Chapter 1

Introduction

Some crucial definitions regarding this thesis which are motivated and inspired us to investigate the nature of the present model of the universe in the framework of Lyra's geometry in the Bianchi type-V cosmological model are as follows:

1.1 Theory of General Relativity

Albert Einstein defined the extensive meaning of relativity. The theory of relativity is the physical theory that depends on a regular physical explanation of ideas of motion, space, and time. This statement is related to the fact that motion from the angle of possible knowledge always seems like the relative motion of one object to another. The advancement of the theory of relativity has emerged in two parts, The Special Theory of Relativity and the General Theory of Relativity. Einstein constituted a new theory in theoretical cosmology as the Special Theory of Relativity in 1905 that describes the relation between space and time connected with objects moving at a regular speed in a straight line. Einstein gave the theory of relativity, which proved that absolute motion could not be detected. Einstein materialized two postulates on his special theory of relativity-

► The nature laws must preserve their forms relative to all observers in a state of relative

uniform motion. This postulate explains that velocity is not absolute but relative.
► The velocity of light is independent of the observer's. This hypothesis does not support Galilean transformations. It is an observational fact that the velocity of light calculated by any method is constant, and this result clearly distinguishes between classical theory and Einstein's theory of relativity. Einstein was motivated to develop these postulates by investigating the properties of Maxwell's equations.

Albert Einstein (1915) states that GR is the study of the geometric theory of gravitation. GR takes a broad view of special relativity and Newton's law of universal gravitation, on the condition that an incorporated representation of gravity as a geometric property of space and time through metric field g_{ij}
In the 20th century, Einstein realized Newton's absolute space was a concept without physical content. Newton's theory, which describes the object's motion with a speed much less than the speed of light, was insufficient to describe the object's motion when its speed becomes very close to the speed of light. As a result, Einstein developed a theory of relativity in 1905, predicting that no object can travel sooner than the speed of light c. Galilean symmetry is recovered from special relativity, for speed v is significantly less than by c. Hence, the special theory of relativity gives an excellent approximation for most everyday physics. This theory fails to study relative motion in accelerated frames of reference and does not apply to kinds of motion. Given these limitations, Einstein generalized the particular theory of relativity treated as the general theory of Relativity or Einstein's theory of gravitation in 1916. Einstein's general theory of relativity has successfully described gravitational phenomena. It is based on the Riemannian metric tensor g_{ij}, which describes the gravitational field and the geometry.

1.2 Cosmology

Cosmology deals with the scientific study of the universe's LSS, evolution, and origin. The large-scale mass distribution in the universe tells us about the nature and evolution of the universe, and the law of gravitational attraction determines the structure described by models. Gravitation is the sole long-range force binding the universe's constituents to understand the nature and evolution of the LSS. Einstein introduced a new influential theory of gravity, known as the general theory of relativity, in 1916. Einstein considered the earliest relevance of the general theory of relativity to study cosmology in 1917. He developed the subject with a static universe, followed by de Sitter's cosmological model. A series of significant ideas have characterized subsequent developments. In particular, cosmological models now form the broad skeleton for astrophysics and provide tests of different aspects of fundamental physics. Investigating the LSS of the physical Universe is the most crucial endeavor of cosmology. Cosmologists assemble mathematical models of the universe and compare these models with the present-day universe as astronomers observe. In 1922, Hubble published his eminent law connecting to apparent luminosities of distant galaxies to their redshifts.

i.e. $V = HD$

where V represents the speed of recession of a galaxy at a distance D from us and H is the Hubble's constant.

Friedmann (1922) was the first to explore the most general non-static, homogeneous, and isotropic spacetime described by the Robertson-Walker metric

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] \tag{1.1}$$

where $R(t)$ is the scale factor and k is the constant curvature of spaces, which takes the values $-1, 0$ or 1 that represent the open, flat, and closed, respectively.

The astronomical observational evidence suggests that the universe on a large scale could be as isotropic and homogeneous, but it is not so on a smaller scale. Misner (1968) proposed the Chaotic Cosmology program according to which an initially highly irregular universe approaches the Friedmann–Lemaître–Robertson–Walker (FLRW) model stage only during cosmological evolution. This program inspired the research of the universe's inhomogeneous and anisotropic cosmological models. Spatially homogeneous and anisotropic spacetime belongs to either the Bianchi class or the Kantowski-Sachs class.

1.3 Bianchi Type Models

Bianchi cosmology is the study of universes that are homogeneous but not isotropic. The present-day universe can be described by spatially homogeneous and isotropic FRW spacetime. However, at a smaller scale, the universe is neither homogeneous nor isotropic, nor do we expect it to possess these properties in its early stages of evolution. Spatially homogeneous and anisotropic cosmological models have been widely studied in general relativity in different contexts in searching for realistic models of the universe in its early stages. Bianchi classified spatially homogeneous and anisotropic models into I-IX types under different conditions on structure constants. Bianchi-type cosmological models are essential because these are homogeneous and anisotropic, from which the process of isotropization of the universe over time. The advantage of the Bianchi type models is that they give a general relativity model with actual dynamics, even though the metric components consistently are only functions of time.

Here, in this thesis, we are interested in Bianchi type-V cosmological model, as the study of Bianchi Type-V cosmological models attracts attention since they include particular isotropic instances and allow for arbitrary tiny anisotropy at some point in cosmic time. Bianchi type-V universes are natural generalizations of open FRW models that eventually become isotropic and homogeneous. They are essential in understanding phenomena like galaxy formation in the early universe [Misner (1968)]. There are substantial theoretical arguments for the existence of an anisotropic phase in the universe's evolution. Many cosmologists believe that before decaying into an isotropic FRW model, the anisotropic model best reproduces the earliest development phase. Several authors are done lots of remarkable results and shown the different physical nature in the framework of Bianchi type-V. Bali and Singh (2005) investigated the Bianchi Type-V bulk viscous fluid string dust cosmological model in GR, while Singh and Chaubey (2007) examined the evolution of a homogeneous, anisotropic viscous universe with cosmological constant Λ, in the same way, Singh and Baghel (2009) investigated with the time-dependent cosmological term $\Lambda(t)$.

