Chapter 7

Bianchi Type-V Dark Energy Model with varying EOS parameter in Lyra Geometry

7.1 Introduction

This chapter deals with the study of Bianchi type-V DE model with varying EOS parameters in Lyra Geometry. Recent acceleration of the universe has been endorsed by several observational data such that SNela [Riess et al. (1998), Perlmutter et al. (1998a, 1999)] CMB [Spergel et al. (2003), Bennett et al. (2003)], LSS [Hawkins et al. (2003), Tegmark et al. (2004)], BAO [Eisenstein et al. (2005), Aubourg et al. (2015)], Cosmic Microwave Radiation [Caldwell & Doran (2004), Huang et al. (2006)] and behind this accelerated expansion is mainly for negative pressure, called DE. The leading aspirants for the DE are modified gravity, cosmological constant, quintessence, phantom, quintom, K-essence, holographic DE, and tachyon. Several researchers have shown the behavior of the model. DE, which merely matters with negative pressure and the accelerating universe, has been investigated in several modified gravity theories. Several researchers have studied the idea and the DE model in different contexts. Because of this context, here in this study, we are trying to explore the characteristics of the DE model with varying EOS parameters based on $f(R,T)$ gravity. Where R is the Ricci scalar and T is the trace of the stress-energy tensor. Generally, barotropic EOS, a combination of pressure and density, has played a vital role in exploring the mysterious characteristics of the universe's current expansion. It is well known that the general form of quadratic EOS is

$$p = p(\rho) = a_1\rho^2 + a_2\rho + a_3 \tag{7.1}$$

where a_1, a_2 and a_3 are all constants such that $a_1 \neq 0$. Here, it is considered that $a_1 < 0$ and the rest are positive for discussion and to explore the relation between the pressure and the density. Other researchers are discussing with a positive value for a_1 and show the behavior of the model as concerned.

Moreover, Anada and Bruni (2006) examined and explored the general relativistic dynamics of Robertson-Walker models and the unique properties of a nonlinear equation of state in the context of GR at the singularity [Reddy et al. (2015)]. For $\rho = 0$, equation (i) shows the first term of Taylor expansion of EOS of the form $p = p(\rho)$. To study the consequence of the quadratic equation, they looked at an EOS in the form $p = \alpha\rho + \frac{\rho^2}{\rho_c}$ in GR. DE with various EOS discussed by several authors [Pradhan et al. (2011), Avelino et al. (2012), Saha (2013), Chavanis (2015), Singh & Bishi (2015), Thakur (2017)] investigated the spatially homogeneous and anisotropic Bianchi I cosmological model in the presence of a cosmological parameter Λ in $f(R,T)$ theory with EOS parameter. Very recently, Dagwal and Pawar (2021) examined the cosmological models in the $f(T)$ theory of gravity with EOS parameters, and the torsion scalar is taken to discuss the cosmic acceleration of the universe without DE.

The above discussion and the theory motivated us to investigate the DE in $f(R,T)$ gravity based on Lyra geometry with EOS parameter in Bianchi type-V model. Brahma and Dewri (2021) have recently investigated the spatially homogeneous and anisotropic

Bianchi type-V model in Lyra geometry.

7.2 Metric and the field equations of $f(R,T)$ gravity

Let us consider the Bianchi type-V space-time in the form

$$ds^2 = -dt^2 + A^2dx^2 + e^{-2mx}(B^2dy^2 + C^2dz^2) \tag{7.2}$$

where A, B, C are functions of cosmic time t and m is a constant.

The EMT is given by

$$T_{ij} = (\rho + p)u_i u_j + pg_{ij} \tag{7.3}$$

where $u^i = (0,0,0,1)$ is the four velocity vector in co-moving co-ordinate system satisfying the condition $u_i u^i = -1$ and $u^i \nabla_j u_i = 0$. Also, ρ and p denotes the energy density for the perfect fluid and pressure of the fluid respectively.

For the metric (7.2), the EFE (1.21) in view of eq. (7.3) reduces as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 = -p + \left(\frac{\rho - p}{2}\right) \tag{7.4}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{C}\dot{A}}{CA} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 = -p + \left(\frac{\rho - p}{2}\right) \tag{7.5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 = -p + \left(\frac{\rho - p}{2}\right) \tag{7.6}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{3m^2}{A^2} - \frac{3}{4}\beta^2 = \rho + \left(\frac{\rho - p}{2}\right) \tag{7.7}$$

$$\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - 2\frac{\dot{A}}{A} = 0 \tag{7.8}$$

Here, $\alpha = \left(\frac{8\pi G - \mu c^2}{\mu c^2}\right)$ is taken as unity.

