Chapter 4

Bianchi Type-V Cosmological Model with Heat Conduction in Lyra Geometry

4.1 Introduction

This chapter deals with the study of Bianchi type-V DE in $f(R,T)$ gravity based on Lyra geometry with heat conduction. It is known that matter does not achieve thermal equilibrium in the current cosmological model cosmology [Reboucas (19820]. As a result, heat would circulate throughout the universe. So far, it has been found that various writers have investigated the effect of heat flow in different approaches of models [Roy & Prasad (1994), Singh (2009), Ram et al. (2008, 2009), Singh (2007)], where we obtained that the viscosity and heat flow produced using Bianchi type-V cosmologies. Hatkar et al. (2019) discussed the cosmological model with bulk viscosity and heat flux in a Bianchi type-III modified theory of gravitation. Currently, many modified theories of general relativity are used to examine the negative pressure (Dark Energy) of the model [$f(R)$, $f(G)$, $f(T)$, $f(R,T)$ gravity]. In this chapter, it is investigated the effect of the heat flow of the model in Bianchi type-V with the occurrence of the most well-known modification of Harko et al. (2011), known as $f(R,T)$ gravity, in which the gravitational Lagrangian contains arbitrary functions of R and T, which denotes the Ricci scalar-tensor and Trace of the EMT, respectively.

4.2 Metric and the field equations of $f(R,T)$ gravity

Let us consider the Bianchi type-V space time in the form

$$ds^2 = -dt^2 + A^2 dx^2 + e^{-2mx}(B^2 dy^2 + C^2 dz^2) \tag{4.1}$$

where A, B, C are functions of cosmic time t alone and m is a constant.
The EMT of the matter with heat conduction is consider as

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij} + h_i u_j + h_j u_i \tag{4.2}$$

Here ρ and p are denotes the energy density and thermodynamic pressure of the matter. On the other hand $u^i = (0,0,0,1)$ is the four velocity vector in co-moving co-ordinate system satisfying the condition $u_i u^i = -1$.
In equation (4.2), h_i represents the heat flow vector of the model in x direction only such that $h_i = (h_1, 0, 0, 0)$ is a function of cosmic time t, which satisfying the condition

$$h^i u_i = 0 \tag{4.3}$$

For the line element (4.1), in view of energy momentum tensor (4.2), [assuming that $\alpha = (\frac{8\pi G - \mu c^2}{\mu c^2})$ is a constant], the EFE (1.21), reduces to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 = -\alpha p + \left(\frac{\rho - p}{2}\right) \tag{4.4}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{C}\dot{A}}{CA}-\frac{m^2}{A^2}+\frac{3}{4}\beta^2 p=-\alpha p+\left(\frac{\rho-p}{2}\right) \tag{4.5}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{m^2}{A^2}+\frac{3}{4}\beta^2=-\alpha p+\left(\frac{\rho-p}{2}\right) \tag{4.6}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}-\frac{3m^2}{A^2}-\frac{3}{4}\beta^2=\alpha\rho+\left(\frac{\rho-p}{2}\right) \tag{4.7}$$

$$m\left(2\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)=h_1 \tag{4.8}$$

The energy conservation equation $T^i_j u^j=0$ takes the form

$$\dot{\rho}+\frac{3}{2}\beta\dot{\beta}+\left[(\rho+p)+\frac{3}{2}\beta^2\right]\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=\frac{2m}{A^2}h_1 \tag{4.9}$$

4.3 Results and Discussion of the cosmological solutions of the field equations

4.3.1 Case-I: Solutions with heat conduction (i.e. $h_1 \neq 0$)

In solving the above field equations (4.4)-(4.8), the following physical parameters are very important and these parameters are defined as follows:

The spatial volume is given by

$$V=a(t)^3=ABC \tag{4.10}$$

The generalized mean HP is defined as

$$H=\frac{1}{3}(H_1+H_2+H_3) \tag{4.11}$$

where the symbols are already defined in introduction section.

Then, from eqs. (4.10) and (4.11), it is obtained that the relation between the HP and the average scale factor as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{4.12}$$

The scalar expansion, shear expansion and the anisotropy parameter are defined as

$$\theta = 3H = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \tag{4.13}$$

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2 - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{C}\dot{A}}{CA}\right] \tag{4.14}$$

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{4.15}$$

The projection tensor P_{ij} of the model has the form given by

$$P_{ij} = g_{ij} - u_i u_j \tag{4.16}$$

In the EFE (4.4) - (4.8), there are five highly non-linear differential equations consisting of six unknown variables, namely A, B, C, p, ρ, and β. In order to find out these six unknowns, another two conditions are assumed as follows:

(i) The scalar expansion of the model is proportional to the shear scalar [i.e. $\frac{\sigma}{\theta}$ = constant], which yields that [Collins et al. (1980)]

$$C = B^n \tag{4.17}$$

where $n \neq 1$ is a positive constant that preserves the anisotropy of space-time (Thorne (1967).

(ii) To find the deterministic solution of the model, it is assumed that the relation between the metric potentials as

$$A = (BC)^l \tag{4.18}$$

where l is a positive constant.

