Chapter 3

Bianchi Type-V Dark Energy Modified $f(R,T)$ Gravity Model in the presence of Massive Scalar Field in Lyra Geometry

3.1 Introduction

This chapter deals with the study of Bianchi type-V DE modified $f(R,T)$ gravity model with a massive scalar field in Lyra Geometry. DE and DM, these two modern concepts, are the main hidden tools in discussing present cosmological models of the universe, and they illustrate how the model behaves. Many researchers have been searching for DE and have proposed a variety of theoretical models in the presence of Quintessence, Phantom; as a result, they have found that the universe, approximately composed of 4% ordinary matter, 23% DM, 73% DE [Spregel et al. (2007)] and from that result, we can say that our universe is filled with DE. Modifying gravity, which appears more appealing in exploring the behavior of DE (Negative Pressure), is one of the finest approaches to

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establishing cosmic acceleration (Late time) in cosmology. Several authors have made a remarkable contribution to explaining the present universe of our accelerating phase and bringing out different theories for relevant theories. The theories are $f(R)$, $f(G)$, $f(T)$, $f(R,T)$, as theoretically and geometrically observations from many researchers found to be $f(R,T)$ gravity is beautiful and suitable among these modified theories for the present cosmological model, in which the EHA replaced by a function of Ricci scalar [Nojiri and Odintsov (2004, 2007), Bertolami et al. (2007), Bamba et al. (2018), Linder (2010), Rodrigues et al. (2014)]. However, Harko et al. (2011) perceived a new modification of $f(R)$, which is term as $f(R,T)$ gravity theory, where gravitational Lagrangian is given by an arbitrary function of Ricci-Scalar and Trace T of the energy-momentum tensor.

Two models of ideas have been considered to reveal relevant facts about DE: one is to create negative pressure (which is called DE), and the other is to modify Einstein's theory of gravitation to discuss the corresponding anisotropic of negative pressure (DE), and we have found that significant modifications have been suggested by several authors [Brans and Dicke (1961), Saez and Ballester (1986), Nojiri and Odintsov (2006), Raju et al. (2020)] in various contexts. Here, we are focusing on the field of $f(R,T)$ gravity model in the framework of Lyra Geometry in Bianchi type –V cosmological models in the presence of an attractive massive scalar field. Many researchers [Mohanty and Pradhan (1992), Singh and Rani (2015), and Naidu et al. (2020)] have examined spatially homogenous and anisotropic cosmological models of the universe based on Lyra Geometry and GR with an attractive massive scalar field in a different physical context, in which special law of variation for HP [Proposed by Berman (1983)] was applied to examine the constant DP. Here, we focus on the DE field based on the Lyra geometry with $f(R,T)$ gravity model and an attractive massive scalar field. In the present decades, several authors have examined the different cosmological models and obtained the different physical features in the framework of an attractive massive scalar field based on different contexts so far [Mohanty et al. (2003), Aygün et al. (2012), Naidu et al. (2019), Rao et al. (2022)].

3.2 Metric and the $f(R,T)$ gravity

Let us consider the Bianchi type-V space time in the form

$$ds^2 = -dt^2 + A^2dx^2 + e^{-2mx}(B^2dy^2 + C^2dz^2) \tag{3.1}$$

where A,B,C are functions of cosmic time t alone and m is a constant.

The EMT for a perfect fluid is given by

$$T_{ij} = (\rho + p)u_iu_j + pg_{ij} \tag{3.2}$$

where ρ and p denote the energy density and pressure of the matter. On the other hand $u^i = (0,0,0,1)$ is the four-velocity vector in the co-moving coordinate system satisfying the condition $u_iu^i = -1$

An attractive massive scalar field, whose EMT is assumed as,

$$T_{ij}^{\varphi} = \varphi_{,i}\varphi_{,j} - \frac{1}{2}\left(\varphi_{,k}\varphi^{,k} - M^2\varphi^2\right) \tag{3.3}$$

where mass of that scalar field is represented by M and which satisfies the Klein- Gordon equation

$$g^{ij}g_{,ij} + M^2\varphi = 0 \tag{3.4}$$

The symbols comma and a semicolon represents respectively the ordinary and covariant differentiation, where φ is a function of time t.

$$L_m = -p, \;\; L_{\varphi} = \frac{1}{2}(M^2\varphi^2 - \dot{\varphi}^2) \tag{3.5}$$

