Chapter 2

Biachi Type-V Modified $f(R,T)$ Gravity Model in Lyra Geometry with Varying Deceleration Parameter

2.1 Introduction

This chapter deals with the study of Bianchi type-V modified $f(R,T)$ gravity model in Lyra Geometry with varying DP. Nevertheless, the main cause of accelerated expansion still doubts the universe's cosmic time acceleration and DM's existence. Generally, modifications of Einstein's theory are attracting further to describe the DE and cosmic time. Various modified theories are $f(R)$ gravity [Chiba et al. (2007), Sharif and Shamir (2009), Capozziello and Vignolo (2011), Sebastiani and Myrzakulov (2015)], $f(G)$ gravity [Sharif and Fatima (2016), Makarenko (2017)], $f(T)$ gravity [Chattopadhyay (2018), Paliathanasis et al. (2016), Sharif and Rani (2011)]. This chapter focuses on the $f(R,T)$ gravity theory, as this is a generalization of $f(R)$ gravity, and this model suggests the best

The arcticle related to this chapter is published in Journal in Mathematical and Computational Sciences way to find out the DE in the cosmic time accelerated expansion of the universe. Also suggested is using a generalization of $f(R)$ gravity. Many researchers have shown the nature of cosmological models in the present context of the modified $f(R,T)$ theory of gravity. There are so many different types of DE in our model of the universe, and we found that many researchers [Godani (2019), Akta (2019), Singh and Beesham (2020)] have tried to investigate challenges of DE with $f(R,T)$ gravity models, by the relevance of [Harko et al. (2011)]. Subsequently, [Rao et al. (2008), Pradhan and Ram (2009), Ram et al. (2010), Nath and Sahu (2018)] are some of the authors who have shown the Bianchi type cosmological models in different physical circumstances and with time-dependent gauge function (β) for perfect fluid distribution in the presence of Lyra Geometry with the occurrence of $f(R,T)$ gravity also. Recently, Desikan (2020) investigated cosmological models with time-varying displacement field. Motivation from the above discussion, here in this chapter, we have investigated the physical nature of the model using Bianchi type-V modified $f(R,T)$ model in the presence of Lyra Geometry with a certain form of DP.

2.2 Metric and the field equations of $f(R,T)$ gravity

Let us consider the Bianchi type-V space-time in the form

$$ds^2 = -dt^2 + A^2dx^2 + e^{-2mx}(B^2dy^2 + C^2dz^2) \tag{2.1}$$

where A, B, and C are functions of cosmic time t alone and m is a constant.
The EMT for a perfect fluid is given by

$$T_{ij} = (\rho + p)u_iu_j + pg_{ij} \tag{2.2}$$

Here ρ and p denotes the energy density and pressure of the matter. On the otherhand $u^i = (0,0,0,1)$ is the four velocity vector in co-moving co-ordinate system satisfying the condition $u_i u^i = -1$.

Then, for the metric (2.1), the EFE (1.21) reduces to the form as [where $(\alpha = \frac{8\pi G - \mu c^2}{c^2})$ is a constant]

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 = -\alpha p + (\frac{\rho - p}{2}) \tag{2.3}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{C}\dot{A}}{CA} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 = -\alpha p + (\frac{\rho - p}{2}) \tag{2.4}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 = -\alpha p + (\frac{\rho - p}{2}) \tag{2.5}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{3m^2}{A^2} - \frac{3}{4}\beta^2 = \alpha \rho + (\frac{\rho - p}{2}) \tag{2.6}$$

$$\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - 2\frac{\dot{A}}{A} = 0 \tag{2.7}$$

The covariant derivative of the field equation (1.20) of RHS gives the energy conservation law as

$$\alpha\,[\dot{\rho} + 3H(\rho + p)] - \frac{1}{2}\,(\dot{\rho} - \dot{p}) = 0 \tag{2.8}$$

and the covariant derivative of the field equation (1.20) of LHS gives energy conservation law as

$$\frac{3}{2}\beta\dot{\beta} + \frac{3}{2}\beta^2\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0 \tag{2.9}$$

2.3 Cosmological solutions of the field equations

Integrating eq. (2.7), we get

$$A^2 = k_1 BC \tag{2.10}$$

where $k_1$is an integrating constant. The constant of integration k_1 can be chosen as unity, so that we obtain

$$A^2 = BC \tag{2.11}$$

In the EFE (2.3) - (2.7), there are five highly non-linear differential equations with six unknown variables, namely A, B, C, p, ρ and β. So Another condition must need to complete the field equations to find these six unknown constants and it is assumed that for spatially homogeneous metrics, the shear scalar (σ) is proportional to the expansion scalar (θ), i.e. $\frac{\sigma}{\theta} =$ constant, which yields that [Collins et al. (1980)]

$$B = C^n \tag{2.12}$$

where n is a non-zero constant.