In a similar context, Ram and Zeyauddin (2008) studied the Bianchi type –V cosmological models in the presence of perfect fluid and heat conduction in the framework of Lyra's geometry. However, Ram et al. (2010) investigated Anisotropic Bianchi type V for a perfect fluid in the same manner.
Yadav (2011) has studied some anisotropic dark energy models by assuming the skewness parameters as time-dependent in Bianchi type-V. On the other hand, Naidu et al. (2012) examined the DE model in a Scalar-Tensor Theory of Gravitation.

Very interestingly, Kumar and Srivastava (2013) have shown the new aspects of the Bianchi type-V spacetime and then discussed the Electric and Magnetic parts of Weyl tensors in terms of tilted congruence.

The existence of the Bianchi-V string cosmological model through power-law expansion is explored by Kumar (2014), based on $f(R,T)$ gravity. In a similar context, Tiwari and Singh (2015) have also studied a new class of spatially homogeneous and anisotropic Bianchi type-V cosmological models based on the perfect fluid distribution in time-varying cosmological and gravitational constants in GR. Apart from that, Tiwari and Mishra (2017) have shown the mysterious behavior of Bianchi's type-V cosmological model in the framework of the $f(R,T)$ theory of gravity.

Recently, Yadav and Bhardwaj (2018) investigated the existence of Lyra cosmology of a hybrid universe with the minor relations between DE and ordinary matter in Bianchi V spacetime.

1.4 ΛCDM Model

The ΛCDM (Lambda- cold dark matter) model is a process of finding the parametric equation s of a curve or a surface of the Big Bang cosmological models, in which the universe contains three major components: first, a cosmological constant associated with DE denoted by Lambda (Greek); second, postulated CDM; and third, ordinary matter. It is sometimes referred to as the standard model of Big Bang cosmology, since it is the simplest model that accounts for the following aspects of the universe relatively well:

- the cosmological microwave background's existence and structure.
- the galaxies' distribution has a large-scale structure.
- hydrogen (including deuterium), helium, and lithium abundances were observed.
- the universe's accelerated expansion is observed in the light of distant galaxies and supernovae.

Lambda-CDM exhibits difficulties at small scales, which might be due to our limited understanding of everything from DM to gravity or to the function of baryon physics, which is poorly understood and incorporated in simulation codes and semi-analytic models. At this point, it is critical to determine if the Lambda-CDM model's flaws are a symptom of the model's limitations or a sign of our inability to correct the finer details.

1.5 Dark Energy and Dark Matter

DE is one of the mysteries of modern science. Unlike any known form of matter or energy, it has been detected solely by its gravitational effect of repulsion. The astronomical observations of SNela, galaxy redshift surveys, cosmic background radiation data, and

LSS convincingly suggest that the observable universes accelerated expansion. Observations also suggest a transition of the universe from the earlier deceleration phase to the recent acceleration phase. Astronomers theorize that this faster expansion rate is due to a mysterious dark force called dark energy. The simplest DE model is the cosmological constant Λ, initially introduced by Einstein in 1917 to construct the Einstein static universe but later abandoned. The cosmological constant agrees with observational data; however, its physical interpretation is unsatisfying when adopting a particle physics point of view. In this context, it is interpreted as a measure of the vacuum energy density, which leads to the well-known cosmological constant problem [Weinberg (1989), Samanta et al. (2014)].

DM plays a vital role in forming structure in the early universe. It is an unknown type of matter distinct from DE, baryonic matter, i.e., ordinary matter and neutrinos. These matters are invisible to the entire electromagnetic spectrum. Its existence and properties are inferred from its gravitational effects, influence on galaxies, and effect on the CMB. The structure of the universe that we observe, galaxies, stars, and other large-scale objects evolved from small fluctuations in the plasma of the early universe that underwent gravitational collapse over the eons. Without DM, a structure can only be formed by ordinary baryonic matter, but up to the recombination era, ordinary matter is coupled to the photons in the universe. This coupling results in a restoring force that acts to prevent further collapse; the result is acoustic oscillations and inhibition of structure formation. The addition of DM changes the picture since DM is free to collapse gravitationally without a restoring force that helps the formation of structure around local concentrations of DM Current results from the WMAP experiment support the existence of dark matter in the early universe in amounts comparable to those today, indicating that DM is a long-lived species. The standard cosmology model indicates that the universe's total mass energy contains 68.3% DE, 26.8% DM, and 4.9% ordinary matter [Ade et al. (2013)].

1.6 Energy-momentum tensors

The EMT is an attribute of matter, radiation, and non-gravitational force fields. The EMT is the source of the gravitational field in the EFE of GR; just a mass density is the source of such a field in Newton's gravity. The EMT involves the use of superscripted variables. If Cartesian coordinates in SI units are used, then the components of the position four-vector are given by $X^0 = t$, $X^1 = x$, $X^2 = y$, and $X^3 = z$, where x, y, z are distance in meters.

The EMT is defined as the tensor $T^{\alpha}\beta$ of order two that gives the flux of αth component of the momentum vector across a surface with constant X^{β} coordinate. In relativity, this momentum vector is taken as the four-momentum, and the stress-energy tensor is symmetric.

