Chapter 6

Bulk Viscous Bianchi Type-V Cosmological Model with Special Type of Scale Factor in Lyra Geometry

6.1 Introduction

This chapter deals with the study of bulk viscous Bianchi type-V cosmological model in $f(R,T)$ theory of gravity. In cosmology, bulk viscosity is very constructive for studying the early stage of the universe's evolution as it plays an essential role in the inflationary phase. Barrow (1986) and Padmanabhan (1987) have pointed out that the universe's expansion causes inflationary solutions in bulk viscosity. Subsequently, in Brans Dicke's theory, Johri and Sudharsan (1988) studied the inflationary solutions in bulk viscosity. The presence of bulk viscosity institutes several intriguing characteristics of the universe's dynamics. he bulk viscosity cosmological models and the nature of the models have been discussed by Singh & Baghel (2009), Yadav & Yadav (2011), Bali et al. (2012), Kiran &

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Reddy (2013), Mahanta (2014), Rao & Rao (2015), Tiwari & Tiwari (2017) and, Gogoi & Goswami(2021) have recently investigated the modeling of accelerating and the dynamical behaviors of Chaplygin gas, cosmological and gravitational 'constants' with cosmic viscous fluid in a different context.

As evidenced by various observational data by Garnavich (1998a, 1998b), Riess et al. (1998), and Perlmutter et al. (1997,1999), the expansion of the universe is accelerating. Among modifications of Einstein's theories are Brans Dicke's theory, $f(R)$ gravity, $f(T)$ gravity, $f(G)$ gravity, and $f(R,G)$ gravity, where R, T and G indicates the scalar curvature, the torsion scalar, and the Gauss-Bonnet scalar respectively. However, in the recent cosmological model, $f(R)$ gravity has become a more attractive theory to represent the behavior of the expansion of the universe, known as $f(R,T)$ gravity. Recently, Arora et al. (2021) discussed the late-time viscous cosmology in $f(R,T)$ gravity considering a bulk viscous fluid with a viscosity coefficient.

As motivated by the above, in this chapter, we have investigated the behavior of the present cosmological model in Lyra Geometry with Bianchi type-V. Many authors have looked into cosmological theories based on Lyra geometry [Pradhan and Vishwakarma (2004), Singh (2008), Singh and Kale (2009), Ram et al. (2010), Adhav (2011), Kumari et al. (2013), Singh and Rani (2015), Maurya and Zia (2019), Ram et al. (2020), Hegazy (2020)], where they investigated the different nature of the model in the different cosmological model so far. Recently Brahma and Dewri (2021) have investigated the $f(R,T)$ gravity for Bianchi type – V metric in Lyra geometry, and one particular form (Harko et al., 2011) is used with linearly varying deceleration parameters to investigate the mysterious nature of the DE.

6.2 Metric and the field equations of $f(R,T)$ gravity

Let us consider the Bianchi type-V space time in the form

$$ds^2 = -dt^2 + A^2dx^2 + e^{-2mx}(B^2dy^2 + C^2dz^2) \tag{6.1}$$

where A,B,C are functions of cosmic time t and m is a constant.
The action of $f(R,T)$ gravity in Lyra geometry is

$$S = \frac{1}{16\pi}\int f(\widetilde{R},T)\sqrt{-g}\ d^4x + \int L_m\sqrt{-g}\ d^4x \tag{6.2}$$

where

$$\widetilde{R} = R + 3\nabla_i\phi^i + \frac{3}{2}\phi^i\phi_i \tag{6.3}$$

In which $\widetilde{R}$, T and L_m are respectively denotes the function of Ricci scalar R, the trace of the stress tensor and Lagrangian density of matter.
The EMT for a perfect fluid is given by

$$T_{ij} = (\rho + \overline{p})u_iu_j + \overline{p}g_{ij} \tag{6.4}$$

Here, ρ denote the energy density and $u^i = (0,0,0,1)$ is the four velocity vector in co-moving co-ordinate system satisfying the condition $u_iu^i = -1$ and $u^i\nabla_ju_i = 0$. Here, it is assumed a perfect fluid matter as $L_m = -\overline{p}$ and the trace of the total EMT [Debnath (2019)] is given by $T = \rho - 3\overline{p}$, so that eq. (1.14) reduces to as follows:

$$\theta_{ij} = -2T_{ij} - \overline{p}g_{ij} \tag{6.5}$$

To obtain the remarkable behaviour of the present model of the universe in occurence with the bulk viscous fluid, one frame of Harko et al. (2011) is considered as given by

$$f(\widetilde{R},T) = f_1(\widetilde{R}) + f_2(T) \tag{6.6}$$

where $f(\widetilde{R},T)$ is an arbitrary function of the trace of the stress tensor.