In present context, several authors have researched cosmological model to draw up the DE by using deceleration parameter is considerd to be time dependent or some functions. Let us consider a generalized linearly varying DP [Akarsu and Dereli (2012)] as given by

$$q = -\frac{\dot{a}\ddot{a}}{a} = -kt + l - 1 \tag{2.13}$$

where $k \geq 0$ and $l \geq 0$ are constants.

Solving eq. (2.13), we get

$$a = \begin{cases} a_1 e^{\frac{2}{\sqrt{l^2 - 2c_1 k}}} \text{arctanh}\left(\frac{kt - l}{\sqrt{l^2 - 2c_1 k}}\right) \\ a_2(lt + c_2)^{\frac{1}{l}} for \;\; k = 0, l > 1 \\ a_3 e^{c_3} t for \;\; k = 0, l = 0 \end{cases} \tag{2.14}$$

For $l = 0$, eq. (2.14) reduces to

$$a = a_1 e^{\frac{2}{l} arctanh\left(\frac{kt}{l} - 1\right)} \tag{2.15}$$

Then from the above relations, we obtain the HP is

$$H = \frac{\dot{a}}{a} = -\frac{2}{t(kt - 2l)} \tag{2.16}$$

The Spatial volume is

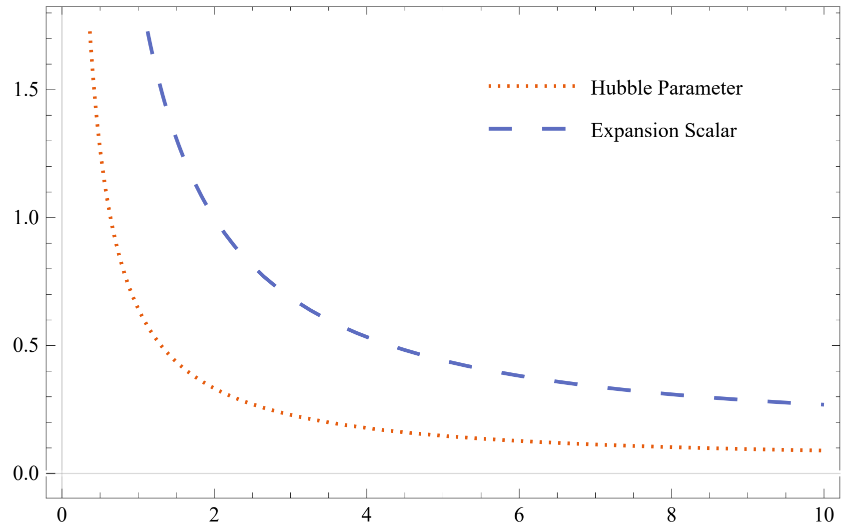

Figure 2.1: Variation of H, θ vs. t

$$V = a^3 = ABC = \left(a_1 e^{\frac{2}{l} arctanh\left(\frac{kt}{l}-1\right)}\right)^3 \tag{2.17}$$

From eqs. (2.11), (2.12) and (2.17) we obtained that the dynamical parameters are as

$$A = a_1 e^{\frac{2}{l} arctanh\left(\frac{kt}{l}-1\right)} \tag{2.18}$$

$$\tag{2.19}$$

$$C = \left[a_1 e^{\frac{2}{l} arctanh\left(\frac{kt}{l}-1\right)}\right]^{\frac{2}{n+1}} \tag{2.20}$$

Then from eq. (2.1), we obtain the metric in of the form

$$\begin{aligned} ds^2 = -dt^2 + \left(a_1 e^{\frac{2}{l} arctanh\left(\frac{kt}{l}-1\right)}\right)^2 dx^2 + e^{-2mx}\left(a_1 e^{\frac{2}{l} arctanh\left(\frac{kt}{l}-1\right)}\right)^{\frac{4n}{n+1}} dy^2 \\ + e^{-2mx}\left(a_1 e^{\frac{2}{l} arctanh\left(\frac{kt}{l}-1\right)}\right)^{\frac{4}{n+1}} dz^2 \end{aligned} \tag{2.21}$$