1.7 Lyra Geometry and the field equations of $f(R,T)$ gravity

Lyra (1951) proposed a modification of Riemannian geometry by introducing an additional gauge function into the structure less manifold, as a result of which the cosmological constant arises naturally from the geometry. This bears a remarkable resemblance to Weyl's (1918) geometry. Lyra defined a displacement vector between two neighbhouring points $P(x^i)$ and $Q(x^i + dx^i)$ as Adx^i where $A = A(x^i)$ is a non zero gauge function of the coordinates. The gauge function $A(x^i)$ together with the coordinate system x^i form a refernece system (A, x^i). The transformation to a new reference system $(\overline{A}, \overline{x^i})$ is given as

$$\overline{A} = \overline{A}(A, x^i) \quad \overline{x^i} = \overline{x}(x^i) \tag{1.2}$$

such that $\frac{\partial \overline{A}}{\partial A} \neq 0$ and $\det\left(\frac{\partial \overline{x}}{\partial x}\right) \neq 0$ The symmetric affine connections Γ^{i}_{jk} on this manifold is given by

$$\widetilde{\Gamma}^{i}_{jk} = \frac{1}{A}\Gamma^{i}_{jk} + \frac{1}{2}\left(\delta^{i}_{j}\phi_{k} + \delta^{i}_{k}\phi_{j} - g_{jk}\phi^{i}\right) \tag{1.3}$$

In which the connection Γ^{i}_{jk} is defined in terms of the metric tensor g_{ij} (as in RG), and $\phi^{i} = g^{ij}\phi_{j}$ is called the displacement vector field of Lyra geometry. Lyra (1951) and Sen (1957) indicate that in any general reference system, the displacement vector field ϕ^{i} arises as a natural consequence of the formal introduction of the gauge function $A(x^{i})$ into the structureless manifold.

The above equation (1.2) indicates that the component of the affine connection, not only depends on metric g_{ij} but also on the displacement vector field ϕ^{i} . In Lyra geometry the line element, given by

$$ds^{2} = A^{2}g_{ij}dx^{i}dx^{j} \tag{1.4}$$

is invariant under both coordinate and gauge transformations.

The infinitesimal parallel transport of a vector field V^{i} is given by

$$\delta V^{i} = \widetilde{\Gamma}^{i}_{jk}V^{j}Adx^{k} \tag{1.5}$$

where

$$\hat{\Gamma}^{i}{}_{jk} = \widetilde{\Gamma}^{i}_{jk} - \frac{1}{2}\delta^{i}_{j}\phi_{k} \tag{1.6}$$

which is not symmetric with respect to j and k, but the Lyra connection Γ^{i}_{jk} is syymetric with respect to the two lower suffixes $(i.e. \Gamma^{i}_{jk} = \Gamma^{i}_{kj})$. In Lyra geometry the connection Γ^{i}_{jk} is metric preserving which indicates that length transfers are integrable and this result indicates the length of a vector is conserved upon parallel transports, as in RG. In the same manner of RG, the curvature tensor of Lyra geometry is given as

$$\widetilde{\Gamma}^{i}_{jkh} = \frac{1}{A^{2}}\left[\frac{\partial}{\partial x^{k}}\left(A\widetilde{\Gamma}^{i}_{jh}\right) - \frac{\partial}{\partial x^{h}}\left(A\widetilde{\Gamma}^{i}_{jk}\right) + A^{2}\left(\hat{\Gamma}^{i}{}_{ak}\hat{\Gamma}^{a}{}_{jh} - \hat{\Gamma}^{i}{}_{ah}\hat{\Gamma}^{a}{}_{jk}\right)\right] \tag{1.7}$$

Then the curvature scalar of Lyra geometry reduces to

$$\widetilde{R} = A^{-2}R + 3A^{-1}\nabla_i\phi^i + \frac{3}{2}\phi^i\phi_i + 2A^{-1}(logA^2)_{,i}\phi^i \tag{1.8}$$

where R indicates the Riemannian curvature scalar, in which the covariant derivative is taken with respect to the Christiffel symbols. Now applying normal gauge as $A = 1$ in (1.7), we have

$$\widetilde{R} = R + 3\nabla_i\phi^i + \frac{3}{2}\phi^i\phi_i \tag{1.9}$$

This curvature scalar of Lyra geometry is like Weyl's geometry as in RG.

Harko et al. [2011] have proposed another extension of $f(R)$ gravity (which provides several viable models and successful alternative theories of GR) which is called the $f(R,T)$ gravity theory, where the gravitational Lagrangian is given by an arbitrary function of Ricci scalar R and trace of the energy-momentum tensor T. In this theory, a covariant of stress-energy is obtained, and the cosmological models depend on a source term. However, the source term is expressed as a function of matter Lagrangian L_m, with that several sets of field equations can be obtained with each choice of L_m. In the framework of this theory, the field equations of $f(R,T)$ gravity are obtained from the Hilbert-Einstein principle by keeping the metric-dependent Lagrangian density L_m. The action for $f(R,T)$ gravity is given by using Lyra geometry as

$$S = \frac{1}{16\pi G}\int f(\widetilde{R},T)\sqrt{-g}\, d^4x + \int L_m\sqrt{-g}\, d^4x \tag{1.10}$$

where the gravitational Lagrangian comprises of an arbitrary functions of Ricci scalar $\widetilde{R}$ and the trace T of the EMT T_{ij} of the matter source, in which L_m represents the usual matter Lagrangian density. The stress- energy tensor T_{ij} for the matter source is given by

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}} \tag{1.11}$$

its trace is $T = g^{ij}T_{ij}$

Here, we considered the matter Lagrangian L_m is depends only on the metric tensor components g^{ij} rather than its derivatives and hence we obtain as

$$T_{ij} = g_{ij}L_m - 2\frac{\partial L_m}{\partial g^{ij}} \tag{1.12}$$

Varying the action S in eq. (1.9) with respect to metric tensor g_{ij}, the gravitational field equations of $f(\widetilde{R},T)$ gravity are obtain as

$$\begin{aligned} f_{\widetilde{R}}(\widetilde{R},T)\widetilde{R}_{ij} - \frac{1}{2}f(\widetilde{R},T)g_{ij} + (g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j)f_{\widetilde{R}}(\widetilde{R},T) \\ = -\frac{8\pi G}{c^2}T_{ij} - f_T(\widetilde{R},T)T_{ij} - f_T(\widetilde{R},T)\Theta_{ij} \end{aligned} \tag{1.13}$$

where

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{lm}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}} \tag{1.14}$$

Here, $f_{\widetilde{R}}(\widetilde{R},T) = \frac{\partial f(\widetilde{R},T)}{\partial \widetilde{R}}$, $f_T(\widetilde{R},T) = \frac{\partial f(\widetilde{R},T)}{\partial T}$ and $\Box \equiv \nabla^i\nabla_i$, where ∇_i is represent the covariant derivative.