The scale factor [Dewri (2020)], is assumed as

$$a(t) = \alpha_2 \left(2\sqrt{t} - \alpha_1\right)^2 e^{\sqrt{t}} \tag{4.19}$$

From eqs. (4.10), (4.17) and (4.18), we obtain

$$A = a^{\frac{3l}{l+1}} = \left[\alpha_2 \left(2\sqrt{t} - \alpha_1\right)^2 e^{\sqrt{t}}\right]^{\frac{3l}{l+1}} \tag{4.20}$$

$$BC = \left[\alpha_2 \left(2\sqrt{t} - \alpha_1\right)^2 e^{\sqrt{t}}\right]^{\frac{3}{l+1}} \tag{4.21}$$

For convenience, this relation is written as

$$B = \left[\alpha_2 \left(2\sqrt{t} - \alpha_1\right)^2\right]^{\frac{3}{l+1}} \tag{4.22}$$

and

$$C = \left[e^{\sqrt{t}}\right]^{\frac{3l}{l+1}} \tag{4.23}$$

Using eq. (4.20) in (4.10), we obtained the volume as

$$V = [\alpha_2 \left(2\sqrt{t} - \alpha_1\right)^2 e^{\sqrt{t}}]^3 \tag{4.24}$$

Also, the other dynamical scalars which are obtained from eqs. (4.11)-(4.15) is given by

$$H = \frac{4 + 2\sqrt{t} - \alpha_1}{2\sqrt{t}(2\sqrt{t} - \alpha_1)} \tag{4.25}$$

$$\theta = \frac{3(4 + 2\sqrt{t} - \alpha_1)}{2\sqrt{t}(2\sqrt{t} - \alpha_1)} \tag{4.26}$$

$$\sigma^2 = \frac{1}{2}\left(-1 + \frac{n^2+1}{l^2(n+1)^2}\right)\left[\frac{4 + 2\sqrt{t} - \alpha_1}{2\sqrt{t}(2\sqrt{t} - \alpha_1)}\right]^2 \tag{4.27}$$

$$\Delta = \frac{1}{3}\left[\left(\frac{1}{l(n+1)} - 1\right)^2 + \left(\frac{n}{l(n+1)} - 1\right)^2\right] \tag{4.28}$$

$$q = -1 - \frac{2\left(2\sqrt{t} - \alpha_1\right)}{\left(4 + 2\sqrt{t} - \alpha_1\right)^2} + \frac{4\sqrt{t} - \alpha_1}{\sqrt{t}\left(4 + 2\sqrt{t} - \alpha_1\right)} \tag{4.29}$$

It is observed that the metric potentials A, B, C are increasing functions of cosmic time t,

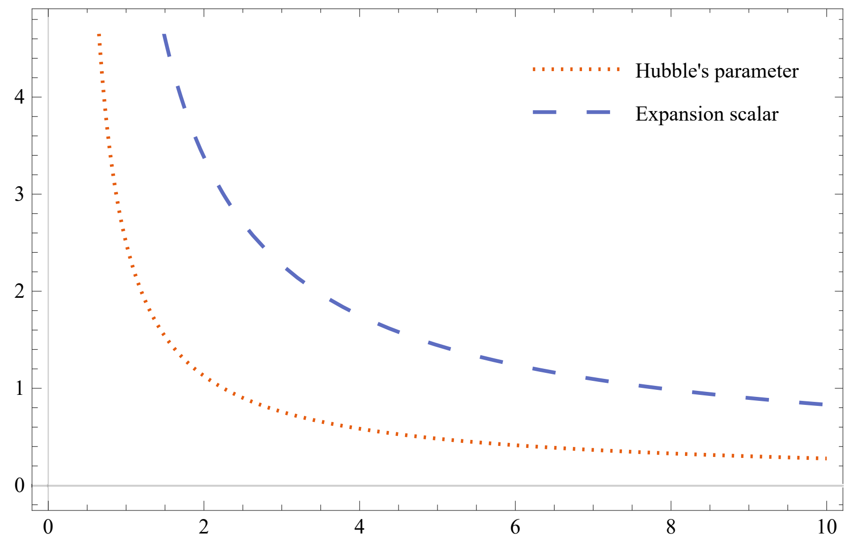

Figure 4.1: Variation of H, θ vs. t

as shown by eqs. (4.20)-(4.23). The metric potentials tend to be a constant value as $t \to \infty$ grows. The metric potentials do not exist as $t \to \infty$, i.e., as $t \to \infty$; all of them tend to infinity, indicating that the model is consistent with the Big Bang hypothesis. However, the metric potentials increase intriguingly with a minimal positive value as time passes. The model's spatial volume also has a constant value for $t \to 0$, but it goes to infinity as $t \to \infty$ approaches. The volume of the $f(\widetilde{R}, T)$ gravity model always increases as cosmic time increases, as given in eq. (4.24).

From Eqs. (4.25) and (4.26) it can be seen that as $t \to 0$ approaches, H and θ always diverge, and as $t \to \infty$ approaches, H and θ approaches zero, as shown in Fig (4.1). The Shear scalar vanishes as $t \to 0$ and diverges at $t \to \infty$, as shown in Eqs. (4.27). In this study, it is obtained that eq. (4.28) never tends to zero with cosmic time t, indicating that

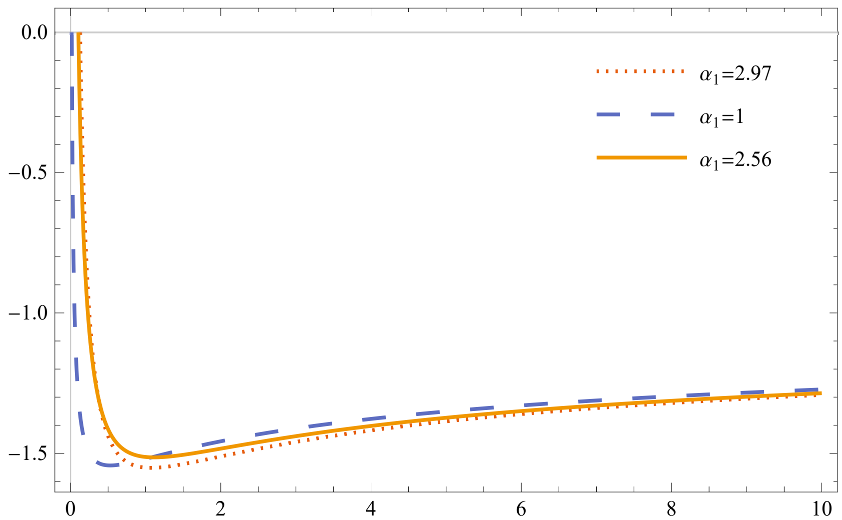

Figure 4.2: Variation of q vs. t

our model is anisotropic in the current model of the universe. It is clear from Eq. (4.29) that for $t \to 0$, the DP always lies between 0 and -1, preserving the model's accelerating universe in the framework of Lyra geometry in $f(R,T)$ gravity, as in Fig (4.2), by the correct choice of constants, such as $\alpha_1 = \alpha_2 = 1$, $l = 2$, $m = 2$, $n = 4$, $\alpha = 5$, $\gamma = -0.5$. As a result of the above explanation, it can be seen that all of the model's metric potentials A, B, C, and its volume are constant at $t \to 0$, indicating that our model is free of initial singularity.