7.3 Solutions and physical properties of the field equations

For Bianchi type-V metric, the average scale factor and the spatial volume are defined as

$$V = a(t)^3 = (ABC) \tag{7.9}$$

and the average HP is given by

$$H = \frac{\dot{a}}{a} \tag{7.10}$$

The directional HP in the directions of x, y, and z respectively are defined as follows:

$$H_1 = \frac{\dot{A}}{A},\ \ H_2 = \frac{\dot{B}}{B},\ \ H_3 = \frac{\dot{C}}{C} \tag{7.11}$$

Then the generalized HP and the scalar expansion are defined as

$$H = \frac{1}{3}(H_1 + H_2 + H_3)\ and\ \theta = 3H = (H_1 + H_2 + H_3) \tag{7.12}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$, $H_3 = \frac{\dot{C}}{C}$ are the directional HP in the directions of x, y and z axes respectively.

Integrating eq. (7.8), we get

$$A^2 = k_1 BC \tag{7.13}$$

where $k_1$is an integrating constant and without loss of generality, the constant of integration k_1 can be chosen as unity

$$A^2 = BC \tag{7.14}$$

The system of eqs. (7.4)–(7.8) consists five independent equations having six unknowns; A, B, C, ρ, p and β. To obtain the determinate solution set for the system, another two conditions is considered to complete the system as follows:

(i) Functional form [Desikan (2020)] for displacement vector as given below

$$\beta^2(t) = \beta_0 H^2 + H \tag{7.15}$$

where β_0 is a constant.

(ii) The shear scalar (σ) is proportional to the expansion scalar (θ) [Collins et al. (1980)]

$$B = C^n \tag{7.16}$$

where n is a non zero constant.

Now, to discuss the logamediate inflation in the context of $f(\widetilde{R}, T)$ gravity in this model, let us consider the generalized version of inflation [Barrow & Nunes (2007)], known as logamediate inflation with a scale factor of the form

$$a(t) = exp[\xi(\ln t)^{\lambda}] \tag{7.17}$$

where $\xi > 0$ and $\lambda > 1$ are constant. The model reduces to power-law inflation with $a = t^p$ for $\lambda = 1$, where $p = \xi$. The logamediate inflationary form is driven by a class of indefinitely expanding cosmological solutions that come from weak general requirements being imposed on the cosmological model [Barrow & Nunes (2007)] .

From eqs. (7.9), (7.14), (7.16) and (7.17), the metric potentials for this model is

$$A(t) = exp[\xi(\ln t)^{\lambda}] \tag{7.18}$$

$$B(t) = exp[\xi(\ln t)^{\lambda}]^{\frac{2n}{n+1}} \tag{7.19}$$

$$C(t) = exp[\xi(\ln t)^{\lambda}]^{\frac{2}{n+1}} \tag{7.20}$$

From eqs. (7.18)-(7.20), the present model is obtained as

$$ds^2 = -dt^2 + \left[\exp(\xi(\ln t)^\lambda)\right]^2 dx^2 + e^{-2mx}\left[\exp(\xi(\ln t)^\lambda)\right]^{\frac{4n}{n+1}} dy^2 \\ + e^{-2mx}\left[\exp(\xi(\ln t)^\lambda)\right]^{\frac{4}{n+1}} dz^2 \tag{7.21}$$

The main physical and geometric parameters of this model, which are the remarkable objects for the discussion, such as the spatial volume, the average HP, the expansion scalar, the anisotropy parameter, the shear scalar and the deceleration parameter are obtain as follows:
The spatial volume is given by

$$V = \left[exp\left(\xi(\ln t)^\lambda\right)\right]^3 \tag{7.22}$$

The average HP and the expansion scalar are

$$H = \xi\lambda(\ln t)^{\lambda-1}\frac{1}{t} \tag{7.23}$$

and

$$\theta = 3\xi\lambda(\ln t)^{\lambda-1}\frac{1}{t} \tag{7.24}$$

The anisotropy parameter of the expansion is obtain from eq. (1.38) as

$$A_m = \frac{2}{3}\left(\frac{n-1}{n+1}\right)^2 \tag{7.25}$$

where for $n \neq 1$, the model approaches anisotropy, whereas for $n = -1$, it deserves as isotropy.
Subsequently, the shear scalar and the DP are obtain from eqs. (7.12) and (1.32) respectively as

$$\sigma^2 = \left(\frac{n-1}{n+1}\right)^2\left(\xi\lambda(\ln t)^{\lambda-1}\frac{1}{t}\right)^2 \tag{7.26}$$

and

$$q = -1 + \frac{1}{\xi\lambda(\ln t)^{\lambda-1}} - \frac{\lambda-1}{\xi\lambda(\ln t)^{\lambda-2}} \tag{7.27}$$