The matter field is essential for the field equations of $f(\widetilde{R},T)$ gravity through the metric tensor Θ_{ij}. Since different cosmological models of $f(\widetilde{R},T)$ gravity are possible depending on the nature of the matter source, Harko et al. (2011) constructed three explicit form of $f(\widetilde{R},T)$ gravity such as follows:

$$f(\widetilde{R},T) = \begin{cases} \widetilde{R} + 2f(T) \\ f_1(\widetilde{R}) + f_2(T) \\ f_1(\widetilde{R}) + f_2(\widetilde{R})f_3(T) \end{cases} \tag{1.15}$$

Here, in this thesis we consider second case of Harko et al. (2011) in all the chapters, to form the field equations of $f(\widetilde{R},T)$ gravity and to obtained the physical nature of the matter field through Θ_{ij}. i.e.

$$f(\widetilde{R},T) = f_1(\widetilde{R}) + f_2(T) \tag{1.16}$$

For matter Lagrangian, the standard stress energy tensor is given by

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij} \tag{1.17}$$

Where $u^i = (0,0,0,1)$ is the four velocity vector in co-moving co-ordinate system satisfying the condition $u_i u^i = -1$ and $u^i \nabla_j u_i = 0$. where, ρ and p denotes the energy density for the perfect fluid and pressure of the fluid respectively. We assume a perfect fluid matter as $L_m = -p$, which yields that

$$\Theta_{ij} = -2T_{ij} - p g_{ij} \tag{1.18}$$

Then by contracting eq. (1.12), we have

$$\begin{aligned} f_1'(\widetilde{R},T)\widetilde{R}_{ij} - \frac{1}{2} f_1(\widetilde{R}) g_{ij} + (g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j) f_1'(\widetilde{R}) = \\ -\frac{8\pi G}{c^2} T_{ij} + f_2'(T) T_{ij} + \left[f_2'(T)p + \frac{1}{2} f_2(T) \right] g_{ij} \end{aligned} \tag{1.19}$$

By choosing $f_1(\widetilde{R}) = \mu\widetilde{R}$ and $f_2(T) = \mu T$ [where μ is taken as arbitrary constant], the field equations of $f(\widetilde{R},T)$ gravity, for a perfect fluid matter source, eq. (1.19) becomes

$$\widetilde{R}_{ij} - \frac{1}{2}\widetilde{R} g_{ij} = -\left(\frac{8\pi G - \mu c^2}{c^2}\right) T_{ij} + \left[p + \frac{1}{2}T\right] g_{ij} \tag{1.20}$$

Now, applying eq. (1.8) in (1.20), the field equation in Lyra geometry is obtained as [Maurya (2020)] :

$$R_{ij} - \frac{1}{2} R g_{ij} + \frac{3}{2}\phi_i\phi_j - \frac{3}{4} g_{ij}\phi_i\phi^j = -\left(\frac{8\pi G - \mu c^2}{c^2}\right) T_{ij} + \left[p + \frac{1}{2}T\right] g_{ij} \tag{1.21}$$

Here, $\phi_i = (0,0,0,\beta(t))$ is a displacement vector field.

1.8 Work Related with Lyra's Geometry

In this thesis, the main reason is to find out the mysterious behavior of the cosmological models of the present universe in $f(R,T)$ gravity based on Lyra Geometry in Bianchi type-V.

In Einstein's General Theory of Relativity, gravitation is described in terms of the geometry of spacetime, and this result motivated him to geometrize other physical fields. Weyl (1918) made one of the best attempts in this direction and introduced a generalization of RG to unify gravitation and electromagnetism. Later, Lyra (1951) suggested a modification of RG, which resembles Weyl's Geometry. Lyra introduced a gauge function, which removed the non-integrability condition of the length of a vector under parallel transport. Thus, Lyra modified Riemannian geometry, named Lyra's Geometry.

In consecutive investigations, Sen (1957), Sen, and Dunn (1971) proposed a new scalar-tensor theory of gravitation and constructed an analog of the EFE based on Lyra's geometry with a normal gauge as

$$R_{ij} - \frac{1}{2}Rg_{ij} + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_i\phi^j = -8\pi G T_{ij} \tag{1.22}$$

where

◇ R_{ij} is the Ricci curvature tensor.

◇ R is the Ricci scalar tensor.

◇ G is Newton's gravitational constant.

◇ g_{ij} is a 4 x 4 symmetric metric tensor.

◇ T_{ij} is the energy-momentum tensor.

In which ϕ_i is the time like displacement field vector defined by $\phi_i = (0,0,0,\beta(t))$.

Subsequently, Halford (1970, 1972) extensively showed that the energy-conservation law does not hold in the cosmological theory. However, the scalar-tensor theory of gravitation predicts the same effects within observational limits in the Lyra manifold as in the Einstein theory.

In his paper, Murphy (1973) shows that the Einstein equations in a manner that easily can be integrable. The solutions are found in two ways-one is a steady-state cosmology (unstable). The other is a start from the steady state, expanding very fast as viscosity vanishes.

Bhamra (1974) offered a spherically symmetric class-one cosmological model based on Lyra's geometry, and the motionless universe is found to be bodily unrealistic.

Ruban and Finkelstein (1975) formulate the dynamics of the homogeneous model of Bianchi type-I based on the general analytic solutions based on the problems of the initial singularity in the scalar-tensor anisotropic cosmology of Jordan Brans-Dicke. Solutions of the model is examined in vacuo and in the presence of matter with equation $p = n\varepsilon$ $(0 \leq n \leq 1)$

Reddy (1977) showed that in a scalar-tensor theory of gravitation, Birkhoff's theorem of GR exists for electromagnetic fields when it is introduced in theory and is independent of time, as proposed by Sen and Dunn (1971).

Karade and Borikar (1978) examined the consequences of the thermodynamic equilibrium of a gravitating fluid sphere; as a result, they found zero redshifts in a static model in Lyra's.