Then, the tensor (1.14) is obtained as below, where the physical nature of model depends on this tensor Θ_{ij} as

$$\Theta_{ij} = -2T_{ij} - \frac{1}{2}g_{ij}\left(2p + \dot{\varphi}^2 - M^2\varphi^2\right) \tag{3.6}$$

In this case, the line element (3.1), with respect to eq. (3.3), the EFE (1.21) reduces to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 + \frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2} = -\alpha p + \left(\frac{\rho - p}{2}\right) \tag{3.7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{C}\dot{A}}{CA} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 + \frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2} = -\alpha p + \left(\frac{\rho - p}{2}\right) \tag{3.8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 + \frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2} = -\alpha p + \left(\frac{\rho - p}{2}\right) \tag{3.9}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{3m^2}{A^2} - \frac{3}{4}\beta^2 + \frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2} = \alpha\rho + \left(\frac{\rho - p}{2}\right) \tag{3.10}$$

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \tag{3.11}$$

The Klein Gordon equation is as

$$\ddot{\varphi} + \dot{\varphi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + M^2\varphi = 0 \tag{3.12}$$

and the the conservation law for the energy momentum tensor of DE fluid is

$$\dot{\varphi} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0 \tag{3.13}$$

3.3 Cosmological solutions of the field equations

In solving the above field eqs. (3.7)-(3.11), the following physical parameters are very important and these parameters are defined as follows:

The spatial volume and the scale factor are given by

$$V = a^3 = ABC \tag{3.14}$$

The generalized HP and the scalar expansion are defined as

$$H = \frac{\dot{a}}{a} = (H_x + H_y + H_z) \tag{3.15}$$

$$\theta = 3H = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \tag{3.16}$$

where the symbols are already defined in the previous chapter.

The shear exapasion and the anisotropy parameter are respectively defined as

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2 - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{C}\dot{A}}{CA}\right] \tag{3.17}$$

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{3.18}$$

Integrating eq. (3.11), we get

$$A^2 = k_1 BC \tag{3.19}$$

where $k_1$is an integrating constant. Without loss of generality, it can be taken as $k_1 = 1$, so that we obtain

$$A^2 = BC \tag{3.20}$$

In the EFE (3.7) - (3.11), there are five highly non-linear differential equations with seven unknown variables, namely A, B, C, p, ρ, β, φ. Thus, in order to find out these seven unknown constants, another two conditions are assumed as follows:

(i) Considering $\frac{\sigma}{\theta} =$ constant, which yields [Throne (1967)]

$$C = B^n \tag{3.21}$$

where $n \neq 1$ is a positive constant.

(ii) Power law relation between the scalar field φ and the average scale factor

$$3\frac{\dot{A}}{A} = -2\frac{\dot{\varphi}}{\varphi} \tag{3.22}$$

From (3.12), (3.20)-(3.22), we obtain the scalar field

$$\varphi = \frac{2M\exp^{Mt}}{\varphi_0\exp^{2Mt} - \varphi_1} \tag{3.23}$$

From eqs. (3.14), (3.21) - (3.23), we obtain

$$A = \left[\frac{\varphi_0 e^{2Mt} - \varphi_1}{2Me^{Mt}}\right]^{\frac{2}{3}}, B = \left[\frac{\varphi_0 e^{2Mt} - \varphi_1}{2Me^{Mt}}\right]^{\frac{4}{3(n+1)}}, C = \left[\frac{\varphi_0 e^{2Mt} - \varphi_1}{2Me^{Mt}}\right]^{\frac{4n}{3(n+1)}} \tag{3.24}$$

where φ_0, φ_1 are constant of integration. Using eq. (1.24) in the metric (3.1), it reduces to

$$ds^2 = -dt^2 + \left[\frac{\varphi_0 e^{2Mt} - \varphi_1}{2Me^{Mt}}\right]^{\frac{4}{3}} dx^2 + \left[\frac{\varphi_0 e^{2Mt} - \varphi_1}{2Me^{Mt}}\right]^{\frac{8}{3(n+1)}} e^{-2mx} dy^2 + \left[\frac{\varphi_0 e^{2Mt} - \varphi_1}{2Me^{Mt}}\right]^{\frac{8n}{3(n+1)}} e^{-2mx} dz^2 \tag{3.25}$$