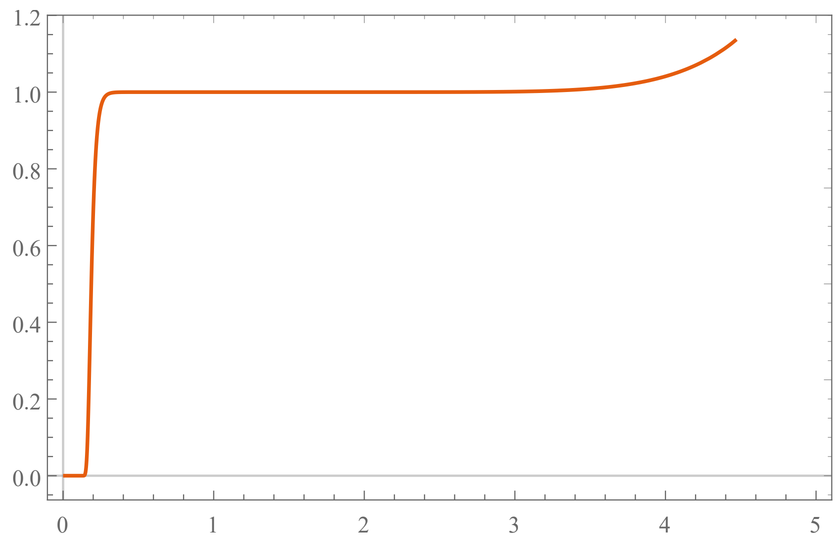

Figure 7.1: Variation of V vs. t, for $\xi = 0.00001, \lambda = 15$

From eqs. (7.4)-(7.6), we obtained that the energy density of this model in $f(\widetilde{R}, T)$ gravity based on Lyra geometry with quadratic EOS parameter as

$$\rho = \frac{-4a_2 \pm \sqrt{(4a_2)^2 - 4(4a_1)(2Z + 3\beta^2 + 4a_3)}}{8a_1} \tag{7.28}$$

where $Z = \left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C}\right)$ is a function of cosmic time t.
Applying (7.15) in (7.28), energy density reduces to

$$\rho = \frac{-4a_2 \pm \sqrt{(4a_2)^2 - 4(4a_1)(2Z + 3\beta_0 H^2 + 3H + 4a_3)}}{8a_1} \tag{7.29}$$

Subsequently, using (7.29) in the general form of quadratic EOS $(p = a_1\rho^2 + a_2\rho +$

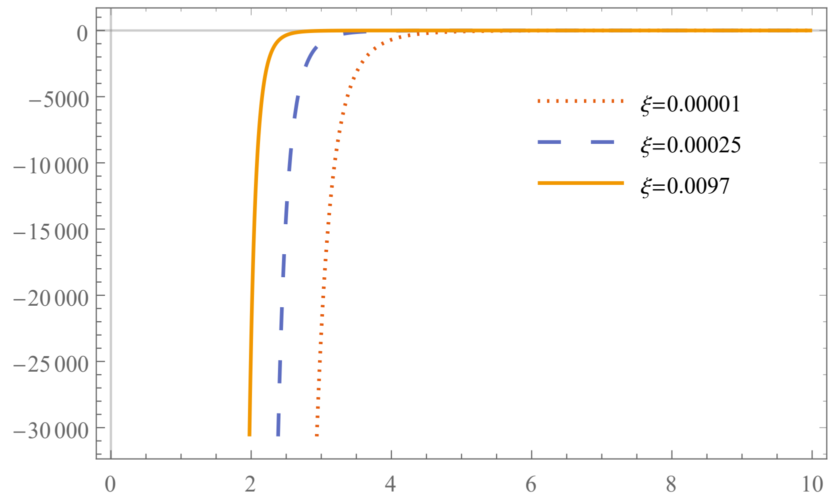

Figure 7.2: Variation of q vs. t, for $\lambda = 15$

a_3), we have the pressure of the model as

$$p = a_1\left(\frac{-4a_2 \pm \sqrt{(4a_2)^2 - 4(4a_1)(2Z + 3\beta_0 H^2 + 3H + 4a_3)}}{8a_1}\right)^2 + a_2\left(\frac{-4a_2 \pm \sqrt{(4a_2)^2 - 4(4a_1)(2Z + 3\beta_0 H^2 + 3H + 4a_3)}}{8a_1}\right) + a_3 \tag{7.30}$$