Kalyanshetti and Waghmode (1982), in Lyra Geometry, a stationary cosmological model is examined in the Einstein-Cartan theory. They observed that the spins of the individual particles composing the fluid are all aligned in the radial direction and surveyed the detail of the static Einstein universe.

Berman (1983), in his paper, presented the law of variation of HP in evolutionary models, which produced a constant value for the DP that leads naturally to the exclusion of open universes.

Reddy and Innaiah (1986) showed that the pressure and the energy density are unique when the source of the gravitational field is in perfect fluid in the framework of Lyra geometry, based on a non-static plane-symmetric cosmological model. In contrast, a minor difference without a cosmological constant (Beesham 1986).

Soleng (1987), in his paper, pointed out that the cosmologies based on Lyra's manifold play an essential role in Hoyle's creation field cosmology, together with the gauge-vector term as a cosmological term.

Beesham (1988) studied the FLRW cosmological models in Lyra's manifold by keeping the time-dependent displacement field, and as a result, in his study, he obtained singularity, entropy, and, finally, horizon problems.

Reddy and Venkateswarlu (1987) explored the exact Bianchi type-I cosmological model in the presence of zero-mass scalar fields obtained when the source of the gravitational field is a perfect fluid with pressure and energy density the same.

In the study of Friedmann universes with bulk viscosity, Johri and Sudharsan (1988) explored the effect of bulk viscosity on the evolution of Friedmann models. As a result, they found the presence of a tiny time-independent component of bulk viscosity, which play a decisive part in motivating the present-day universe into a steady state. Singh and Singh (1991) investigated the behavior of the model of the Bianchi types of V and VI_0 with the gauge function β as a time-dependent and constant function in the framework of Lyra Geometry.

Singh and Singh (1991) showed the exact solutions for the anisotropic Bianchi type-I model in normal gauge based on Lyra's geometry; they also discussed the physical behavior of the models in a vacuum in the presence of perfect fluids.

Ram and Singh (1992) studied spatially homogeneous cosmological models of types III and V in the normal gauge to obtain the exact solutions of EFE in a vacuum in the presence of stiff matter.

In the study of Lyra's Geometry and Cosmology: A Review, Singh, and Singh (1993) thoroughly reviewed Lyra's geometry. Further, they also investigated the cosmological models with constant and time-dependent displacement fields.

To produce several solutions to the EFE, they use a form of HP with variable cosmological and gravitational constants. Moreover, these parameters offer an array of solutions in the Robertson-Walker spacetimes and Bianchi type-I [Maharaj and Naidu (1993)].
Johri and Desikan (1994) have found a specific study of the cosmological model with constant DPs undertaken in Brans-Dicke's theory. These models are (i) Singular models with expansion driven by Big- Bang impulse and (ii) Nonsingular models with expansion driven by the creation of matter particles.
Singh and Desikin (1997) studied FRW models have been in the cosmological theory based on Lyra's geometry by considering a time-dependent displacement.

Pradhan et al. (2001) studied an isotropic homogeneous FRW universe in the presence of a bulk viscous fluid within the framework of Lyra's geometry. They obtained the exact solutions of the Sen equations assuming the constant DP.
On the other hand, Rahaman and Bera (2001), the Kaluza-Klein cosmological model is explored within the framework of Lyra geometry, and the physical behavior of the model is examined in a vacuum and the presence of perfect fluids.
Pradhan and Vishwakarma (2004) studied the LRS Bianchi type-I cosmological model based on Lyra's geometry with a new class of exact solutions considering a time-dependent displacement field for constant DP models of the universe.
Rahaman et al. (2005) discussed the two cosmological models as Bianchi-I and Kantowski Sachs models with constant DPs within the framework of Lyra geometry. The study has taken ad hoc EMT components and the field-theoretic approach with a flat potential.
Singh and Dewri (2006) investigated the Robertson–Walker model universe with a hybrid scale factor using flat and open models interacting with the Brans–Dicke field and electromagnetic field, respectively.
Singh and Chaubey (2006) have studied a self-consistent system of the gravitational field with a binary mixture of perfect fluid and DE through a cosmological constant based on

Bianchi's type-V universe.

On the other hand, Ram et al. (2008) studied the EFE with a time-dependent displacement vector field in Bianchi type-V with a perfect fluid and heat flow based on Lyra geometry.

In the same way, Kumar and Singh (2008) explored a spatially homogeneous and anisotropic Bianchi type-I spacetime within the framework of Lyra's geometry with a time-dependent gauge function.

Singh and kale (2009) obtained a new class of exact solutions given various well-known power-law relations among scale factors, cosmological and gravitational constants, and cosmic time in the Bianchi type-V cosmological model filled with a bulk viscous cosmic and fluid in GR.

Ram et al. (2010) discussed the law of variation for mean HP with average scale factor in an anisotropic Bianchi type V cosmological spacetime within the framework of Lyra's manifold.

Yadav (2011) examined Bianchi type V universe by introducing three different skewness parameters along with spatial directions and obtaining the dynamical DE's anisotropic nature. The skewness parameters are assumed to be time-dependent; as a result, he showed that the universe achieves flatness in the quintessence model.

Bali et al. (2012) examined the Bianchi type V viscous fluid cosmological model for barotropic fluid distribution with varying cosmological term Λ; also, they explored a cosmological scenario by assuming a variation law for HP in the background of homogeneous Bianchi type V spacetime.

In the same way, Kumari et al. (2013) explored an anisotropic Bianchi type-III cosmological model in the occurrence of a bulk viscous fluid based on Lyra geometry through a time-dependent displacement vector.

On the other hand, Singh and Sharma (2014), here in his paper, the spatially homogeneous and anisotropic Bianchi type-II cosmological model has been discussed in GR in the presence of a hypothetical anisotropic DE fluid with constant DP within the framework of the Lyra's manifold with uniform and time-varying displacement field vector.