Now using eqs. (4.20)-(4.23) in the metric (4.1), we obtain

$$ds^2 = -dt^2 + \left[\alpha_2\left(2\sqrt{t}-\alpha_1\right)^2 e^{\sqrt{t}}\right]^{\frac{6l}{l+1}} dx^2 + \left[\alpha_2\left(2\sqrt{t}-\alpha_1\right)^2 e^{\sqrt{t}}\right]^{\frac{6}{l+1}} e^{-2mx} dy^2 + \left[e^{\sqrt{t}}\right]^{\frac{6l}{l+1}} dz^2 \tag{4.30}$$

provided $l \neq -1$. This equation indicates the spatially homogeneous and anisotropic Kantowski - Sachs DE cosmological model of the present model of universe.

Adding eqs. (4.4) -(4.6), we get

$$\alpha p = (2q-1)H^2 - \sigma^2 + \frac{m^2}{A^2} - \frac{3}{4}\beta^2 + \frac{1}{2}(\rho - p) \tag{4.31}$$

Also eq. (4.31) reduces to

$$\alpha\rho = 3H^2 - \sigma^2 - \frac{3m^2}{A^2} - \frac{3}{4}\beta^2 - \frac{1}{2}(\rho - p) \tag{4.32}$$

Using eqs. (4.20), (4.25) and (4.29) in eq. (4.32), it is obtained that by considering the relation of equation of state parameter $p = \gamma\rho$, to determine the extensive behavior of pressure and density of the model as

$$\begin{aligned}\rho = C_1 &\left[-\frac{3}{2}\frac{\left(4+2\sqrt{t}-\alpha_1\right)^2}{t\left(2\sqrt{t}-\alpha_1\right)^2} - \frac{1}{t\left(2\sqrt{t}-\alpha_1\right)} + \frac{\left(4+2\sqrt{t}-\alpha_1\right)\left(4\sqrt{t}-\alpha_1\right)}{2t\sqrt{t}\left(2\sqrt{t}-\alpha_1\right)^2}\right] \\ &+ \frac{4m^2C_1}{\left[\alpha_2(2\sqrt{t}-\alpha_1)^2 e^{\sqrt{t}}\right]^{\frac{6l}{l+1}}}\end{aligned} \tag{4.33}$$

Then the pressure of model is obtained as

$$\begin{aligned}p = C_1\gamma &\left[-\frac{3}{2}\frac{\left(4+2\sqrt{t}-\alpha_1\right)^2}{t\left(2\sqrt{t}-\alpha_1\right)^2} - \frac{1}{t\left(2\sqrt{t}-\alpha_1\right)} + \frac{\left(4+2\sqrt{t}-\alpha_1\right)\left(4\sqrt{t}-\alpha_1\right)}{2t\sqrt{t}\left(2\sqrt{t}-\alpha_1\right)^2}\right] \\ &+ \frac{4m^2C_1\gamma}{\left[\alpha_2(2\sqrt{t}-\alpha_1)^2 e^{\sqrt{t}}\right]^{\frac{6l}{l+1}}}\end{aligned} \tag{4.34}$$

where $C_1 = \frac{1}{(\gamma-1)(\alpha+1)}$ is a constant, which bears a important role in this study by considering different values it shows the different behavior of the $f(\widetilde{R},T)$ gravity model.

As $t \to 0$ approaches infinity, the model's energy density and pressure tend to infinity, as shown in Eqs. (4.33) and (4.34), indicating that the universe evolved from a singularity point. The density and pressure of the $f(\widetilde{R},T)$ model reach zero as time progresses, as in Fig. (4.3). It's worth noting that the $\lim_{t\to 0}(\frac{\rho}{\theta^2})$ spread out is constant in this case. As

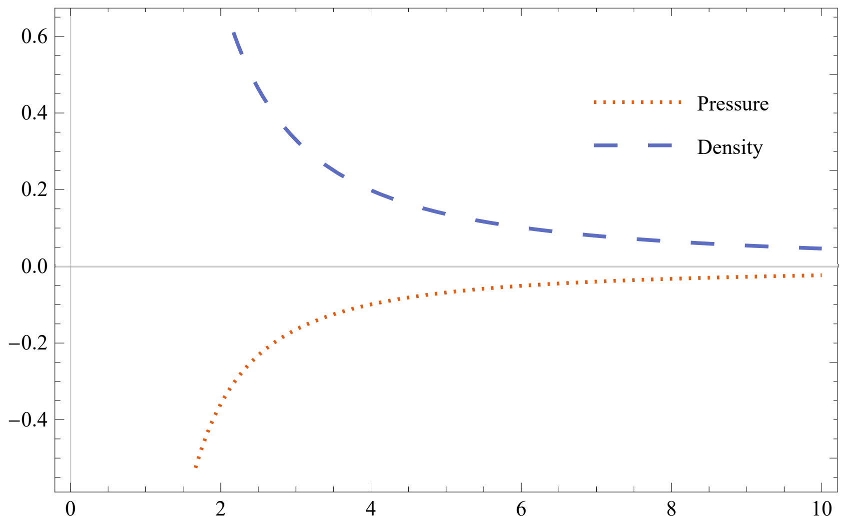

Figure 4.3: Variation of p and ρ vs t

a result, the universe's model ascends inhomogeneity, and matter becomes dynamically negligible near the origin. Collins (1977) earlier came up with a similar finding.