3.4 Physical and dynamical parameters of the model

The following are the dynamical parameters of this model of the universe, which are necessary for discussion in cosmology:

$$V = \left[\frac{\varphi_0 e^{2Mt} - \varphi_1}{2Me^{Mt}}\right]^2, H = \frac{2M}{3}\left[\frac{\varphi_0 e^{2Mt} + \varphi_1}{\varphi_0 e^{2Mt} - \varphi_1}\right], \theta = 2M\left[\frac{\varphi_0 e^{2Mt} + \varphi_1}{\varphi_0 e^{2Mt} - \varphi_1}\right] \tag{3.26}$$

and

$$\sigma^2 = \frac{4M^2}{9}\left(\frac{n-1}{n+1}\right)^2\left[\frac{\varphi_0 e^{2Mt} + \varphi_1}{\varphi_0 e^{2Mt} - \varphi_1}\right]^2, \quad \Delta = \frac{2}{3}\left(\frac{n-1}{n+1}\right)^2 \tag{3.27}$$

The DP is obtained as

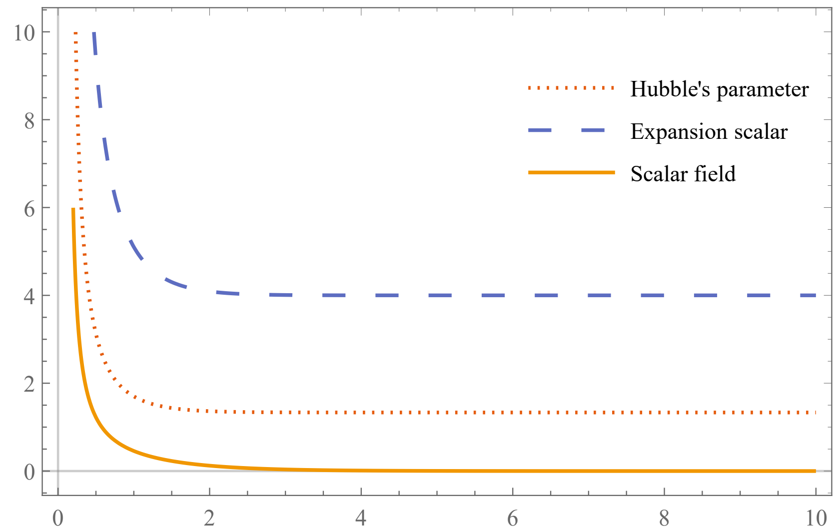

Figure 3.1: Variation of H, θ, φ vs. t

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$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) = -1 + \frac{6\varphi_0\varphi_1 e^{2Mt}}{(\varphi_0 e^{2Mt}+\varphi_1)^2} \tag{3.28}$$

Now, from eq. (3.10), we obtain

$$\alpha\rho = 3H^2 - \sigma^2 - \frac{3M^2}{A^2} - \frac{3}{4}\beta^2 - \frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2} - \left(\frac{\rho - p}{2}\right) \tag{3.29}$$

Adding (3.7)-(3.9), it becomes

$$\alpha p = H^2(2q-1) - \sigma^2 + \frac{m^2}{A^2} - \frac{3}{4}\beta^2 + \left(\frac{\rho - p}{2}\right) - \frac{\dot{\varphi}^2}{2} - \frac{M^2\varphi^2}{2} \tag{3.30}$$

Applying eq. (3.29) in (3.30), we get

$$K_1\rho = -\frac{8}{3}M^2\left(\frac{\varphi_0 e^{2Mt}+\varphi_1}{\varphi_0 e^{2Mt}-\varphi_1}\right)^2 + \frac{48M^2}{9}\frac{\varphi_0\varphi_1 e^{2Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^2} + 4m^2\left(\frac{2Me^{Mt}}{\varphi_0 e^{2Mt}-\varphi_1}\right)^{\frac{4}{3}}$$
$$-4M^2\frac{e^{2Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^2} \tag{3.31}$$

