M(K) + M(L) = M(K) + M(L)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2.14)$$

2.4.1.2. Proposition: Addition of complement of the two imprecise matrices of same order is commutative.

Proof:

We need to prove,

$$M^C(V) + M^C(W) = M^C(V) + M^C(W) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.15)$$

It is obvious that complement of imprecise matrix is again an imprecise matrix. And addition of two imprecise matrices is commutative.

So, the commutatively law also hold good in case of compliment of imprecise matrix.

This theorem helps us to know the complement of maximum service provided between the objects at the different locations.

2.4.2. Application

If $M(K) = \begin{bmatrix} (0.5,0) & (0.4,0) \\ (0.3,0) & (0.1,0) \end{bmatrix}$ and $M(L) = \begin{bmatrix} (0.2,0) & (0.3,0) \\ (0.3,0) & (0.7,0) \end{bmatrix}$ be two imprecise matrices having each element is measured from the reference function 0. Then,

$$M(K) + M(L) = \begin{bmatrix} (\max(0.5,0.2), \min(0,0)) & (\max(0..4,0.3), \min(0,0)) \\ (\max(0.3,0.3), \min(0,0)) & (\max(0.1,0.7), \min(0,0)) \end{bmatrix}$$

$$= \begin{bmatrix} (0.5,0) & (0.4,0) \\ (0.3,0) & (0.7,0) \end{bmatrix}$$

In physical significance, it is a transportation problem, where two buses $M(A)$ and $M(B)$ are providing services at the same center 1 and 2, in such a way that bus $M(A)$ can provide service 40% at center 2,when 50% at the center 1 and 10% at center 2, when 30% at the center 1. Similarly, bus $M(B)$ Can provide service 30% at center 2 when 20% at the center 1 and 70% in 2 when 30% at the center 1. The maximum possibility of services between the two Buses at the center 1 and 2 are found that 40% at center 2 when 50% at center1 and 70% at center 2 when 30% at center 1 and is derived from the definition of the addition of imprecise matrices.

2.4.3. Subtraction: Subtraction of any two imprecise matrices $M(V)$ and $M(W)$ is defined by

$$M(V) - M(W) = \left(\min(v_{ij}, w_{ij}), \max(r_{ij}, r'_{ij})\right) \ldots\ldots\ldots\ldots\ldots (2.16)$$

Provided both the matrices are conformable for subtraction. Where v_{ij} , w_{ij} and r_{ij} , r'_{ij} are membership functions and reference functions of the given imprecise matrices respectively.

This theorem helps us to identify what the minimum service is provided between the two objects at the different locations.

2.4.3.1. Proposition: Two imprecise matrices of same type is commutative under the operation of subtraction.

Let, $M(K) = \begin{bmatrix} (0.1,0) & (0.2,0) \\ (0.3,0) & (0.4,0) \end{bmatrix}$ and $M(L) = \begin{bmatrix} (0.2,0) & (0.4,0) \\ (0.3,0) & (0.1,0) \end{bmatrix}$ be two imprecise matrices having each element is measured from the reference function 0. Then,

$$M(K) - M(L) = \begin{bmatrix} (0.1,0) & (0.2,0) \\ (0.3,0) & (0.1,0) \end{bmatrix}$$

$$M(K) - M(L) = \begin{bmatrix} (0.1,0) & (0.2,0) \\ (0.3,0) & (0.1,0) \end{bmatrix}$$

So we have,

$$M(K) - M(L) = M(L) - M(K) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.17)$$

2.4.3.2. Proposition: If $M^C(V)$ and $M^C(W)$ are Complement of same type imprecise matrix $M(A)$ and $M(B)$, Then

$$M^C(V - W) = M^C(V) - M^C(W) \ldots\ldots\ldots\ldots\ldots\ldots(2.18)$$

where $M^C(V - W)$ is complement of the difference between the two imprecise matrices. This theorem helps to identify the complement of minimum service provided between the two objects at the different locations.

Let, $M(V) = \begin{bmatrix} (0.4,0) & (0.3,0) \\ (0.5,0) & (0.2,0) \end{bmatrix}$ and $M(W) = \begin{bmatrix} (0.4,0) & (0.5,0) \\ (0.6,0) & (0.7,0) \end{bmatrix}$ be two imprecise matrices with each element is measured from the reference function 0. Then,

$$M^C(V) = \begin{bmatrix} (1,0.4) & (1,0.3) \\ (1,0.5) & (1,0.2) \end{bmatrix}$$

$$M^C(W) = \begin{bmatrix} (1,0.4) & (1,0.5) \\ (1,0.6) & (1,0.7) \end{bmatrix}$$

And $$M(V) - M(W) = \begin{bmatrix} (0.4,0) & (0.3,0) \\ (0.5,0) & (0.2,0) \end{bmatrix}$$

$$M^C(V - W) = \begin{bmatrix} (1,0.4) & (1,0.3) \\ (1,0.5) & (1,0.2) \end{bmatrix}$$

$$M^C(V) - M^C(W) = \begin{bmatrix} (1,0.4) & (1,0.3) \\ (1,0.5) & (1,0.2) \end{bmatrix}$$

So we have,

$$M^C(V - W) = M^C(V) - M^C(W)$$

2.4.4. Application

Let $M(K) = \begin{bmatrix} (0.2,0) & (0.3,0) \\ (0.4,0) & (0.5,0) \end{bmatrix}$ and $M(L) = \begin{bmatrix} (0.3,0) & (0.4,0) \\ (0.2,0) & (0.1,0) \end{bmatrix}$

be two imprecise matrices having each elements is measured from the reference function 0. Then,

$$M(K) - M(L) = \begin{bmatrix} \begin{Bmatrix} \min(0.2,0.3), \\ \max(0,0) \end{Bmatrix} & \begin{Bmatrix} \min(0..3,0.4), \\ max(0,0) \end{Bmatrix} \\ \begin{Bmatrix} \min(0.4,0.2), \\ \max(0,0) \end{Bmatrix} & \begin{Bmatrix} \min(0.5,0.1), \\ \max(0,0) \end{Bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} (0.2,0) & (0.3,0) \\ (0.2,0) & (0.1,0) \end{bmatrix}$$

In physical significance, it is a transportation problem where two buses $M(A)$ and $M(B)$ are providing services at the same center 1 and 2 in such a way that bus $M(A)$ can provide service 30% at center 2,when 20% at the center 1 and 50% at the center 2, when 40% at the center 1. Similarly, bus $M(B)$ can provide service 40% at center 2 when 30% at the center 1 and 10% at center 2 when 20% at the center 1. The minimum possibility of services at the center 1 and 2 are 30% at center 2 when 20% at center 1 and 10% at center 2 when 20% at center 1 and is derived from the definition of subtraction of two imprecise matrices.

2.5. Conclusions

Many real-life problems are complex in nature. Those are very much related to fuzziness. Imprecise number definition is the feasible definition of fuzzy number in terms of the complement of the fuzzy problem. So the involving of fuzzy number concept in the study of various set theories is discussed in the imprecise number definition. Different imprecise matrices related to fuzzy problems are presented in the diagram. Arithmetic operations of the imprecise matrices are represented with the maximum and minimum operator. With these operators, properties of imprecise matrices are discussed with examples. Finally, the imprecise matrices addition and the imprecise matrices subtraction have obtained an application in the field of transportation problems.