The displacement field vector of $f(\widetilde{R},T)$ gravity model is obtained from eq. (4.35) as given by

$$
\begin{aligned}
(1-\gamma)\frac{3}{4}\beta^2 = &-3\left[\frac{4+2\sqrt{t}-\alpha_1}{2\sqrt{t}(2\sqrt{t}-\alpha_1)}\right]^2 - \frac{1}{t\left(2\sqrt{t}-\alpha_1\right)} + \frac{\left(4+2\sqrt{t}-\alpha_1\right)\left(4\sqrt{t}-\alpha_1\right)}{2t\sqrt{t}\left(2\sqrt{t}-\alpha_1\right)^2} \\
&-3\left[\frac{4+2\sqrt{t}-\alpha_1}{2\sqrt{t}(2\sqrt{t}-\alpha_1)}\right]^2 - \frac{1}{2}(1-\gamma)\left[-1+\frac{n^2+1}{l^2(n+1)^2}\right]\left[\frac{4+2\sqrt{t}-\alpha_1}{2\sqrt{t}(2\sqrt{t}-\alpha_1}\right] \\
&+\frac{m^2(1+3\gamma)}{\left[\alpha_2(2\sqrt{t}-\alpha_1)^2 e^{\sqrt{t}}\right]^{\frac{6l}{l+1}}} + \frac{(\gamma+1)}{2(\alpha+1)}\left[\frac{3(4+2\sqrt{t}-\alpha_1)^2}{2(2\sqrt{t}-\alpha_1)^2} + \frac{1}{t\left(2\sqrt{t}-\alpha_1\right)}\right] \\
&-\frac{(\gamma+1)}{2(\alpha+1)}\left[\frac{\left(4+2\sqrt{t}-\alpha_1\right)\left(4\sqrt{t}-\alpha_1\right)}{2t\sqrt{t}\left(2\sqrt{t}-\alpha_1\right)^2} + \frac{4m^2}{\left[\alpha_2(2\sqrt{t}-\alpha_1)^2 e^{\sqrt{t}}\right]^{\frac{6l}{l+1}}}\right]
\end{aligned}
\tag{4.35}
$$

Using eqs. (4.20) - (4.23) in (4.8), we obtained the heat conduction of Lyra geometry

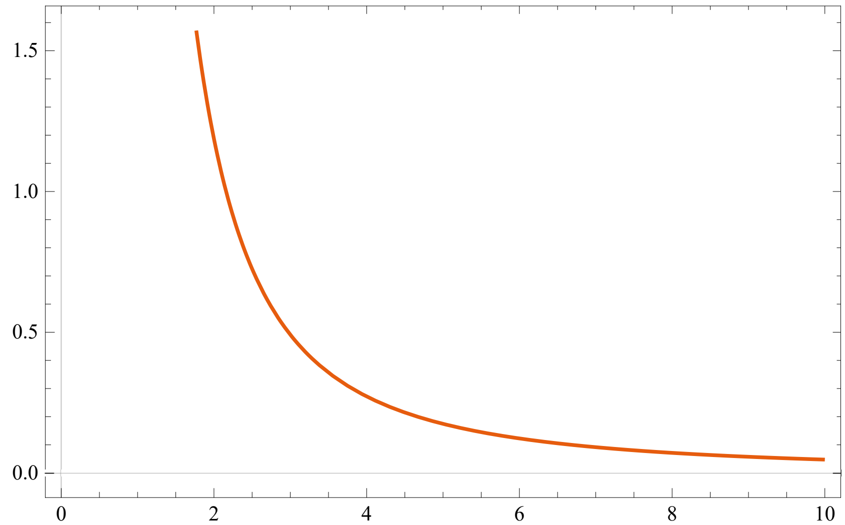

Figure 4.4: Variation of β^2 vs. t

is

$$h_1 = \frac{3m(2m-1)}{m+1}\left[\frac{4+2\sqrt{t}-\alpha_1}{2\sqrt{t}(2\sqrt{t}-\alpha_1)}\right] \tag{4.36}$$

When $t = 0$, the displacement vector and the heat conduction (h_1) both diverge [Fig.

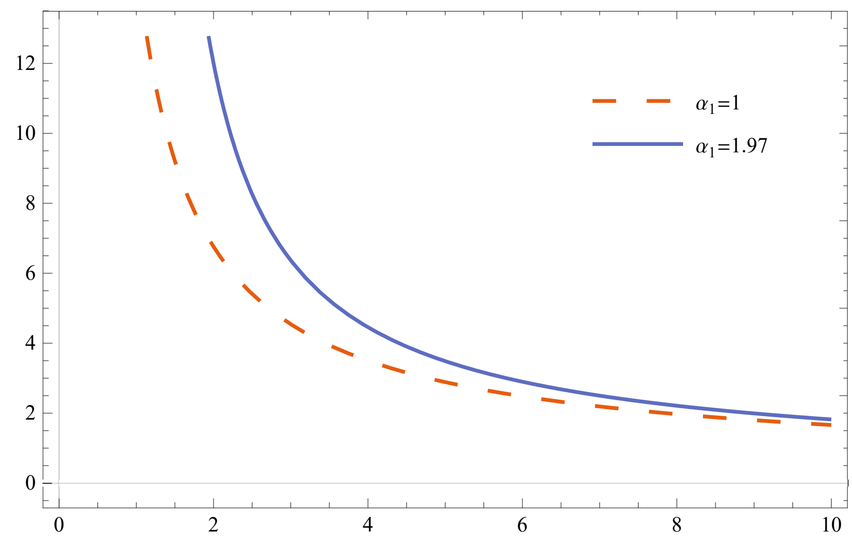

Figure 4.5: Variation of h_1 vs. t

(4.4) and Fig. (4.5)]. However, the displacement vector and the heat conduction may eventually vanish as time passes. Heat conduction decreases with time and is greatest in the beginning epoch. It is observed that heat flow is maximum along the direction of the x axis in the early universe.