The shear expansion scalar and anisotropy parameter are obtained from eqs. (1.35) and (1.38) as

$$\sigma^2 = \frac{4}{t^2(kt-2l)^2}\frac{(n-1)^2}{(n+1)^2}, \quad \Delta = \frac{2}{3}\frac{(n-1)^2}{(n+1)^2} \tag{2.22}$$

Adding eqs. (2.3) - (2.5) and then using in eq. (2.6), it is found the energy density, pressure and displacement vector are, where for convenience we assumed that the the energy density and pressure in $f(\widetilde{R},T)$ gravity of the Bianchi type-V model in Lyra geometry is considered as

$$p = \gamma\rho \tag{2.23}$$

that yields

$$\rho = \frac{12}{6\alpha+\gamma+6\alpha\gamma-1}\left[\frac{4(-kt+l)}{(t^2(kt-2l)^2} - \frac{4}{t^2(kt-2l)^2}\left(\frac{n-1}{n+1}\right)^2 - \frac{m^2}{a_1^2 e^{\frac{4}{l}\operatorname{arctanh}(\frac{kt}{l}-1)}}\right] \tag{2.24}$$

$$p = \frac{12\gamma}{6\alpha+\gamma+6\alpha\gamma-1}\left[\frac{4(-kt+l)}{(t^2(kt-2l)^2} - \frac{4}{t^2(kt-2l)^2}\left(\frac{n-1}{n+1}\right)^2 - \frac{m^2}{a_1^2 e^{\frac{4}{l}\operatorname{arctanh}(\frac{kt}{l}-1)}}\right] \tag{2.25}$$

$$\frac{3}{4}\beta^2 = \frac{4(-2kt+2l-3)}{t^2(kt-2l)^2} - \frac{4}{t^2(kt-2l)^2}\left(\frac{n-1}{n+1}\right) + \frac{m^2}{a_1^2 e^{\frac{4}{l}\operatorname{arctanh}(\frac{kt}{l}-1)}} + \frac{(1-\gamma-2\alpha\gamma)}{(6\alpha+\gamma+6\alpha\gamma-1)}\left[\frac{4(-kt+l)}{(t^2(kt-2l)^2} - \frac{4}{t^2(kt-2l)^2}\left(\frac{n-1}{n+1}\right)^2 - \frac{m^2}{a_1^2 e^{\frac{4}{l}\operatorname{arctanh}(\frac{kt}{l}-1)}}\right] \tag{2.26}$$

Model of that trace in $f(\widetilde{R},T)$ gravity is given by

$$T = \frac{12(1-3\gamma)}{(6\alpha+\gamma+6\alpha\gamma-1)}\left[\frac{4(-kt+l)}{(t^2(kt-2l)^2} - \frac{4}{t^2(kt-2l)^2}\left(\frac{n-1}{n+1}\right)^2 - \frac{m^2}{a_1^2 e^{\frac{4}{l}\operatorname{arctanh}(\frac{kt}{l}-1)}}\right] \tag{2.27}$$

The Riemannian curvature R of $f(\widetilde{R},T)$ gravity of that model is given by

$$R = \frac{6m^2}{a_1^2 e^{\frac{4}{l}\operatorname{arctanh}(\frac{kt}{l}-1)}} - \frac{8\left(kt-l+1\right)}{t^2\left(kt-2l\right)^2} - \frac{16}{t^2\left(kt-2l\right)^2} - \frac{16\left(3n^2+4n+3\right)}{(n+1)^2 t^2\left(kt-2l\right)^2} \tag{2.28}$$