Then, for the metric (6.1), the EFE (1.21) reduces to the form as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 = -\overline{p} + \left(\frac{\rho - \overline{p}}{2}\right) \tag{6.7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{C}\dot{A}}{CA} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 = -\overline{p} + \left(\frac{\rho - \overline{p}}{2}\right) \tag{6.8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 = -\overline{p} + \left(\frac{\rho - \overline{p}}{2}\right) \tag{6.9}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{3m^2}{A^2} - \frac{3}{4}\beta^2 = \rho + \left(\frac{\rho - \overline{p}}{2}\right) \tag{6.10}$$

$$\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - 2\frac{\dot{A}}{A} = 0 \tag{6.11}$$

6.3 Solutions of the field equations

The spatial volume and the scale factor are given by

$$V = a^3 = ABC \tag{6.12}$$

The generalized HP and the scalar expansion are defined as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}(H_1 + H_2 + H_3); \quad \theta = 3H \tag{6.13}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$, $H_3 = \frac{\dot{C}}{C}$ are the directional HP in the directions of x, y and z axes respectively.

Integrating eq. (6.11), we get

$$A^2 = k_1 BC \tag{6.14}$$

where k_1 is an integrating constant and without loss of generality, the constant of integration k_1 can be chosen as unity as

$$A^2 = BC \tag{6.15}$$

In the field equations (6.7) - (6.11), it is found that there are five equations involving seven unknowns. To solve the highly non-linear field equations, the following conditions are considered:

- The shear scalar is proportional to the expansion scalar [Collins et al. (1980)]

$$B = C^n \tag{6.16}$$

where n is a non zero constant.

- The combined effect of the proper pressure and the bulk viscous pressure, for a barotropic fluid is:

$$\overline{p} = p - 3\xi H = \varepsilon\rho \ ; \ \ p = \varepsilon_0 \rho \tag{6.17}$$

such that $\varepsilon = \varepsilon_0 - \eta (0 \leq \varepsilon_0 \leq 1)$ and ε, ε_0 and η are constant. The symbols ξ and p are respectively known as the coefficient of bulk viscosity and proper pressure of the model.

Let us consider a scale factor [Pradhan et al. (2006)] as given by

$$a(t) = \alpha e^{\alpha_1 t} \tag{6.18}$$

where α and α_1 are constants.

From eqs. (6.12), (6.15), and (6.16), we get the metric potentials of the model as

$$A = \alpha e^{\alpha_1 t},\ B = \left(\alpha e^{\alpha_1 t}\right)^{\frac{2n}{n+1}},\ C = \left(\alpha e^{\alpha_1 t}\right)^{\frac{2}{n+1}} \tag{6.19}$$

Then eq. (6.1) reduces to

$$ds^2 = -dt^2 + (\alpha e^{\alpha_1 t})^2 dx^2 + e^{-2mx}\left[\left(\alpha e^{\alpha_1 t}\right)^{\frac{4n}{n+1}} dy^2 + \left(\alpha e^{\alpha_1 t}\right)^{\frac{4}{n+1}} dz^2\right] \tag{6.20}$$

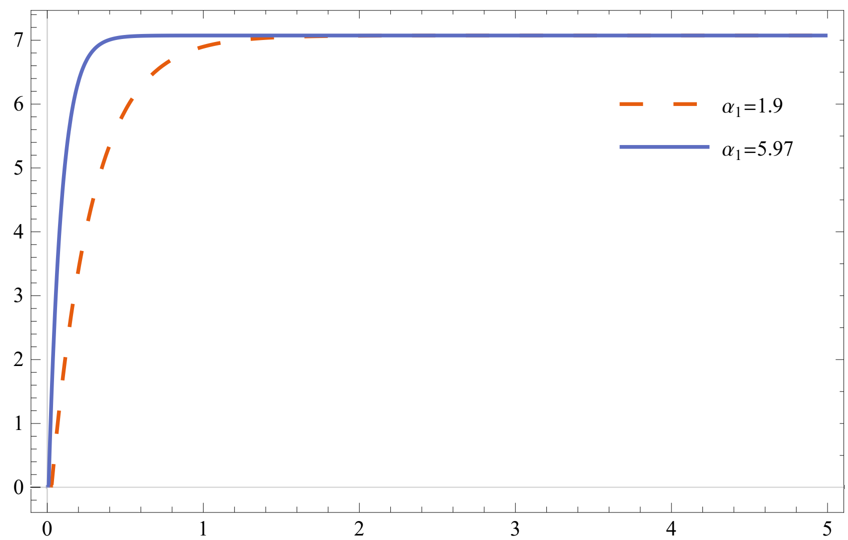

Figure 6.1: Variation of ρ vs. t

6.4 Physical properties of the model in $f(R,T)$ gravity

Spatial Volume:

$$V = a^3(t) = \left(\alpha e^{\alpha_1 t}\right)^3 \tag{6.21}$$

Hubble's Parameter:

$$H = \alpha\alpha_1 \tag{6.22}$$

The Expansion Scalar:

$$\theta = 3\alpha\alpha_1 \tag{6.23}$$

The Shear Scalar:

$$\sigma^2 = \left(\frac{n-1}{n+1}\right)^2 (\alpha\alpha_1)^2 \tag{6.24}$$

Anisotropy parameter:

$$A_m = \frac{2}{3}\left(\frac{n-1}{n+1}\right)^2 \tag{6.25}$$

Deceleration parameter:

$$q = -1 \tag{6.26}$$

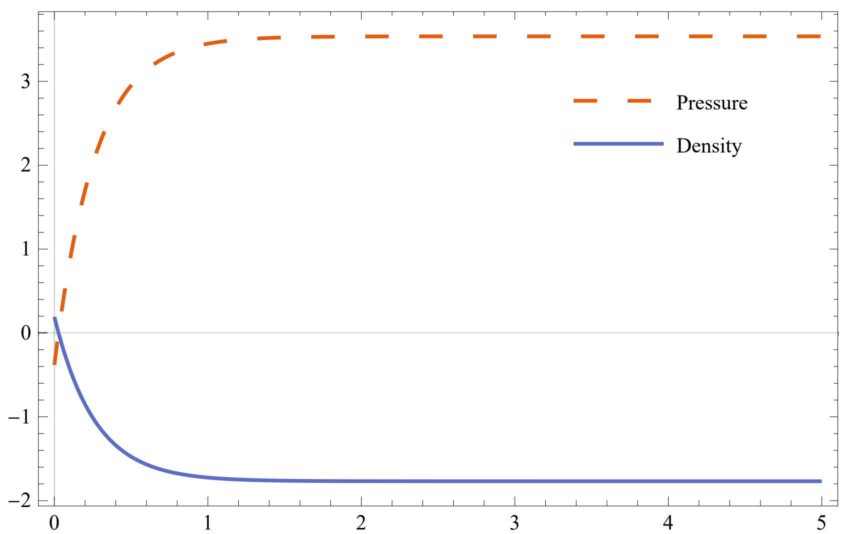

Figure 6.2: Variation of $\overline{p}$ and p vs. t

Adding eqs. (6.7) - (6.9) and applying in (6.10), we get

$$\rho = \frac{1}{2\varepsilon - 1}\left[-6(\alpha\alpha_1)^2 + \frac{4m^2}{(\alpha e^{\alpha_1 t})^2}\right] \tag{6.27}$$

Total pressure:

$$\overline{p} = \frac{\varepsilon}{2\varepsilon - 1}\left[-6\,(\alpha\alpha_1)^2 + \frac{4m^2}{(\alpha e^{\alpha_1 t})^2}\right] \tag{6.28}$$

Proper pressure:

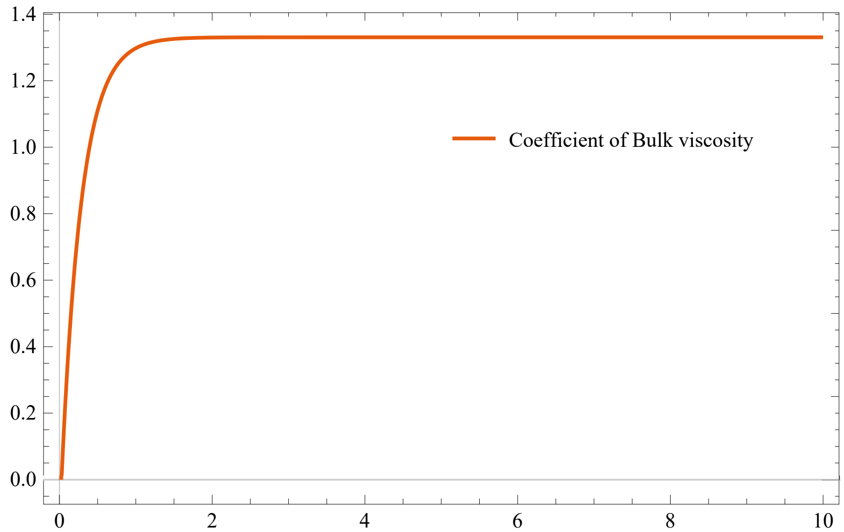

Figure 6.3: Variation of ξ vs. t

$$p = \frac{\varepsilon_0}{2\varepsilon - 1}\left[-6(\alpha\alpha_1)^2 + \frac{4m^2}{(\alpha e^{\alpha_1 t})^2}\right] \tag{6.29}$$

Co-efficient of bulk viscosity:

$$\xi = \frac{\varepsilon_0 - \varepsilon}{3(2\varepsilon - 1)(\alpha\alpha_1)}\left[-6(\alpha\alpha_1)^2 + \frac{4m^2}{(\alpha e^{\alpha_1 t})^2}\right] \tag{6.30}$$

Displacement vector:

$$\begin{aligned}\frac{3}{4}\beta^2 = -3\,(\alpha\alpha_1)^2 - \frac{m^2}{(\alpha e^{\alpha_1 t})^2} + \frac{1 - 3\varepsilon}{2\,(2\varepsilon - 1)}\left[-6(\alpha\alpha_1)^2 + \frac{4m^2}{(\alpha e^{\alpha_1 t})^2}\right] \\ + \left(\frac{n-1}{n+1}\right)^2 (\alpha\alpha_1)^2\end{aligned} \tag{6.31}$$

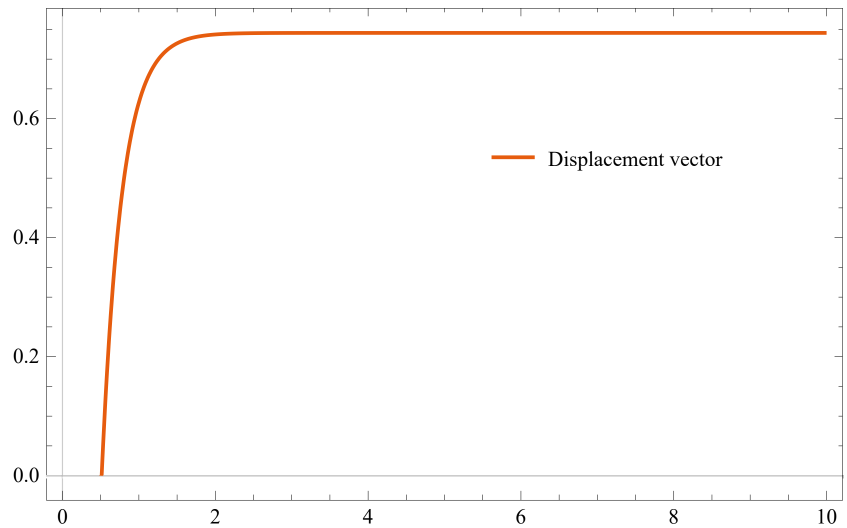

Figure 6.4: Variation of β^2 vs. t

The trace $(T = \rho - 3\overline{p})$, function of Ricci-Scalar and the $f(\widetilde{R},T)$ gravity are given by

$$T = \frac{1-3\varepsilon}{2\varepsilon-1}\left[-6\left(\alpha\alpha_1\right)^2 + \frac{4m^2}{\left(\alpha e^{\alpha_1 t}\right)^2}\right] \tag{6.32}$$

$$\begin{aligned}\widetilde{R} = &\left(\frac{\varepsilon}{1-2\varepsilon}\right)\frac{4m^2}{(\alpha e^{\alpha_1 t})^2} - 6\left(\frac{3\varepsilon-2}{2\varepsilon-1}\right) + \frac{18}{\sqrt{3}}\left(\alpha\alpha_1\right)Z_1 \\ &+2\sqrt{3}m^2\left(\frac{8\varepsilon-3}{2\varepsilon-1}\right)\left(\frac{\alpha\alpha_1 e^{\alpha_1 t}}{(\alpha\alpha_1 e^{\alpha_1 t})^3}\right)Z_2\end{aligned} \tag{6.33}$$