The displacement vector is obtained by applying eq. (7.23) in (7.15) as given by

$$\beta^2(t) = \beta_0 \xi^2 \lambda^2 (\ln t)^{2(\lambda-1)} \frac{1}{t^2} + \xi \lambda (\ln t)^{\lambda-1} \frac{1}{t} \tag{7.31}$$

From eqs. (7.29) and (7.30), the trace $(T = \rho - 3p)$ of this model is

$$T = \frac{-4a_2 \pm \sqrt{(4a_2)^2 - 4(4a_1)(2Z + 3\beta_0 H^2 + 3H + 4a_3)}}{8a_1} - 3a_1\left(\frac{-4a_2 \pm \sqrt{(4a_2)^2 - 4(4a_1)(2Z + 3\beta_0 H^2 + 3H + 4a_3)}}{8a_1}\right)^2 - 3a_2\left(\frac{-4a_2 \pm \sqrt{(4a_2)^2 - 4(4a_1)(2Z + 3\beta_0 H^2 + 3H + 4a_3)}}{8a_1}\right) - 3a_3 \tag{7.32}$$

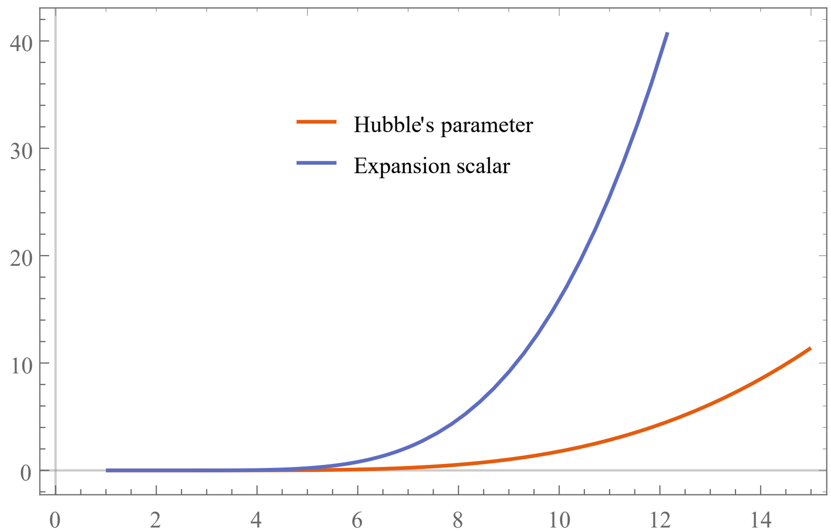

Figure 7.3: Variation of H and θ vs. t

From (7.18)-(7.20), we get the Riemannian curvature scalar as follows:

$$R=\frac{6m^2}{\left[\exp\xi(\ln t)^{\lambda}\right]^2}-6\xi\lambda(\lambda-1)(\ln t)^{\lambda-2}\frac{1}{t^2}-6\xi\lambda(\ln t)^{\lambda-1}\frac{1}{t^2}+6\left(\frac{n-1}{n+1}\right)^2\left(\xi\lambda(\ln t)^{(\lambda-1)}\frac{1}{t}\right)^2 \tag{7.33}$$

Subsequently, using eq. (7.33) in (1.8), the Ricci scalar tensor is obtained as:

$$\begin{aligned}\widetilde{R}=&\frac{6m^2}{\left[\exp\xi(\ln t)^{\lambda}\right]^2}-6\xi\lambda(\lambda-1)(\ln t)^{\lambda-2}\frac{1}{t^2}-6\xi\lambda(\ln t)^{\lambda-1}\frac{1}{t^2}\\&+6\left(\frac{n-1}{n+1}\right)^2\xi^2\lambda^2(\ln t)^{2(\lambda-1)}\frac{1}{t^2}+3W_2\beta_0\xi^2\lambda^2\frac{1}{t^3}\left[(\lambda-1)(\ln t)^{2\lambda-3}-(\ln t)^{\lambda-1}\right]\\&+\frac{3}{2}W_2\xi\lambda\frac{1}{t^2}\left[(\lambda-1)(\ln t)^{\lambda-2}-(\ln t)^{\lambda-1}\right]+3W_1\xi\lambda(\ln t)^{\lambda-1}\frac{1}{t}\\&+\frac{3}{2}\beta_0\xi^2\lambda^2(\ln t)^{2(\lambda-1)}\frac{1}{t^2}+\frac{3}{2}\xi\lambda(\ln t)^{\lambda-1}\end{aligned} \tag{7.34}$$

where

$W_1(t)=\beta(t)=\left[\beta_0\xi^2\lambda^2(\ln t)^{2(\lambda-1)}\frac{1}{t^2}+\xi\lambda(\ln t)^{\lambda-1}\frac{1}{t}\right]^{\frac{1}{2}}$,