and, the function of Ricci Scalar tensor is given by

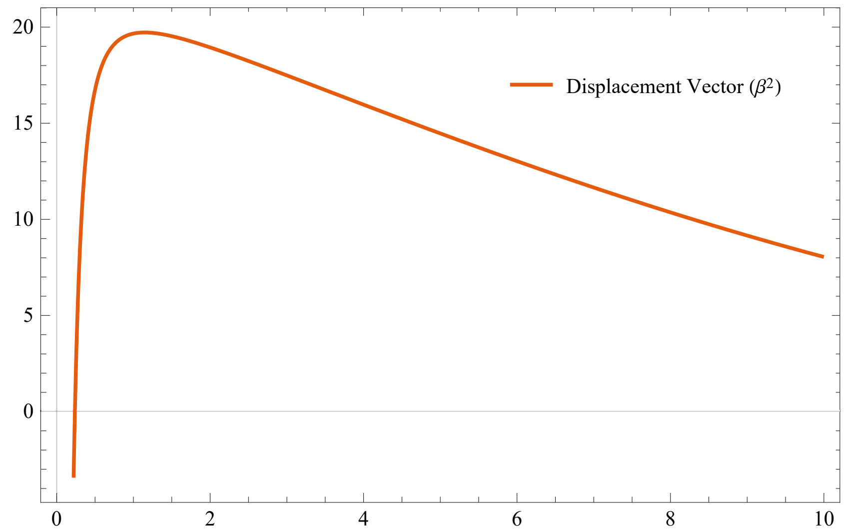

Figure 2.2: Variation of β^2 vs. t

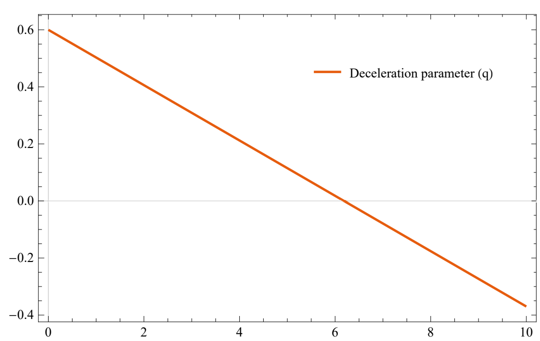

Figure 2.3: Variation of q vs. t

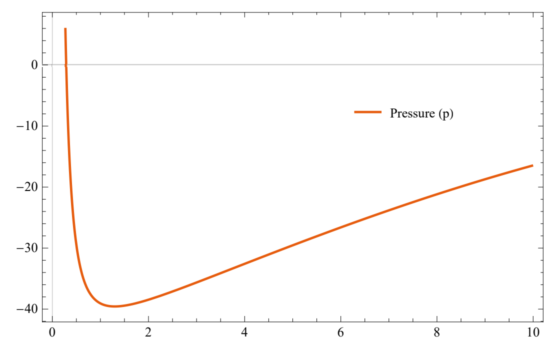

Figure 2.4: Variation of p vs. t

$$
\begin{aligned}
\widetilde{R} = {} & \frac{6m^2}{a_1^2 e^{\frac{4}{l}\operatorname{arctanh}\left(\frac{kt}{l}-1\right)}} - \frac{8\left(kt-l+1\right)}{t^2\left(kt-2l\right)^2} - \frac{16}{t^2\left(kt-2l\right)^2} - \frac{16\left(3n^2+4n+3\right)}{(n+1)^2 t^2\left(kt-2l\right)^2} \\
& + 2W_2\left[\frac{-8K}{t^2(kt-2l)^2} - \frac{4(kt-l)}{t^3(kt-2l)^3} + \frac{16kt}{t^3(kt-2l)^3}\left(\frac{n-1}{n+1}\right)^2\right] \\
& - 2W_2\left[\frac{m^2}{a_1^2 e^{\frac{4}{l}\operatorname{arctanh}\left(\frac{kt}{l}-1\right)}}\frac{4}{t(2l-kt)} + \frac{4KZ}{t^2(kt-2l)^2} + \frac{8Z(l-kt)}{t^3(kt-2l)^3}\right] \\
& + 2W_2 Z\left[\frac{16kt}{t^3(kt-2l)^3}\left(\frac{n-1}{n+1}\right)^2 + \frac{m^2}{a_1^2 e^{\frac{4}{l}\operatorname{arctanh}\left(\frac{kt}{l}-1\right)}}\frac{4}{t(2l-kt)}\right] - \frac{18W_1}{t(kt-2l)} \\
& + \frac{8(-2kt+2l-3)}{t^2(kt-2l)^2} - \frac{8}{t^2(kt-2l)^2}\left(\frac{n-1}{n+1}\right) + \frac{2m^2}{a_1^2 e^{\frac{4}{l}\operatorname{arctanh}\left(\frac{kt}{l}-1\right)}} \\
& + 2Z\left[\frac{4(-kt+l)}{(t^2(kt-2l)^2} - \frac{4}{t^2(kt-2l)^2}\left(\frac{n-1}{n+1}\right)^2 - \frac{m^2}{a_1^2 e^{\frac{4}{l}\operatorname{arctanh}\left(\frac{kt}{l}-1\right)}}\right]
\end{aligned}
\tag{2.29}
$$

where $Z = \frac{1-\gamma-2\alpha\gamma}{6\alpha+\gamma+6\alpha\gamma-1}$ is a constant.