The trace and the Ricci Scalar tensor of $f(\widetilde{R},T)$ gravity are obtained from the above eqs. as

$$\begin{aligned}
T &= \rho - 3p \\
&= \frac{1-3\gamma}{(\gamma-1)(\alpha+1)}\left[-\frac{3}{2}\frac{\left(4+2\sqrt{t}-\alpha_1\right)^2}{t\left(2\sqrt{t}-\alpha_1\right)^2} - \frac{1}{t\left(2\sqrt{t}-\alpha_1\right)}\right] \\
&+ \frac{1-3\gamma}{(\gamma-1)(\alpha+1)}\left[\frac{\left(4+2\sqrt{t}-\alpha_1\right)\left(4\sqrt{t}-\alpha_1\right)}{2t\sqrt{t}\left(2\sqrt{t}-\alpha_1\right)^2} + \frac{4m^2}{\left[\alpha_2(2\sqrt{t}-\alpha_1)^2 e^{\sqrt{t}}\right]^{\frac{6l}{l+1}}}\right]
\end{aligned} \tag{4.37}$$

and

$$\begin{aligned}
R &= \frac{6m^2}{A^2} - 2\left(\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}\right) - 2\left(\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{B}}{B}\frac{\dot{C}}{C}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}\right) \\
&= \frac{6m^2}{\left[\alpha_2(2\sqrt{t}-\alpha_1)^2 e^{\sqrt{t}}\right]^{\frac{6l}{l+1}}} - \frac{3}{t\left(2\sqrt{t}-\alpha_1\right)} + \frac{6\left(4+2\sqrt{t}-\alpha_1\right)\left(4\sqrt{t}-\alpha_1\right)}{\sqrt{t}\left(4t-2\alpha_1\sqrt{t}\right)^2} \\
&- \frac{C_2\left(2\sqrt{t}-\alpha_1+4\right)^2}{4t\left(2\sqrt{t}-\alpha_1\right)^2} - \frac{C_3\left(2\sqrt{t}-\alpha_1+4\right)^2}{4t\left(2\sqrt{t}-\alpha_1\right)^2}
\end{aligned} \tag{4.38}$$

where $C_2 = \frac{18\left(n^2-ln^2-2ln-l^2-2l+1\right)}{(l+1)^2(n+1)^2}$ and $C_3 = \frac{18\left(ln^2+2ln+l+n\right)}{(l+1)^2(n+1)^2}$ both are constants.

The Ricci scalar-tensor R and the model's trace initially expand the rate and eventually vanish for large values of t, as shown in Fig. (4.6).

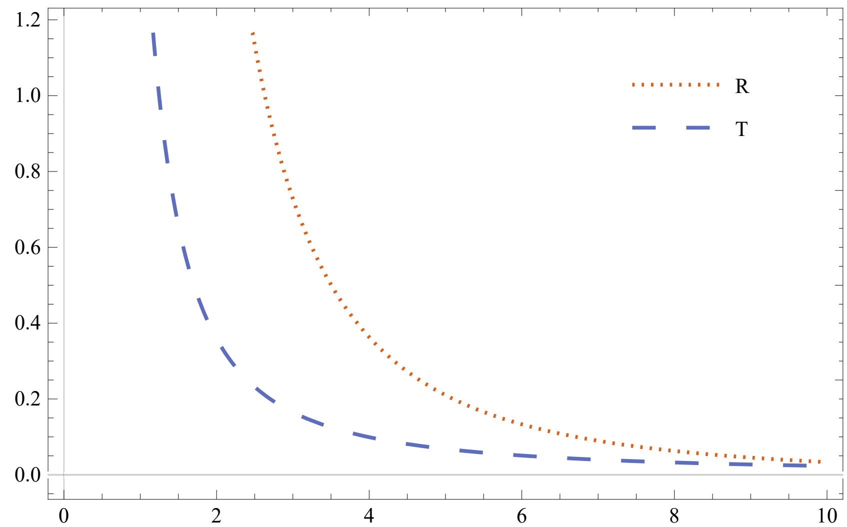

Figure 4.6: Variation of R and T vs. t

4.3.2 Case-II: Solutions without heat conduction (i.e. $h_1 = 0$)

In this case, integrating eq. (4.8), it gives

$$A^2 = kBC \tag{4.39}$$

Here $k = 1$ can be taken. Then

$$A^2 = BC \tag{4.40}$$

In this scenario the metric potentials are obtain as by taking the condition ($C = B^n$)

$$A(t) = \alpha_2 \left(2\sqrt{t} - \alpha_1\right)^2 e^{\sqrt{t}} \tag{4.41}$$

$$B(t) = \left[\alpha_2 \left(2\sqrt{t} - \alpha_1\right)^2 e^{\sqrt{t}}\right]^{\frac{2}{n+1}} \tag{4.42}$$

$$C(t) = \left[\alpha_2 \left(2\sqrt{t} - \alpha_1\right)^2 e^{\sqrt{t}}\right]^{\frac{2n}{n+1}} \tag{4.43}$$

Now using eqs. (4.41)-(4.43) in the metric (4.1), we obtained the model in this case as

$$\begin{aligned} ds^2 = & -dt^2 + \left(\alpha_2\left(2\sqrt{t}-\alpha_1\right)^2 e^{\sqrt{t}}\right)^2 dx^2 + e^{-2mx}\left(\alpha_2\left(2\sqrt{t}-\alpha_1\right)^2 e^{\sqrt{t}}\right)^{\frac{4}{n+1}} dy^2 \\ & + e^{-2mx}\left(\alpha_2\left(2\sqrt{t}-\alpha_1\right)^2 e^{\sqrt{t}}\right)^{\frac{4}{n+1}} dy^2 \end{aligned} \tag{4.44}$$

provided $n \neq -1$.