$W_2(t)=\frac{1}{\beta(t)}=\left[\beta_0\xi^2\lambda^2(\ln t)^{2(\lambda-1)}\frac{1}{t^2}+\xi\lambda(\ln t)^{\lambda-1}\frac{1}{t}\right]^{\frac{-1}{2}}$

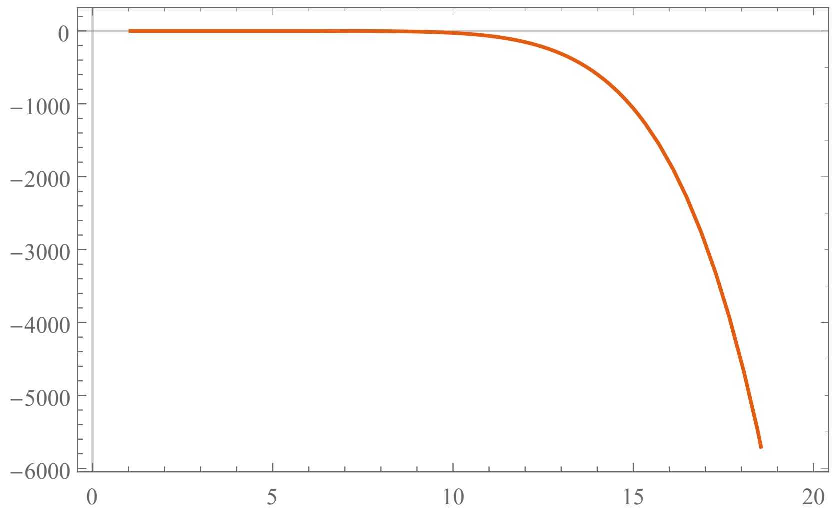

Figure 7.4: Variation of p vs. t for $a_1 = -5, a_2 = 6, a_3 = 10, \beta_0 = 4, \xi = 0.00001, \lambda = 15$

In this study, to obtain the field equations in $f(\widetilde{R}, T)$ gravity [$\widetilde{R}$ is a function of Ricci scalar R and T are the trace of the EMT], we have taken the one frame of Harko et al. (2011), by choosing $f_1 = \mu\widetilde{R}$ and $f_2(T) = \mu T$, where μ is an arbitrary constant. So that it reduces to

$$f(\widetilde{R}, T) = \mu\widetilde{R} + \mu T \tag{7.35}$$

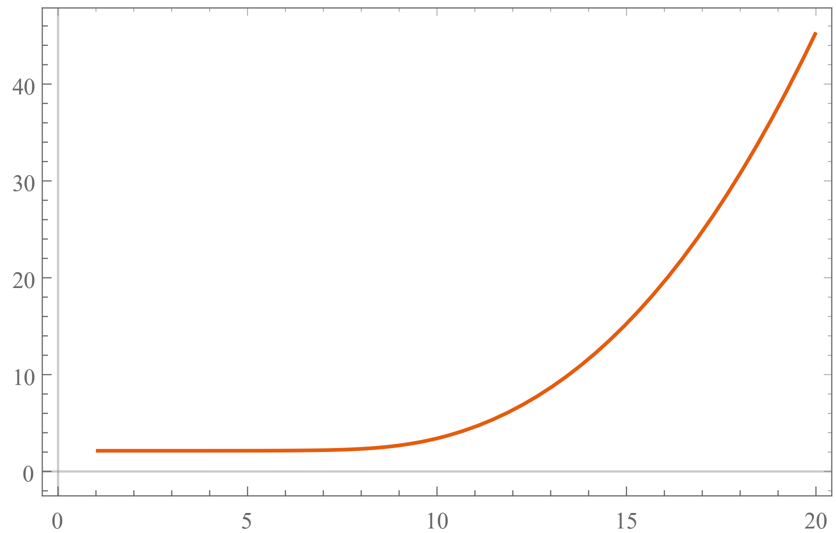

Figure 7.5: Variation of ρ vs. t for $a_1 = -5, a_2 = 6, a_3 = 10, \beta_0 = 4, \xi = 0.00001, \lambda = 15$