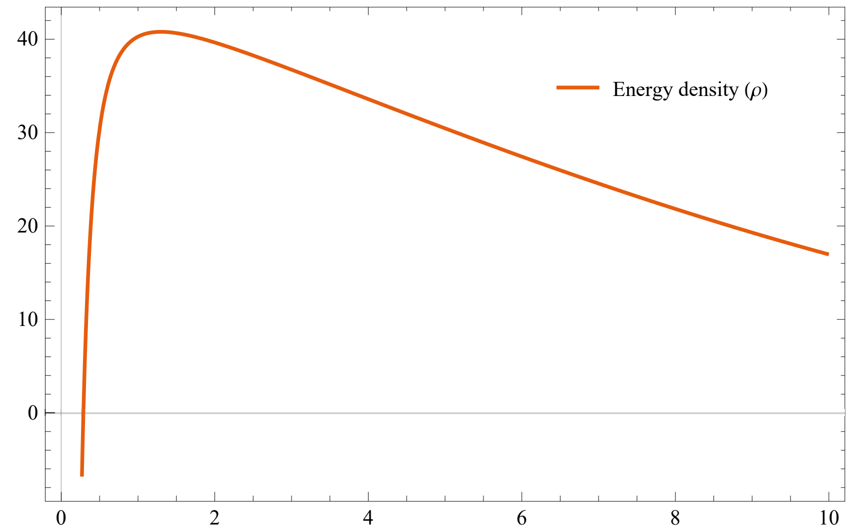

Figure 2.5: Variation of ρ vs. t

$$W_1(t) = \beta(t) = \frac{2}{\sqrt{3}}\left[\frac{4(-2kt+2l-3)}{t^2(kt-2l)^2} - \frac{4}{t^2(kt-2l)^2}\left(\frac{n-1}{n+1}\right) + \frac{m^2}{a_1^2 e^{\frac{4}{l}\operatorname{arctanh}(\frac{kt}{l}-1)}}\right]^{\frac{1}{2}} + \frac{2Z}{\sqrt{3}}\left[\frac{4(-kt+l)}{(t^2(kt-2l)^2} - \frac{4}{t^2(kt-2l)^2}\left(\frac{n-1}{n+1}\right)^2 - \frac{m^2}{a_1^2 e^{\frac{4}{l}\operatorname{arctanh}(\frac{kt}{l}-1)}}\right]^{\frac{1}{2}} \tag{2.30}$$

$$W_2(t) = \frac{1}{\beta(t)} = \frac{2}{\sqrt{3}}\left[\frac{4(-2kt+2l-3)}{t^2(kt-2l)^2} - \frac{4}{t^2(kt-2l)^2}\left(\frac{n-1}{n+1}\right) + \frac{m^2}{a_1^2 e^{\frac{4}{l}\operatorname{arctanh}(\frac{kt}{l}-1)}}\right]^{\frac{-1}{2}} + \frac{2Z}{\sqrt{3}}\left[\frac{4(-kt+l)}{(t^2(kt-2l)^2} - \frac{4}{t^2(kt-2l)^2}\left(\frac{n-1}{n+1}\right)^2 - \frac{m^2}{a_1^2 e^{\frac{4}{l}\operatorname{arctanh}(\frac{kt}{l}-1)}}\right]^{\frac{-1}{2}} \tag{2.31}$$