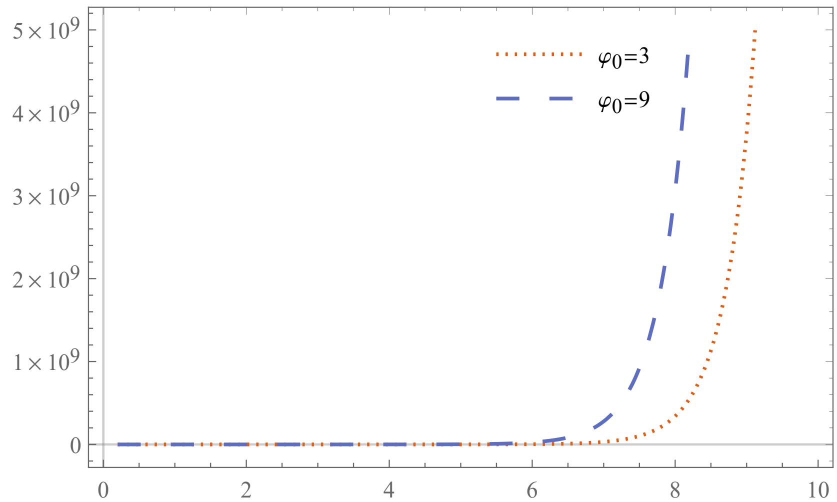

Figure 3.2: Variation of V vs. t

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where $K_1 = (\alpha\gamma - \alpha - 1 + \gamma)$ is constant.

Also, it is assumed the relation $p = \gamma\rho$ and using this condition in the above eq. (3.29), the pressure of the model is obtained as

$$p = \frac{\gamma}{K_1}\left[-\frac{8}{3}M^2\left(\frac{\varphi_0 e^{2Mt} + \varphi_1}{\varphi_0 e^{2Mt} - \varphi_1}\right)^2 + \frac{48M^2}{9}\frac{\varphi_0\varphi_1 e^{2Mt}}{(\varphi_0 e^{2Mt} - \varphi_1)^2} + 4m^2\left(\frac{2Me^{Mt}}{\varphi_0 e^{2Mt} - \varphi_1}\right)^{\frac{4}{3}}\right] - \frac{\gamma}{K_1}\left[4M^2\frac{e^{2Mt}}{(\varphi_0 e^{2Mt} - \varphi_1)^2}\right] \tag{3.32}$$