Applying eqs. (7.32) and (7.34) in (7.35), we get

$$\begin{aligned}
\frac{1}{\mu} f(\widetilde{R},T) = & \frac{6m^2}{\left[\exp \xi (\ln t)^{\lambda}\right]^2} - 6\xi\lambda(\lambda-1)(\ln t)^{\lambda-2}\frac{1}{t^2} - 6\xi\lambda(\ln t)^{\lambda-1}\frac{1}{t^2} \\
& + 6\left(\frac{n-1}{n+1}\right)^2 \xi^2\lambda^2(\ln t)^{2(\lambda-1)}\frac{1}{t^2} + 3W_2\beta_0\xi^2\lambda^2\frac{1}{t^3}\left[(\lambda-1)(\ln t)^{2\lambda-3} - (\ln t)^{\lambda-1}\right] \\
& + \frac{3}{2}W_2\xi\lambda\frac{1}{t^2}\left[(\lambda-1)(\ln t)^{\lambda-2} - (\ln t)^{\lambda-1}\right] + 3W_1\xi\lambda(\ln t)^{\lambda-1}\frac{1}{t} \\
& + \frac{3}{2}\beta_0\xi^2\lambda^2(\ln t)^{2(\lambda-1)}\frac{1}{t^2} + \frac{3}{2}\xi\lambda(\ln t)^{\lambda-1} \\
& - \frac{-4a_2 \pm \sqrt{(4a_2)^2 - 4(4a_1)\left(2Z + 3\beta_0 H^2 + 3H + 4a_3\right)}}{8a_1} \\
& - 3a_1\left(\frac{-4a_2 \pm \sqrt{(4a_2)^2 - 4(4a_1)\left(2Z + 3\beta_0 H^2 + 3H + 4a_3\right)}}{8a_1}\right)^2 \\
& - 3a_2\left(\frac{-4a_2 \pm \sqrt{(4a_2)^2 - 4(4a_1)\left(2Z + 3\beta_0 H^2 + 3H + 4a_3\right)}}{8a_1}\right) - 3a_3
\end{aligned} \tag{7.36}$$

It is observed from eq. (7.22) the spatial volume of the present model diverges at $t = 0$, but it begins to increase positively for $t > 1$. It is observed that the spatial volume increases

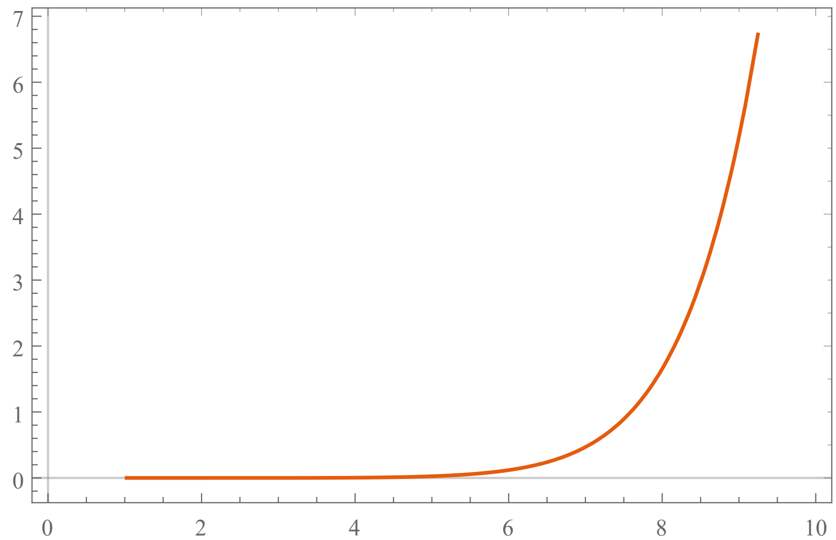

Figure 7.6: Variation of β^2 vs. t for $\xi = 0.00001, \lambda = 15$

as t increases, and this result shows us that this present model is expanding with cosmic time (t).

To recent observations of SNe Ia data, the present model is accelerating, and the DP lies in the range -1 and 0. Furthermore, this value is firmly maintained in this present universe model also, for $\xi > 0$ and $\lambda \geq 1$.