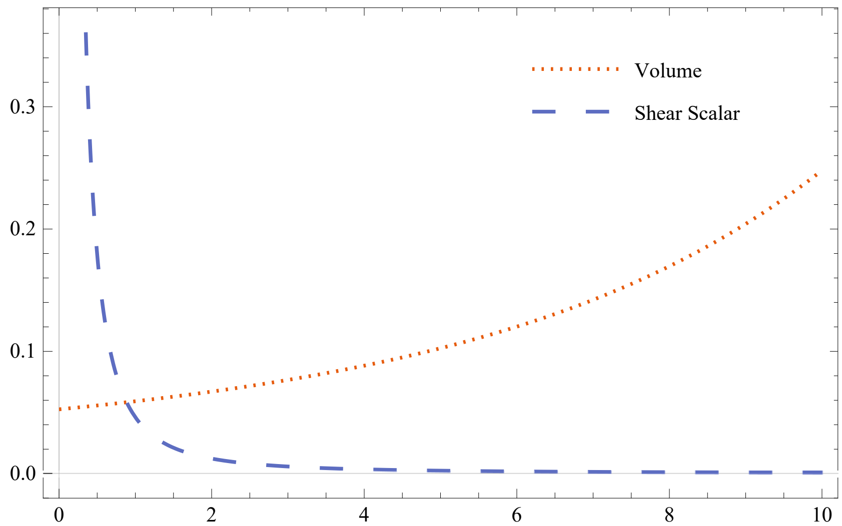

Figure 2.6: Variation of σ^2, V vs. t

In this model $f(\widetilde{R},T)$ and (r,s) are obtained respectively as

$$
\begin{aligned}
\frac{1}{\mu}f(\widetilde{R},T) &= \frac{6m^2}{a_1^2 e^{\frac{4}{l}\operatorname{arctanh}\left(\frac{kt}{l}-1\right)}} - \frac{8\left(kt-l+1\right)}{t^2\left(kt-2l\right)^2} - \frac{16}{t^2\left(kt-2l\right)^2} - \frac{16\left(3n^2+4n+3\right)}{(n+1)^2 t^2\left(kt-2l\right)^2} \\
&+ 2W_2\left[\frac{-8K}{t^2(kt-2l)^2} - \frac{4(kt-l)}{t^3(kt-2l)^3} + \frac{16kt}{t^3(kt-2l)^3}\left(\frac{n-1}{n+1}\right)^2\right] \\
&- 2W_2\left[\frac{m^2}{a_1^2 e^{\frac{4}{l}\operatorname{arctanh}\left(\frac{kt}{l}-1\right)}}\frac{4}{t(2l-kt)} + \frac{4KZ}{t^2(kt-2l)^2} + \frac{8Z(l-kt)}{t^3(kt-2l)^3}\right] \\
&+ 2W_2 Z\left[\frac{16kt}{t^3(kt-2l)^3}\left(\frac{n-1}{n+1}\right)^2 + \frac{m^2}{a_1^2 e^{\frac{4}{l}\operatorname{arctanh}\left(\frac{kt}{l}-1\right)}}\frac{4}{t(2l-kt)}\right] - \frac{18W_1}{t(kt-2l)} \\
&+ \frac{8(-2kt+2l-3)}{t^2(kt-2l)^2} - \frac{8}{t^2(kt-2l)^2}\left(\frac{n-1}{n+1}\right) + \frac{2m^2}{a_1^2 e^{\frac{4}{l}\operatorname{arctanh}\left(\frac{kt}{l}-1\right)}} \\
&+ 2Z\left[\frac{4(-kt+l)}{(t^2(kt-2l)^2} - \frac{4}{t^2(kt-2l)^2}\left(\frac{n-1}{n+1}\right)^2 - \frac{m^2}{a_1^2 e^{\frac{4}{l}\operatorname{arctanh}\left(\frac{kt}{l}-1\right)}}\right] \\
&+ \frac{12(1-3\gamma)}{(6\alpha+\gamma+6\alpha\gamma-1)}\left[\frac{4(-kt+l)}{(t^2(kt-2l)^2} - \frac{4}{t^2(kt-2l)^2}\left(\frac{n-1}{n+1}\right)^2\right] \\
&- \frac{12(1-3\gamma)}{(6\alpha+\gamma+6\alpha\gamma-1)}\left[\frac{m^2}{a_1^2 e^{\frac{4}{l}\operatorname{arctanh}\left(\frac{kt}{l}-1\right)}}\right]
\end{aligned}
\tag{2.32}
$$

$$r = 1 + 3(kt - l) + 2(kt - l)^2 - \frac{kt(kt - 2l)}{2} \tag{2.33}$$

and,

$$s = \frac{3(kt - l) + 2(kt - l)^2 - \frac{kt(kt-2l)}{2}}{3\left(-kt + l - \frac{3}{2}\right)} \tag{2.34}$$