Das and Sarma (2014) have derived the mysterious DE and some exact solutions for Bianchi type V string cosmological model in Lyra geometry.
However, considering a particular form and linearly varying deceleration parameter, Bishi and Mahanta (2015) studied the Bianchi type-V string cosmological model with bulk viscosity in the $f(R,T)$ theory of gravity.
Rani, Singh & Sharma (2015) have investigated the spatially homogeneous and anisotropic Bianchi type-III string cosmological models in the presence and without magnetic field within the framework of $f(R,T)$ gravity. The orientation of the string has been chosen in the z-direction along with the magnetic field.
Similarly, Singh et al. (2016) investigated the Bianchi type-I spacetime in the presence of bulk viscosity and Chaplygin gas based on Lyra geometry.
By the relevance of the above work, Sahoo et al. (2017) studied the Bianchi type-III cosmological model with bulk viscosity for a cloud of string in Lyra geometry.
Interestingly, Tiwari and Mishra (2017) have studied the Bianchi type V cosmological models in the $f(R,T)$ modified theory of gravity thoroughly.
As we found from the recent observations that the expansion rate of our universe is decelerating and accelerating in early and present epochs, respectively, which is a mysterious unsolved matter, and from this motivation, Zia and Murya (2018) have developed a new modification of Einstein theory of gravity, one is a geometrical modification, and another is an EMT, and using these two modifications they have obtained the exact solutions of Einstein Brans-Dicke field equations for a spatially homogeneous Bianchi type-I spacetime with time-dependent DP in Lyra's geometry.

Later, using the above-motivated result, Maurya and Zia (2019) investigated spatially homogeneous and anisotropic Bianchi type-I cosmological models with Brans-Dicke theory in Lyra geometry.
Apart from that, in the context of GR, Naidu, et al. (2019) have discussed spatially homogeneous and anisotropic Bianchi type-V DE models in an attractive massive scalar field.

Godani (2019) has presented his work to focus on studying the LRS Bianchi type-II model with perfect fluid in $f(R,T)$ gravity. The EOS is used to derive physical parameters concerning cosmic time and redshift using the first frame of Harko et al. (2011) [i.e., $f(R,T) = R + 2f(T)$. The current values of the DP, HP, and universe age are computed and compared to the ΛCDM model findings.

Yadav et al. (2020) have studied a bulk viscous universe in $f(R,T)$ gravity. As a result of these observations, they have obtained the explicit solutions of field equations by considering the power-law form of scale factor in modified gravity.

Similarly, Sharma et al. (2020) investigated the viability of Bianchi type V universe in $f(R,T)$ theory of gravity, and to solve the deterministic equations of EFE, they considered the power law for scale factor and constructed a singular Lagrangian model based on the coupling between Ricci scalar R and trace of energy-momentum tensor T.

Gogoi and Goswami (2020) have introduced a new $f(R)$ gravity model to create a model with further parametric control to explain the existing problems and explore new directions in the physics of gravity. They also looked at the potential and mass of scalar gravitons in both Jordan and Einstein frames to learn more about the model's properties.

On the other hand, in Brans-Dicke's scalar-tensor theory of gravity, Dewri (2020) investigated the spatially homogeneous Robertson-Walker cosmological models with magnetized isotropic DE like fluid. Exact solutions of models with volumetric expansion and power-law connection have been found using a variable cosmological constant and the Polytropic equation of state.

Recently, in the occurrence of DM and HDE model components, Gusu and Santhi (2021) have examined an anisotropic and homogeneous Bianchi type V spacetime based on GR and Lyra's geometry.

But, Chundawat and Mehta (2021) studied the Bianchi type-III string cosmological model with bulk viscous fluid in $f(R,T)$ gravity theory, and the barotropic equation of state for bulk viscous pressure is proportional to pressure density to get the deterministic solutions of the model.

Very recently, Singh and Devi (2022) examined the LRS Bianchi type-I cosmological models in $f(R,T)$ gravity with HEL for the average scale factor. These results have obtained that the universe becomes anisotropic in the early phase and later transforms into an isotropic universe at the late epoch.

1.9 $f(R)$ and Gauss - Bonnet gravity

Generally, $f(R)$ gravity is a modified theory of Einstein's theory of gravitation, which is a family of theories by replacing Ricci scalar R in the EHA with a general function $f(R)$. $f(R)$ gravity is vital in explaining inflation, DE, and cosmological perturbations. There are many ways to approach gravity modification in place of a scalar field addition into the EHA. The EHA can be modified by replacing the Ricci- Scalar R with an arbitrary function of $f(R)$, and this modification is called the $f(R)$ gravity theory. $f(R)$ gravity was first proposed by Buddhal (1970). Nojiri and Odinstov (2007, 2008) proved that the f(R) theory of gravity provides a natural unification of early-time inflation and late-time acceleration. Bertolami et al. (2007), Capozziello et al. (2007), Capozziello et al. (2008), Felice, and Tsujikawa (2010), Capozziello and Laurentis (2011) are some of the authors that studied thoroughly in the area of $f(R)$ gravity in different context also studied in extended theories of gravity.

Thus, the action for $f(R)$ gravity theory is given by

$$S = \frac{1}{16\pi G}\int f(R)\sqrt{-g}\; d^4x + \int L_m\sqrt{-g}\; d^4x \tag{1.23}$$

where $f(R)$ is the general function of Ricci scalar R and L_m is the matter Lagrangian.
By varying the action, the field equations can be obtained. One could contrast the exploit concerning the metric connection [alternatively concerning the metric]. The metric by variation principle variation is called metric $f(R)$ gravity. Similarly, variation concerning metric and connection yields the palatini $f(R)$ gravity, where the connection is indepen-

dent of metric and vice versa. These two approaches lead to the same field equation in the usual EHA. The metric-affine $f(R)$ gravity is the last and most important general manner. We use the palatini formalism abandoning the assumption that the matter action is independent of the connection. Performing the variation of action, the field equation is

$$FR_{ij}-\frac{1}{2}fg_{ij}=g_{ij}\nabla_i\nabla_jF-\nabla_i\nabla_jF=8\pi GT_{ij} \tag{1.24}$$

where, $\Box=\nabla_i\nabla^j$ and $F=f'(R)$, where ∇_i is the covariant derivative and T_{ij} is the standard matter EMT.