7.4 Energy Conditions

ECs are a combination of pressure and energy density constructed from the Raychaudhuri equation. Apart from that, it is found that [Sharif & Zubair (2012), Moraes & Sahoo (2017), and Arora et al. (2021)] these tools are essential to understanding the mysterious characteristics of timelike, lightlike, or spacelike curves and singularities.

We depict the variation of ECs of WEC, SEC, and DEC in Fig. 7.7, and thus, from that, we see the evolution of these three ECs with cosmic time t. As a result, only WEC is violet in this present universe model. However, both SEC and DEC are satisfied with the whole evolution of the universe in the context of the EOS parameter with $f(R,T)$ gravity in Lyra geometry.

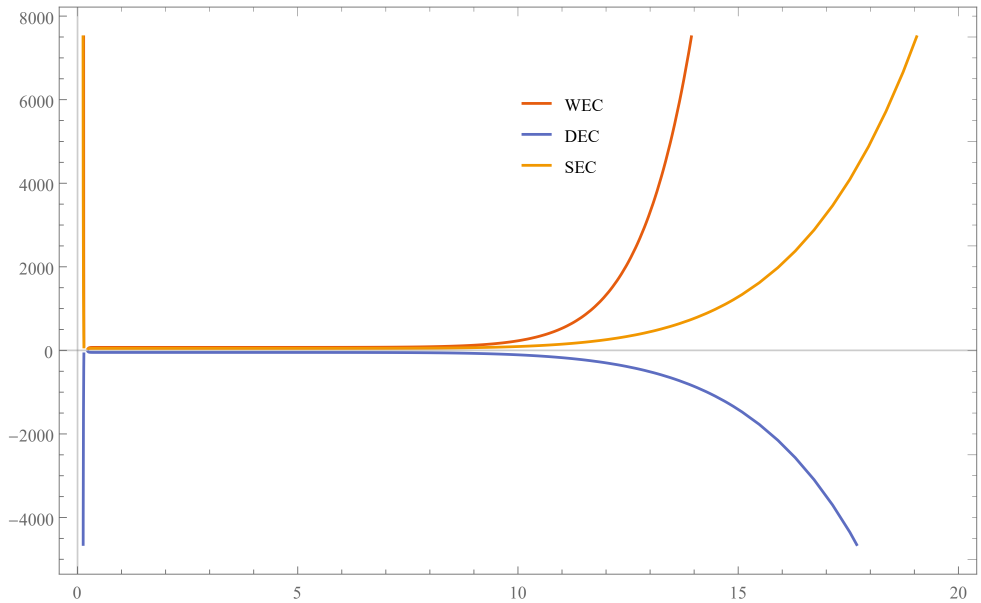

Figure 7.7: Variation of ECs vs. t for $a_1 = -5, a_2 = 6, a_3 = 10, \beta_0 = 4, \xi = 0.00001, \lambda = 15$

7.5 Statefinder parameters

As it is well known that in modern cosmology, Statefinder is also an important tool in representing the geometrical diagnostic and allow us to investigate the DE model in an independent manner, among them such models are ΛCDM, HDE, Quintessence etc., where it is found that r and s ; and r and q are the main tools, where q is the DP. Sahni et al. (2003), Alam et al. (2003) are the main authors who have discovered these two cosmological diagnostic pair (r,s) and according to them the two parameters r and s are defined as

$$r = 1 + 3\frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3} \;\; ; s = \frac{r-1}{3(q-\frac{1}{2})} \tag{7.37}$$