All the graphs are drawn with $\alpha = 1, \gamma = -0.97, k = 0.097, n = 2, a_1 = m = 1, l = 1.6$

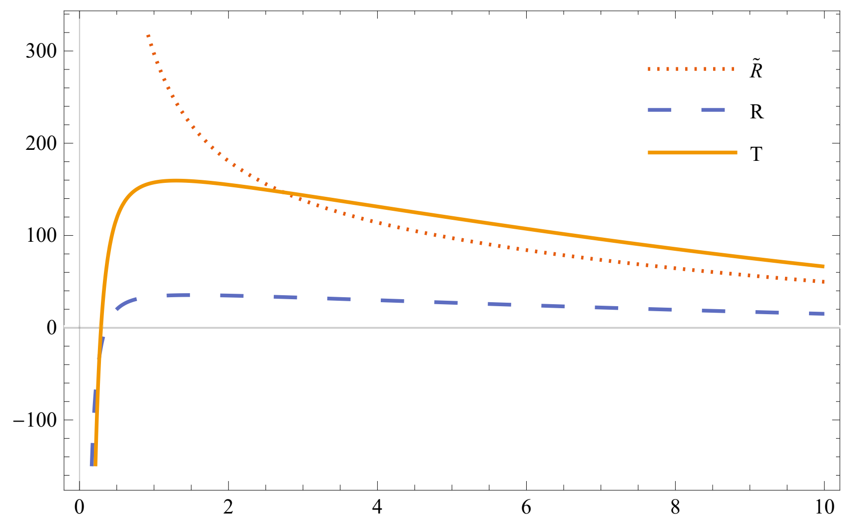

Figure 2.7: Variation of $\widetilde{R}$, R, T vs. t

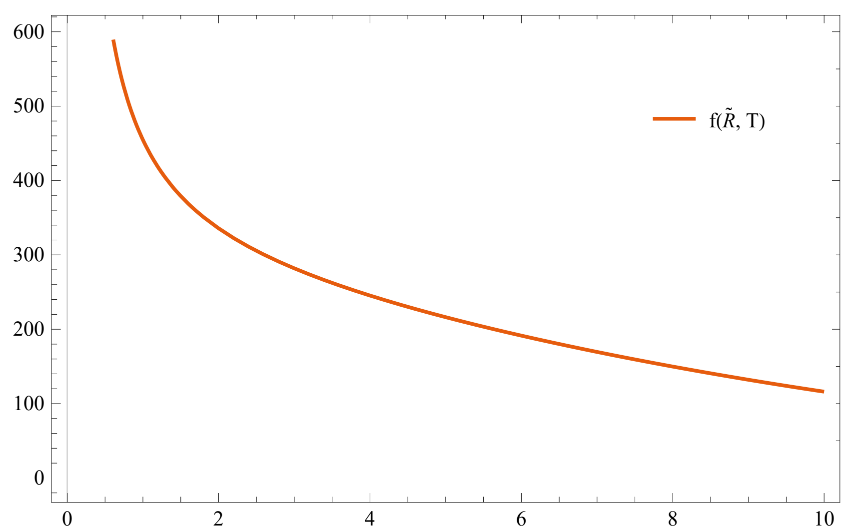

Figure 2.8: Variation of $f(\widetilde{R}, T)$ gravity vs. t

2.4 Conclusion

In this model study, the energy density, HP, shear scalar, and displacement vector gradually decrease over time (Figs. 2.1–2.2). It is observed that the DP type of time variations is positive to negative (i.e., early deceleration to late time acceleration) (Fig. 2.3). Pressure is negative and varies with time, as in Fig. 2.4, and from this figure, it is clear that the model exists DE that satisfies the recent observational data. Spatial volume and shear scalar increase with time's evolution (Fig. 2.6), i.e., the present model of the universe starts expanding with a finite volume. This model is anisotropic as the average anisotropy parameter is non-zero. From Fig. 2.7, it is observed that $\widetilde{R}$, R, and T all increase positively, and finally, it will vanish with the evolution of time. In this model $f(\widetilde{R},T)$ tends to zero with evolution of cosmic time t (Fig. 2.8). Statefinder parameters (r,s) do not tend to $(1,0)$, so the present cosmological model does not satisfy the ΛCDM model. A DE model with acceleration has been studied with $f(\widetilde{R},T)$ gravity in Lyra Geometry.