$$\begin{aligned}\frac{1}{\mu}f(\widetilde{R},T) = &\left(\frac{\varepsilon}{1-2\varepsilon}\right)\frac{4m^2}{(\alpha e^{\alpha_1 t})^2} - 6\left(\frac{3\varepsilon-2}{2\varepsilon-1}\right) + \frac{18}{\sqrt{3}}\left(\alpha\alpha_1\right)Z_1 \\ &+2\sqrt{3}m^2\left(\frac{8\varepsilon-3}{2\varepsilon-1}\right)\left(\frac{\alpha\alpha_1 e^{\alpha_1 t}}{(\alpha\alpha_1 e^{\alpha_1 t})^3}\right)Z_2 \\ &+\frac{1-3\varepsilon}{2\varepsilon-1}\left[-6(\alpha\alpha_1)^2 + \frac{4m^2}{(\alpha e^{\alpha_1 t})^2}\right]\end{aligned} \tag{6.34}$$

where $Z_1(t) = \left[\frac{3\varepsilon}{2\varepsilon-1}(\alpha\alpha_1)^2 + \frac{1-8\varepsilon}{2\varepsilon-1}\frac{m^2}{(\alpha\alpha_1 e^{\alpha_1 t})^2}\right]^{\frac{-1}{2}}$

and $Z_2(t) = \left[\frac{3\varepsilon}{2\varepsilon-1}(\alpha\alpha_1)^2 + \frac{1-8\varepsilon}{2\varepsilon-1}\frac{m^2}{(\alpha\alpha_1 e^{\alpha_1 t})^2}\right]^{\frac{1}{2}}$

From eqs. (6.19) and (6.22), we get the statefinder parameters, which is defined as $r = \frac{\dddot{a}}{aH^3}$ and $s = \frac{r-1}{3(q-\frac{1}{2})}$, that exactly gives the value 1 and 0 respectively.

Energy Conditions: The ECs are constructed from the Raychaudhuri equation, an essential tool for describing the compatibility of timelike, lightlike, or spacelike curves and singularities. Very recently, Alvarenga (2012), Moraes & Sahoo (2017), Zubair et al. (2018), and Ahmed & Abbas (2020) have tested the energy conditions in the $f(R,T)$ theory of gravity and to check the ECs for the present model; they defined the WEC, DEC and the SEC as given by

(i) $\rho \geq 0, \ \ \rho + p \geq 0$ (ii) $\rho - p \geq 0$ (iii)$\rho + 3p \geq 0$

To observe the absolute observational data, the graphs are plotted for energy conditions in terms of ρ and $\overline{p}$, and we extensively observe from the graph that all three energy conditions are satisfied in the present model.

Figure 6.5: Variation of ECs vs. t

6.5 Conclusion

This chapter discusses a completely spatially homogeneous and anisotropic bulk viscous Bianchi type-V cosmological model in Lyra geometry, with an exponential form of the scale factor. A barotropic equation of state is employed to determine the nature and deterministic solution of the highly nonlinear differential equation. Furthermore, bulk viscous pressure is assumed to be proportional to energy density [Naidu et al. (2012)]. According to recent observational data in combination with BAO and CMB from SNeIa, the model found in this study is confirmed. The model (6.20) found here is shearing, expanding, and anisotropic, which is similar to [Zia et al. (2018), Tiwari (2020), Desikan (2020)]. At $t = 0$, It is obtained that the model has no singularity. Subsequently, we obtained from eqs. (6.22) and (6.23) that HP and the expansion scalar remain constant throughout the expansion, implying that model (6.20) represents a uniform expansion. It is evident from eq. (6.21) that the volume of the universe rises with cosmic time t, and that as V approaches infinity, for $t \to \infty$. We observed from eq. (6.30), the bulk viscosity coefficient increases with time and approaches infinity as $t \to \infty$. ρ, $\bar{p}$, ξ, and β^2 all rise positively, but they all yield a constant value for $t \to \infty$ [Figs. (6.1 - 6.4)]. The model anticipated an accelerating phase of the universe for $q = -1$, [Eq. (6.26)]. In the current universe model, there is DE due to negative pressure with bulk viscous fluid [Fig. (6.2)]. All three ECs are satisfied, as in Fig. (6.5). The trace and the function of the Ricci scalar are always positive throughout the cosmic time t, and for $t \to \infty$ offer a constant value, as shown in Eqs. (6.32) and (6.33). Furthermore, r and s tend to be 1 and 0, respectively, indicating that the current universe model approaches the ΛCDM model. Researchers in Lyra geometry have done many works, but Lyra geometry with $f(R,T)$ gravity is a new concept, and there is scope for continuing work.