which are respectively obtained from eqs. (7.23) and (7.27) as

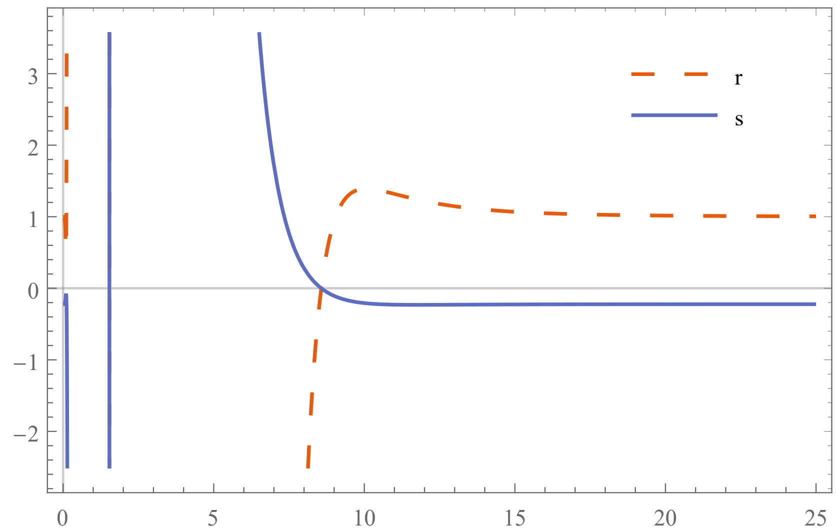

Figure 7.8: Variation of r and s vs. t for $\xi = 0.00001, \lambda = 15$

$$r = 1 + \frac{3(\lambda-1)}{\xi\lambda(\ln t)^{\lambda}} - \frac{3}{\xi\lambda(\ln t)^{\lambda-1}} + \frac{(\lambda-1)(\lambda-2)}{\xi^2\lambda^2(\ln t)^{2\lambda}} - \frac{3(\lambda-1)}{\xi^2\lambda^2(\ln t)^{2\lambda-1}} + \frac{2}{\xi^2\lambda^2(\ln t)^{2\lambda-2}} \tag{7.38}$$

and

$$s = \frac{\frac{3(\lambda-1)}{\xi\lambda(\ln t)^{\lambda}} - \frac{3}{\xi\lambda(\ln t)^{\lambda-1}} + \frac{(\lambda-1)(\lambda-2)}{\xi^2\lambda^2(\ln t)^{2\lambda}} - \frac{3(\lambda-1)}{\xi^2\lambda^2(\ln t)^{2\lambda-1}} + \frac{2}{\xi^2\lambda^2(\ln t)^{2\lambda-2}}}{3\left(\frac{1}{\xi\lambda(\ln t)^{\lambda-1}} - \frac{(\lambda-1)}{\xi\lambda(\ln t)^{\lambda-1}} - \frac{3}{2}\right)} \tag{7.39}$$

7.6 Conclusion

This chapter investigates the logamediate inflation in the universe in the framework of $f(R,T)$ gravity theory with a time-dependent DP. Using the model (7.21), the field equations in $f(R,T)$ gravity are found by assuming a specific model [proposed by Harko et al. (2011)] as $f(R,T) = \mu R + \mu T$. Also we obtained with this special frame that the universe expands for logamediate inflation as $a(t) = exp[\xi(\ln t)^{\lambda}]$, being $\xi > 0$ and $\lambda > 1$. When $r > 1, s < 0$, then it behaves like a Chaplygin gas; also, when $r < 1, s > 0$, then it behaves like a Quintessence [Arora et al. (2021)]. It is clear from the Fig. 7.8, that in the present model, the cosmological diagnostic pair (r,s) does not exactly tends to 1 and

0 respectively, i.e. $(r,s) \nrightarrow (1,0)$, from which we can say that the present modified model of $f(\widetilde{R},T)$ does not satisfied the ΛCDM model.

Summary of the model:

• The spatial volume of the present model diverges at $t = 0$, and it initially begins to increase for $t > 1$ [Fig. 7.1].

• For a suitable choice of constants such as $a_1 = -5, a_2 = 6, a_3 = 10, \beta_0 = 4, \xi = 10, \lambda = 4$, the model (7.21) found that, for $n = 1$ it becomes isotropic, whereas it become anisotropic for $n \neq 1$.

• As the DP lies in the range -1 to 0, Bianchi type-V model is accelerating for $\xi > 1$ and $\lambda \geq 1$, as in Fig 7.1.

• From Fig. 7.6, It is clear that the displacement field vector gradually decreases with cosmic time t, but due to the increase in time, it starts to expand positively, and this concept is satisfied the Lyra manifold for a considerable time.

• Moreover, the energy density of the present model remains positive throughout the universe's evolution and is found to gradually increase with cosmic time t as shown in Fig. 7.5, which geometrically satisfies the present observational data.

• With a suitable choice of the constants, we obtained that the current model represents an expanding, shearing, non-rotating universe [Maurya (2020)]

• From Fig. 7.7, It is clear that only WEC is satisfied in our present model, but both SEC and DEC are violets of the whole evolution of the universe in the context of the EOS parameter with $f(R,T)$ gravity in Lyra geometry.

• Fig. 7.8 shows that the cosmological diagnostic pair $(r,s) \nrightarrow (1,0)$ so that the present model does not satisfy the ΛCDM model as recent observational data.

• It is clear from the eqs. (7.33) and (7.36) that the Riemannian curvature scalar R and $f(\widetilde{R},T)$,the functions of Ricci scalar are all vanishes for $t \to \infty$.