Also, the physical and the dynamical parameters of this present case of the model are given by

Spatial Volume:

$$V(t) = \left[\alpha_2\left(2\sqrt{t}-\alpha_1\right)^2 e^{\sqrt{t}}\right]^3 \tag{4.45}$$

Hubble's Parameter:

$$H = \frac{4+2\sqrt{t}-\alpha_1}{2\sqrt{t}(2\sqrt{t}-\alpha_1)} \tag{4.46}$$

Expansion Scalar:

$$\theta = 3\left(\frac{4+2\sqrt{t}-\alpha_1}{2\sqrt{t}(2\sqrt{t}-\alpha_1)}\right) \tag{4.47}$$

Shear Scalar:

$$\sigma^2 = \left(\frac{n-1}{n+1}\right)^2\left(\frac{4+2\sqrt{t}-\alpha_1}{2\sqrt{t}(2\sqrt{t}-\alpha_1)}\right)^2 \tag{4.48}$$

Anisotropy parameter:

$$A_m = \frac{2}{3}\left(\frac{n-1}{n+1}\right)^2 \tag{4.49}$$

Deceleration parameter:

$$q = -1 - \frac{2\left(2\sqrt{t}-\alpha_1\right)}{\left(4+2\sqrt{t}-\alpha_1\right)^2} + \frac{4\sqrt{t}-\alpha_1}{\sqrt{t}\left(4+2\sqrt{t}-\alpha_1\right)} \tag{4.50}$$

The pressure, energy density and the displacement field vector are respectively obtain [eqs. (4.4) - (4.7)]

$$\rho = C_1\left[-\frac{3}{2}\frac{\left(4+2\sqrt{t}-\alpha_1\right)^2}{t\left(2\sqrt{t}-\alpha_1\right)^2}-\frac{1}{t\left(2\sqrt{t}-\alpha_1\right)}+\frac{\left(4+2\sqrt{t}-\alpha_1\right)\left(4\sqrt{t}-\alpha_1\right)}{2t\sqrt{t}\left(2\sqrt{t}-\alpha_1\right)^2}\right] + \frac{4m^2C_1}{\left(\alpha_2(2\sqrt{t}-\alpha_1)^2e^{\sqrt{t}}\right)^2} \tag{4.51}$$

$$p = C_1\gamma\left[-\frac{3}{2}\frac{\left(4+2\sqrt{t}-\alpha_1\right)^2}{t\left(2\sqrt{t}-\alpha_1\right)^2}-\frac{1}{t\left(2\sqrt{t}-\alpha_1\right)}+\frac{\left(4+2\sqrt{t}-\alpha_1\right)\left(4\sqrt{t}-\alpha_1\right)}{2t\sqrt{t}\left(2\sqrt{t}-\alpha_1\right)^2}\right] + \frac{4m^2C_1\gamma}{\left(\alpha_2(2\sqrt{t}-\alpha_1)^2e^{\sqrt{t}}\right)^2} \tag{4.52}$$

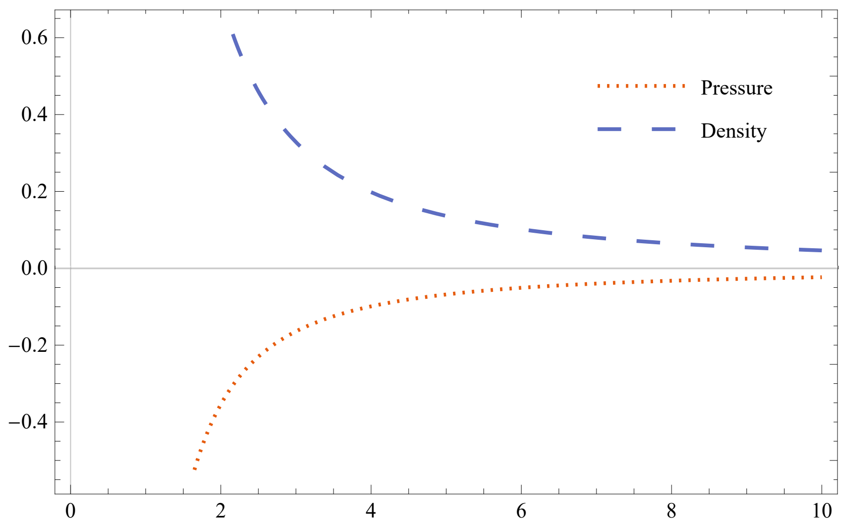

Figure 4.7: Variation of ρ and p vs. t

$$(1-\gamma)\frac{3}{4}\beta^2 = -3\left[\frac{4+2\sqrt{t}-\alpha_1}{2\sqrt{t}(2\sqrt{t}-\alpha_1)}\right]^2 - \frac{1}{t\left(2\sqrt{t}-\alpha_1\right)} + \frac{\left(4+2\sqrt{t}-\alpha_1\right)\left(4\sqrt{t}-\alpha_1\right)}{2t\sqrt{t}\left(2\sqrt{t}-\alpha_1\right)^2}$$
$$-3\gamma\left[\frac{4+2\sqrt{t}-\alpha_1}{2\sqrt{t}(2\sqrt{t}-\alpha_1)}\right]^2 - (1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(\frac{4+2\sqrt{t}-\alpha_1}{2\sqrt{t}(2\sqrt{t}-\alpha_1)}\right)^2$$
$$+\frac{m^2(1+3\gamma)}{\left[\alpha_2(2\sqrt{t}-\alpha_1)^2e^{\sqrt{t}}\right]^2} + \frac{(\gamma+1)}{2(\alpha+1)}\left[\frac{3(4+2\sqrt{t}-\alpha_1)^2}{2t(2\sqrt{t}-\alpha_1)^2} + \frac{1}{t\left(2\sqrt{t}-\alpha_1\right)}\right]$$
$$-\frac{(\gamma+1)}{2(\alpha+1)}\left[\frac{\left(4+2\sqrt{t}-\alpha_1\right)\left(4\sqrt{t}-\alpha_1\right)}{2t\sqrt{t}\left(2\sqrt{t}-\alpha_1\right)^2} + \frac{4m^2}{\left(\alpha_2(2\sqrt{t}-\alpha_1)^2e^{\sqrt{t}}\right)^2}\right] \tag{4.53}$$