1.10 $f(\mathscr{G})$, $f(R,\mathscr{G})$, $f(\mathscr{G},T)$, $f(\mathscr{T})$ gravity

. With the help of Gauss-Bonnet invariant, some other important modified gravity theories are investigated and these theories are $f(\mathscr{G})$, $f(R,\mathscr{G})$ and $f(\mathscr{G},T)$. In cosmology, the Lagrangian is modified with an addition of arbitrary function of $f(\mathscr{G})$ in Einstein Hilbert action [Nojiri and Odintsov (2005), Nojiri et al. (2006), Amendola et al. (2007)], and this theory is an alternate study of DE model, like other modified theories. The action of this theory is

$$S=\frac{1}{16\pi G}\int\sqrt{-g}\,(R+f(\mathscr{G})\,d^4x+\int\sqrt{-g}L_md^4x \tag{1.25}$$

In the same manner, we have obtained a further specific modified gravity that reduces a general class of non-linear gravity model having the action in the following form as given by

$$S=\frac{1}{16\pi G}\int\sqrt{-g}\,(R,(\mathscr{G}))\,d^4x+\int\sqrt{-g}L_md^4x \tag{1.26}$$

where R and $\mathscr{G}$ denotes respectively the Ricci scalar and Gauss-Bonnet. In the similar way recently, shamir and Sadiq (2018) introduced modified Gauss–Bonnet gravity with radiating fluids in general form for $f(\mathscr{G},T)$ gravity as

$$S=\frac{1}{16\pi G}\int\sqrt{-g}\,(R+f(\mathscr{G},T))\,d^4x+\int\sqrt{-g}L_m d^4x \tag{1.27}$$

In the above equation, $\mathscr{G}$ and T represent the Gauss-Bonnet invariant and trace of the stress energy-momentum tensor , respectively.

A different fascinating kind of modified theory is the so-called $f(\mathscr{T})$ gravity ($\mathscr{T}$ is called torsion). Recently, it is revealed that without resorting to DE $f(\mathscr{T})$ gravity theories also admit the accelerated expansion of the universe [Ferraro and Fiorini (2007), Bengochea and Ferraro (2009)]. Remarkably, their equations of motion are always second-order in contrast with GR, whereas the field equations are fourth-order equations. The action of this gravity is given by

$$S=\frac{1}{16\pi G}\int d^4xef(\mathscr{T})+\int d^4xeL_m \tag{1.28}$$

1.11 Cosmological Parameters

The following parameters are vital in describing the nature of the models.

1.11.1 Hubble Parameter

Hubble's discovered that the galaxies recede from the earth with a velocity, i.e., proportional to their distance. i.e., the recession velocity is proportional to the mass. In cosmology, it is denoted and defined by

$$H=\frac{\dot{a}}{a} \tag{1.29}$$

Here, a is the average scale factor, which describes the universe's present expansion rate.

As we know that the relation between the recessional velocity and the Hubble constant together with distance is $V = H \times D$. From this relation, we can also conclude that the HP is not a constant but can be a function of time. In recent years the value of the HP has been considerably refined, and the current value given by WMAP mission is 71 km/sec/M.Pc. From the above relation, we know that the HP or Hubble constant H defines the rate of cosmic expansion.

1.11.2 Co-moving co-ordinate systems

The co-ordinate system x^{μ} is said to be co-moving coordinate system if, $g_{44} = 1$, $\frac{dx^i}{ds} = 0$, $\frac{dt}{ds} = 1$, for $i = 1,2,3$ co-moving coordinates are natural coordinates. They assign constant spatial coordinate values to observers who distinguish the universe as isotropic.
A co-moving observer is the only observer that will distinguish the universe and the CMB radiation to be isotropic. The co-moving time coordinate is the elapsed time since the Big Bang according to a clock of a co-moving observer and is a measure of cosmological time. Space in co-moving coordinates is called static, as most bodies on the scale of galaxies or larger are approximately co-moving, and co-moving bodies have static, unchanging co-moving coordinates.

1.11.3 Deceleration Parameter

H and q play a vital role in describing the character of the universe's evolution. In cosmology, the DP, denoted by q, is a dimensionless parameter of the cosmic acceleration of the expansion. The universe is accelerating if $0 \leq q \leq 1$. And if $q = 0$, then the universe expands at a constant rate. While the universe decelerates when q is more significant than zero, the universe also has an exponential expansion and super-exponential expansion for $q = -1$ and $q \leq -1$, respectively. To obtain the cosmological solutions, Berman (1983) proposed a particular law of variation of HP by assuming a DP is constant, and it is defined as follows:

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{1.30}$$

where a is the average scale factor of the universe and the dot denotes derivatives concerning the proper time. We have found that Reddy et al. (2012) examined a spatially homogeneous Bianchi type-III spacetime with a perfect fluid source in $f(R,T)$ gravity with the help of a special law of variation for HP Similarly, Mahanta et al. (2014) have studied the DE models in self-creation cosmology.

In terms of H, the DP is expressed as follows:

$$q = -1 - \frac{\dot{H}}{H^2} \tag{1.31}$$

We found that Berman and Gomide (1988) have explored the concept of the theory of constant DP. Rao et al. (2008) studied the Bianchi type-V cosmological model with the occurrence of perfect fluid by using a constant negative DP in a scalar-tensor theory based on the framework of Lyra Manifold. In the same manner, by taking a particular form of DP, Singha and Debnath (2009) have explored the quintessence model with a minimally coupled scalar, which provides an early deceleration and late time acceleration for barotropic fluid and Chaplygin gas-dominated models, whereas Tiwari (2009) has explored the Bianchi type-V spacetime within the framework of the scalar-tensor theory of gravitation [proposed by Saez and Ballester (1986)] and obtained a constant value of the DP, by using a particular law of variation for HP, Pradhan et al. (2011) studied the evolution of the DE parameter with a spatially homogeneous and isotropic FRW universe filled with barotropic fluid, and DE with a constant DP Mamon and Das (2017) reconstructed DP in a model flat FRW universe filled with DE and non-relativistic matter.