Now from (3.29), the displacement vector field is obtained as

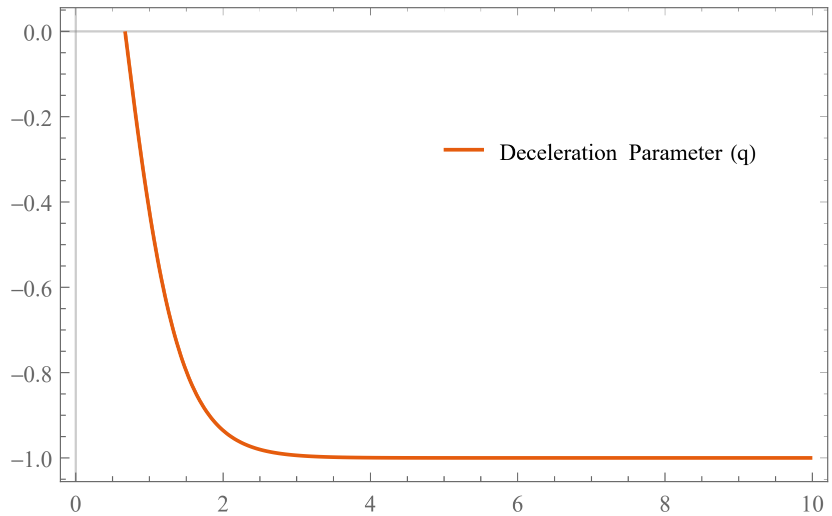

Figure 3.3: Variation of q vs. t

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\begin{aligned}
(1-\gamma)\frac{3}{4}\beta^2 = & -\frac{4}{3}M^2\left(\frac{\varphi_0 e^{2Mt}+\varphi_1}{\varphi_0 e^{2Mt}-\varphi_1}\right)^2+\frac{16}{3}\frac{M^2\varphi_0\varphi_1 e^{2Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^2}-\frac{4}{3}\gamma M^2\left(\frac{\varphi_0 e^{2Mt}+\varphi_1}{\varphi_0 e^{2Mt}-\varphi_1}\right)^2 \\
& -\frac{4}{9}M^2(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(\frac{\varphi_0 e^{2Mt}+\varphi_1}{\varphi_0 e^{2Mt}-\varphi_1}\right)^2+(1+3\gamma)m^2\left(\frac{2Me^{Mt}}{\varphi_0 e^{2Mt}-\varphi_1}\right)^{\frac{4}{3}} \\
& +(\gamma-1)\frac{\varphi_0^2 M^4 e^{6Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^4}+(\gamma-1)\frac{M^4\varphi_1^2 e^{2Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^4}+(\gamma-1)\frac{2\varphi_0\varphi_1 M^4 e^{4Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^4} \\
& -2(\gamma+1)\frac{M^4 e^{2Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^2}-\frac{4}{3}\frac{(1+\gamma)(1-\gamma)M^2}{K_1}\left(\frac{\varphi_0 e^{2Mt}+\varphi_1}{\varphi_0 e^{2Mt}-\varphi_1}\right)^2 \\
& +\frac{24}{9}\frac{(1+\gamma)(1-\gamma)M^2}{K_1}\frac{\varphi_0\varphi_1 e^{2Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^2}+2m^2\frac{(1+\gamma)(1-\gamma)}{K_1}\left(\frac{2Me^{Mt}}{\varphi_0 e^{2Mt}-\varphi_1}\right)^{\frac{4}{3}} \\
& -2M^2\frac{(1+\gamma)(1-\gamma)}{K_1}\frac{e^{2Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^2}
\end{aligned}
\tag{3.33}
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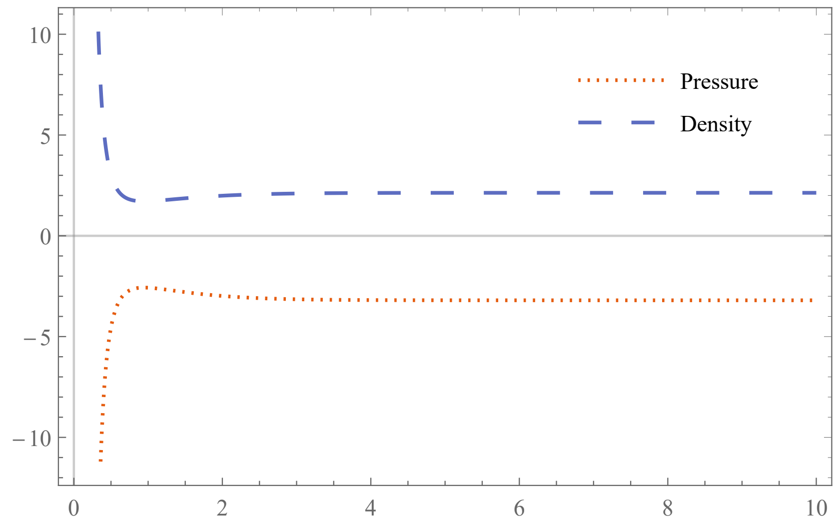

Figure 3.4: Variation of p and ρ vs. t

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We obtain the trace of the model $(T = \rho - 3p)$ from eqs. (3.31) and (3.32) as

$$\begin{aligned} T = &\left(\frac{1-3\gamma}{K_1}\right)\left[-\frac{8}{3}M^2\left(\frac{\varphi_0 e^{2Mt}+\varphi_1}{\varphi_0 e^{2Mt}-\varphi_1}\right)^2 + \frac{48M^2}{9}\frac{\varphi_0\varphi_1 e^{2Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^2}\right] \\ &+\left(\frac{1-3\gamma}{K_1}\right)\left[4m^2\left(\frac{2Me^{Mt}}{\varphi_0 e^{2Mt}-\varphi_1}\right)^{\frac{4}{3}} - 4M^2\frac{e^{2Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^2}\right] \end{aligned} \tag{3.34}$$

Subsequently, the Riemannian curvature scalar is obtained as

$$\begin{aligned} R = &\frac{6m^2}{A^2} - 2\left(\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}\right) - 2\left(\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{B}}{B}\frac{\dot{C}}{C}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}\right) \\ &= 6m^2\left(\frac{2Me^{Mt}}{\varphi_0 e^{2Mt}-\varphi_1}\right)^{\frac{4}{3}} + \frac{16}{9}\left(\frac{n^2-14n+1}{(n+1)^2}\right)\left(\frac{M\varphi_0 e^{2Mt}}{\varphi_0 e^{2Mt}-\varphi_1}\right)^2 \\ &+\frac{64}{9}\left(\frac{n^2+4n+1}{(n+1)^2}\right)\left(\frac{M^2\varphi_0 e^{2Mt}}{\varphi_0 e^{2Mt}-\varphi_1}\right) \end{aligned} \tag{3.35}$$