Similarly, from the above eqs., the trace and the Ricci Scalar tensor, in this case, are

$$T = \frac{1-3\gamma}{(\gamma-1)(\alpha+1)}\left[-\frac{3}{2}\frac{\left(4+2\sqrt{t}-\alpha_1\right)^2}{t\left(2\sqrt{t}-\alpha_1\right)^2} - \frac{1}{t\left(2\sqrt{t}-\alpha_1\right)}\right]$$
$$+\frac{1-3\gamma}{(\gamma-1)(\alpha+1)}\left[\frac{\left(4+2\sqrt{t}-\alpha_1\right)\left(4\sqrt{t}-\alpha_1\right)}{2t\sqrt{t}\left(2\sqrt{t}-\alpha_1\right)^2} + \frac{4m^2}{\left(\alpha_2(2\sqrt{t}-\alpha_1)^2e^{\sqrt{t}}\right)^2}\right] \tag{4.54}$$

and

$$R = \frac{6m^2}{\left[\alpha_2(2\sqrt{t}-\alpha_1)^2e^{\sqrt{t}}\right]^2} - \frac{3}{t\left(2\sqrt{t}-\alpha_1\right)} + \frac{3\left(4+2\sqrt{t}-\alpha_1\right)\left(4\sqrt{t}-\alpha_1\right)}{t\sqrt{t}\left(2\sqrt{t}-\alpha_1\right)^2}$$
$$-\frac{3}{2}\frac{\left(4+2\sqrt{t}-\alpha_1\right)^2}{t\left(2\sqrt{t}-\alpha_1\right)^2} - \frac{n(n-1)}{(n+1)^2}\frac{\left(4+2\sqrt{t}-\alpha_1\right)^2}{t\left(2\sqrt{t}-\alpha_1\right)^2} - \frac{(n^2+4n+2)}{(n+1)^2}\frac{\left(4+2\sqrt{t}-\alpha_1\right)^2}{t\left(2\sqrt{t}-\alpha_1\right)^2} \tag{4.55}$$

In this model of case, the metric potentials $A(t)$, $B(t)$, and $C(t)$ show that they are a positive and increasing function of cosmic time (t), and the model is expanding (i.e., $\theta \geq 0$) as in case-I also. The model starts expanding with Big-Bang at $t = 0$, which shows a point singularity of the model. It is found that the pressure, energy density, displacement field vector, Ricci scalar, and a trace of the energy-momentum tensor all vanish for $t \to \infty$ in both cases. As a result, the large-scale predictions of the universe's model would be virtually empty. The rate of expansion of the model stops when for $t \to \infty$. As $\frac{d\theta}{dt} < 0$, which indicates that the model begins expanding from its singular state and the rate of expansion decreases to 0 for $t \to \infty$.

The ECs play a vital role in understanding the geodesics of the universe. These conditions are derived from the well-known Raychaudhuri equations. Recently, Sahoo et al. (2020) has derived the for a perfect fluid matter distribution to check the ECs in the present model of universe as given by

- Strong energy conditions (SEC): $\rho + 3p \geq 0$;
- Weak energy conditions (WEC): $\rho \geq 0,\ \ \rho + p \geq 0$;
- Null energy condition (NEC): $\rho + p \geq 0$;
- Dominant energy conditions (DEC): $\rho \geq 0$, $|p| \leq \rho$.

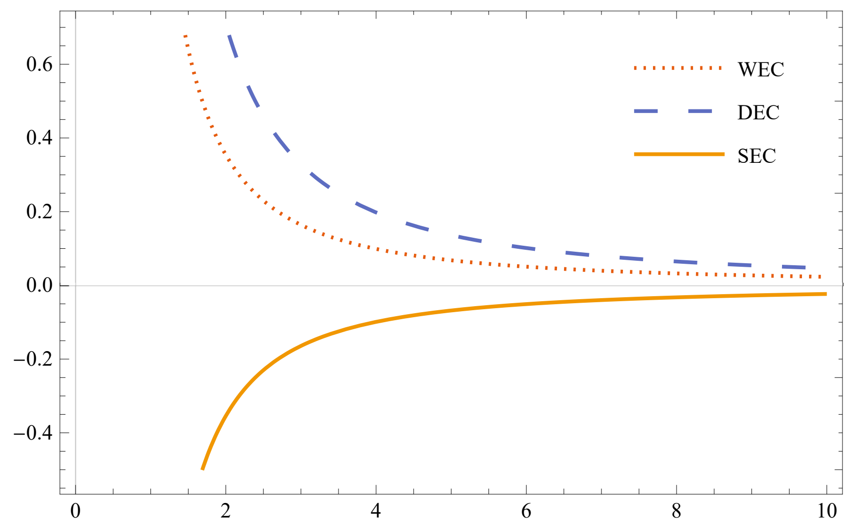

Figure 4.8: Variation of ECs vs. t

The above ECs are identically satisfied provided $\gamma \neq 1$ and $\alpha \neq -1$ for both cases. It is clear from Fig. 4.8 that WEC, NEC, and DEC are satisfied, while SEC violated the present value of choice, which agrees with the current scenario of the universe.