1.12 Dynamical Parameters

Dynamical Parameters are also plays an important role in cosmology and the some of the relevant paramaters are defined as follows:

1.12.1 Expansion scalar

The expansion scalar of the cosmological model is denoted by θ, which shows the expansion of the universe and is given by (in tensor form)

$$\theta = u^{i}_{;j} \tag{1.32}$$

1.12.2 Shear scalar

It palys a vital role in general relativistic and stellar cosmological models, which is defined as

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \tag{1.33}$$

where the shear tensor σ^{i}_{j} or σ_{ij} is given by

$$\sigma_{ij} = \frac{1}{2}\left(u_{i;\alpha}P^{\alpha}_{j} + u_{j;\alpha}P^{\alpha}_{i}\right) - \frac{1}{3}\theta P_{ij} \tag{1.34}$$

where the projection P_{ij} has the form given by

$$P_{ij} = g_{ij} - u_i u_j \tag{1.35}$$

1.12.3 Anisotropy Parameter

The anisotropy parameter (denoted by A_m or Δ), is defined as:

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right) \tag{1.36}$$

where $\Delta H_i = H_i - H(i = 1,2,3)$, Then this eq. can be written as

$$A_m = \frac{1}{3}\left[\left(\frac{H_x - H}{H}\right)^2 + \left(\frac{H_y - H}{H}\right)^2 + \left(\frac{H_z - H}{H}\right)^2\right] \tag{1.37}$$

where $H_x = \frac{\dot{A}}{A}$, $H_y = \frac{\dot{B}}{B}$ and $H_z = \frac{\dot{C}}{C}$ are the directional Hubble's prameters in the directions of x, y and z respectively.

1.13 Observational Constraints

1.13.1 Type Ia Supernovae Observation

A Type Ia supernova occurs in binary systems where one star is a white dwarf, and the other star is anything from a giant to a smaller white dwarf. Supernova Cosmology Project and High-redshift Supernova Search Team are two observable SNe Ia Supernovae in late time acceleration in cosmology. 42 redshift range supernovae and 16 high redshift supernovae were discovered around 34 supernovae by Perlmutter et al. (1999) and Riess et al. (1998). Supernovae have a dazzling history and generate a detonation of radiations. Researchers classify different types of supernovae. If there are no hydrogen lines in their spectra, they are categorised as Type I supernovae; otherwise, they are classified as Type II supernovae. Then each of the two types is divided into subcategories based on the existence of the absorption line of the other element. Those having a strong silicon line at 615 nm in their spectra are classified as type Ia, those with strong helium lines are classified as type Ib, and those without helium lines are classified as type Ic. Type Ia supernovae have become essential as the most accurate cosmic distance measurement, usable for distances greater than 1000 M.Pc.

1.13.2 Cosmic Microwave Background Radiation

It is a valuable source of knowledge about the early Universe 32. The CMB appears to have formed due to thermal equilibrium between radiation and matter during a period when the universe's contents were ionised and photons scattered readily off free electrons, resulting in a nearly uniform and isotropic radiation field with the same temperature as baryonic matter. Penzias and Wilson (1965) discovered the CMB, which is today one of the most important foundations of modern cosmology, offering a wealth of knowledge about the parameters that describe our Universe [White et al. (1994), Hu and Dodelson (2002)]. The CMB photons discovered today came from the last scattering surface (LSS), when the universe was around 380000 years old and had cooled down sufficiently (about 3000K) to allow the production of neutral atoms owing to its expansion. 1 sec after the Big Bang, the universe's temperature would have dropped by around ten thousand million degrees. Photons, electrons, and neutrinos and their antiparticles, as well as protons and neutrons, would have been present in the universe at the time. In 1992, the Cosmic Background Explorer satellite analyzed the spectrum of CMB radiation and discovered minor temperature changes.

1.13.3 Planck's observation

Planck's high-precision cosmic microwave background map has allowed scientists to choose the most accurate value of the universe's constituents [Ade et al. (2014)]. Cosmology comprises 4.9 percent of simple matter, creating galaxies and stars. Cosmology is made up of 26.8% dark matter. For DE, the remaining section of the cosmology is 68.3

percent. PLANCK Collaboration has gone beyond the traditional cosmological scenario to investigate the implications of cosmic data for DE models and their modified gravity [Ade et al. (2016)]. They put various models to the test, including k-essence, $f(R)$ theories, and related DE. They enhanced the current limitations and revealed that the initial predicted DE density must be less than 2% of the critical density.

1.14 Aims and Objectives

The aims and objectives of the research work are

• To study Bulk Viscous Bianchi Type-V DE model with charged fluid distribution in Lyra Geometry.

• To study Bianchi Type-V cosmological models with perfect fluid and Heat conduction in Lyra Geometry.

• To explore anisotropic Bianchi Type-V perfect fluid cosmological models in Lyra's Geometry with kinematics test.

• To explore Bianchi Type-V DE cosmological models in Lyra geometry in the presence of a massive scalar field.

• To investigate Bianchi Type-V anisotropic DE model with varying EOS parameter in Lyra Geometry.

• To examine Bianchi Type-V model in Lyra Geometry in the presence of a magnetic field.

1.15 Methodology and tools

Here, secondary data is used for this research work, referencing books, journals, and published materials on this topic available at the library and on the internet. The field equations will be generated in Lyra geometry using the Bianchi type V metric, and the issues will be solved using various examples. After solving the field equations, the cosmological parameter found will be compared to observational data. Also, use software like Python, Maple, and Reduce-Algebra to solve differential equations of field equations and make graphs to visualize the various cosmological parameters. The primary tool for tackling the problem is tensor algebra.

1.16 Importance of proposed study

Generally, the study of Bianchi type V cosmological models plays a vital role in studying the universe. It creates more interest as these models contain exceptional isotropic cases and permit arbitrary small anisotropy levels at some instant of time. This property makes them suitable as a model of our universe. The spatially homogeneous Bianchi Type V cosmological models also create more interest in studying physical and geometrical properties. It is a natural generalization of the FRW model.