Also, the state finder parameter of the model is obtained as

$$r = 1 - \frac{18\varphi_0^2\varphi_1 e^{2Mt}}{(\varphi_0 e^{2Mt}+\varphi_1)^3} \tag{3.36}$$

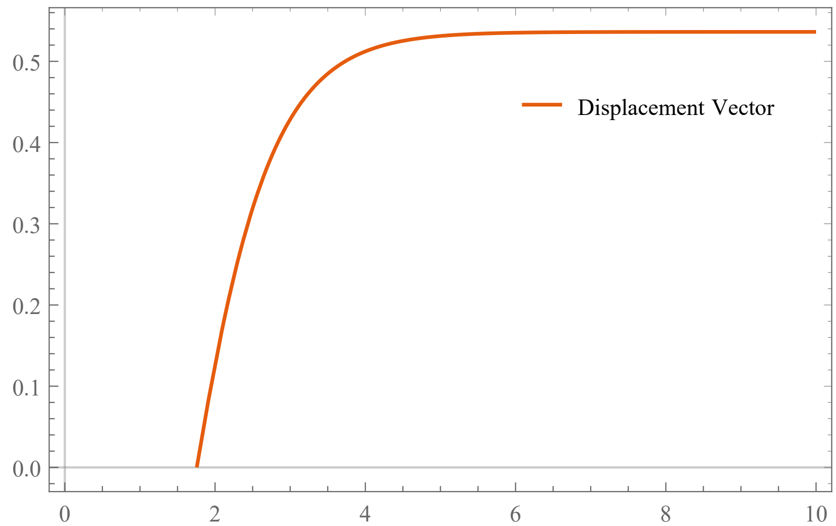

Figure 3.5: Variation of β^2 vs. t

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and

$$s = \frac{2\varphi_0^2\varphi_1 e^{2Mt}}{(\varphi_0 e^{2Mt} - \varphi_1)^2(\varphi_0 e^{2Mt} + \varphi_1)} \tag{3.37}$$

Discussion of the behavior of the $f(\widetilde{R},T)$ model

The main behavior of the model is obtained from the above equations, with the proper choice of the values of the constants, which are the suitable values for this model as we found in the framework of Lyra Geometry in $f(\widetilde{R},T)$ gravity model.

• It is observed from the eq. (3.26) that the spatial volume does not zero at $t = 0$, while as it diverges, which shows that in this $f(\widetilde{R},T)$ model, the universe does not start evolving with zero volume at the initial epoch, i.e., the universe starts with an infinite rate of expansion. Also, the spatial volume does not tend to zero at the initial epoch, so the model is free from the initial singularity at this initial epoch. Subsequently, the HP, Expansion Scalar, and Scalar fields also diverge when $t \to \infty$. From Fig. 3.1, we observed that the scalar field finally decreases the function of cosmic time t, representing the corresponding kinetic energy increases. Here in this model [as eq. (3.28)], we found that the DP always lies in the range -1 to 0. This result indicates that the DP satisfies the present observa-