Statefinder parameter: At the present stage of cosmic time acceleration, we found that many DE models have been constructed in the universe's accelerated expansion to examine the same phenomenon. As a result, we need better relationships between these two

DE models to explain the current stage of cosmic time acceleration. Furthermore, many DE models almost bear the same result among deceleration and HP. So we can not predict the extensive DE model, i.e., it is challenging to differentiate the DE model completely. To remove this critical problem, two new dimensionless parameters have been introduced by Sahni et al. (2003), which term as State finder parameters and it is the second and third derivatives of the radius scale factor as defined by

$$r = \frac{\dddot{a}}{a\dot{H}} = 1 + 3\frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3} \tag{4.56}$$

and,

$$s = \frac{r-1}{3(q-\frac{1}{2})} \tag{4.57}$$

where, the symbols have their usual meaning. From eq. (4.39), it is obtained that the

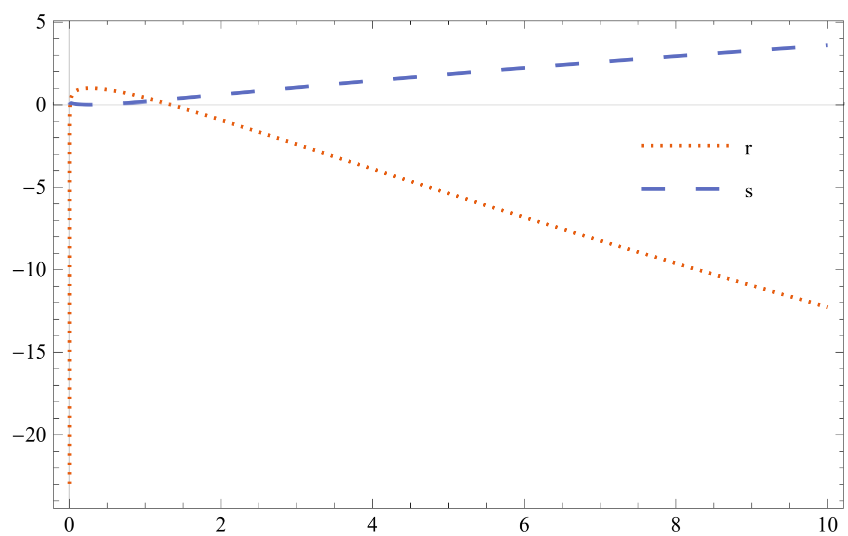

Figure 4.9: Variation of r and s vs. t

statefinder parameters are as follows:

$$\begin{aligned} r = 1 &+ \frac{6(2\sqrt{t}-\alpha_1)}{(4+2\sqrt{t}-\alpha_1)^2} - \frac{6\sqrt{t}(4\sqrt{t}-\alpha_1)(2\sqrt{t}-\alpha_1)}{(4+2\sqrt{t}-\alpha_1)^2} \\ &- \frac{4(2\sqrt{t}-\alpha_1)^2}{\sqrt{t}(4+2\sqrt{t}-\alpha_1)^3} - \frac{8(2\sqrt{t}-\alpha_1)}{(4+2\sqrt{t}-\alpha_1)^3} \end{aligned} \tag{4.58}$$

and

$$s=\frac{\left[\frac{6(2\sqrt{t}-\alpha_1)}{(4+2\sqrt{t}-\alpha_1)^2}-\frac{6\sqrt{t}(4\sqrt{t}-\alpha_1)(2\sqrt{t}-\alpha_1)}{(4+2\sqrt{t}-\alpha_1)^2}-\frac{4(2\sqrt{t}-\alpha_1)^2}{\sqrt{t}(4+2\sqrt{t}-\alpha_1)^3}-\frac{8(2\sqrt{t}-\alpha_1)}{(4+2\sqrt{t}-\alpha_1)^3}\right]}{3\left[-\frac{3}{2}-\frac{2(2\sqrt{t}-\alpha_1)}{(4+2\sqrt{t}-\alpha_1)^2}+\frac{4\sqrt{t}-\alpha_1}{\sqrt{t}(4+2\sqrt{t}-\alpha_1)}\right]} \tag{4.59}$$

4.4 Conclusion

By using the condition of Harko et al. (2011) as $f(\widetilde{R},T)=f_1(\widetilde{R})+f_2(T)$, we have provided the EFE of Bianchi type -V DE in $f(R,T)$ gravity based on Lyra geometry with heat conduction. Using an ad-hoc relationship among the metric potentials, It is obtained that our model is free of initial singularity, with all metric potentials A,B,C, and the volume remaining constant for $t\to 0$. In this scenario, the universe starts to expand quickly from the beginning and gradually shrinks to a significant value of t. With cosmic time t, the expanding universe also shows anisotropy. It is found that the universe's model ascends inhomogeneity, and matter becomes dynamically negligible near the origin. Also, it is observed that heat flow is maximum along the direction of the x axis in the early universe. The temperature and heat conduction have the greatest values during the early phases of expansion and are diminishing functions of cosmic time t. It is shown that the pressure of the model is always negative, indicating that there is DE in our current $f(\widetilde{R},T)$ model in both cases, of which we are investigating the main behavior. This result indicates that our model is an accelerated phase of the universe based on Lyra geometry with and without heat conduction in the $f(R,T)$ gravity model. Furthermore, the energy density will always be positive with cosmic time t and eventually zero as time increases. It is shown that the WEC, NEC, and DEC are satisfied, while SEC violated the present value of choice for the case of with and without heat conduction, which bears a good agreement with the current scenario of the universe. We also found that the state finder parameters r and s tend to be 0 and 1, respectively, indicating that our model approaches ΛCDM, as shown in Fig. (4.8).