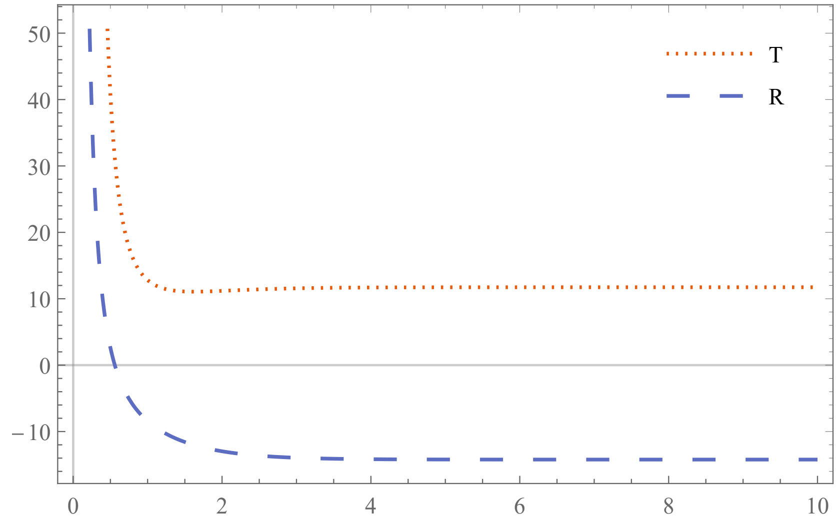

Figure 3.6: Variation of R and T vs. t

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tional data like SNeIa [Perlmutter et al. (1999), Riess et al. (1998)], CMB [Spergel et al. (2003), Hinshaw et al. (2007)], BAO [Battye and Moss (2014), Aubourg et al. (2015)] and thus for this the present $f(\widetilde{R},T)$ model expands at an accelerated rate.

• Further from Eqs. (3.26) and (3.27), we found that $\frac{\sigma^2}{\theta^2} \neq 0$, and this condition shows that this model does not approach isotropy with the cosmic time t with the current observations.

• From Fig. 3.4, the model's behavior of pressure vs. cosmic time t. It is clear from that by the choice of the values $\varphi_0 = 3$, $\varphi_1 = 4$, $\alpha = 1$, $M = 1.2$, $m = 2$, $n = 2$, $\gamma = -1.5$, the pressure remains negative throughout the evolution of time and this condition satisfy the present observational data that there is DE in this $f(\widetilde{R},T)$ gravity model. While we observed that the density of the model decreases with the increases in time, it finally has a minimum positive value.

• It is observed from Fig. 3.5 that the displacement vector (β^2) gradually decreases in the beginning but increases with the evolution of cosmic time t.

• From eq. (3.23), it is found that the scalar field (φ) and the kinetic energy (φ^2) both are

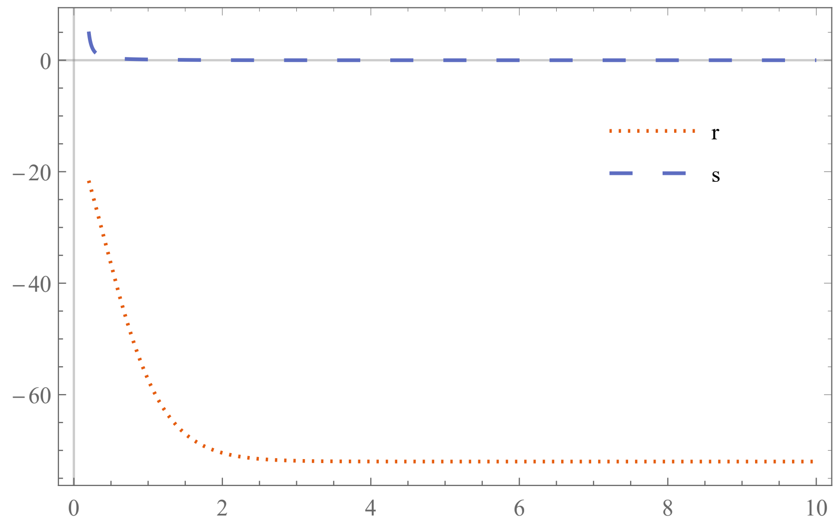

Figure 3.7: Variation of r and s vs. t

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nonsingular, as this model does not tend to zero with the evolution of time.

• It is clear from Fig. 3.6 that the Riemannian curvature and the model trace decrease with time evolution. Finally, it has a constant negative value. However, a trace of the model gradually increases, which has a constant positive value with cosmic time t in our accelerated universe expansion in the $f(\widetilde{R},T)$ model in the framework of Lyra geometry.

• Fig. 3.7 represents the nature of the state finder parameters r and s, which do not tend to 0 and 1, and this suggests that $f(\widetilde{R},T)$ gravity, in this case, does not satisfy the ΛCDM model.

3.5 Conclusion

In this chapter, the equation indicates the spatially homogenous and anisotropic cosmological model with an attractive massive scalar field (3.25). The model is nonsingular, shearing, and non rotating. The volume increases exponentially concerning cosmic time (t), which indicates that the universe starts expanding with finite volume from the infinite past. The displacement vector field in eq. (3.33) gradually increases from a small negative value to a positive value at an initial time and begins with large positive values for $\gamma = -1.5$. This model expands with acceleration for a significant role of $f(R,T)$ gravity with displacement vector field β. It is clear that the present expanding universe model is anisotropic, and the ΛCDM model is not found as in recent observational data.