Chapter 5

Bianchi Type-V Dark Energy model in Lyra Geometry in presence of Magnetic Field

5.1 Introduction

This chapter deals with the study of Bianchi type-V DE model with electromagnetic field based on Lyra Geometry. The study of Bianchi Type-V cosmological models attracts attention since they include particular isotropic instances and allow for arbitrary tiny anisotropy at some point in cosmic time. As demonstrated by numerous high-redshift supernovae discoveries, the early universe is expanding at a faster rate than previously anticipated [Riess et al. (1998), Perlmutter et al. (1999), Bennett et al. (2003), Peebles & Ratra (2003)]. In addition to these tests, observations such as CMB radiations and LSS [Spergel et al. (2003), Hinshaw et al. (2007), Tegmark et al. (2004)] suggest that the universe is expanding at a quicker rate. Because of their capacity to explain the observed

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faster expansion of the universe, modified gravity theories have sparked much attention in recent years. The essential theories among them are $f(R)$, $f(G)$, $f(T)$, and $f(R,T)$ gravity [Sarmah et al. (2022), Sharif & Saba (2020), Bamba et al. (2017), Sahoo et al. (2016), Nath & Sahu (2019), Singh & Beesham (2020), Tiwari, R. K. et al. (2021), Arora et al. (2021)].

The most prevailing significant theory in a current cosmological model of the universe is $f(R,T)$, where the Lagrangian is an arbitrary function of the Ricci scalar and the trace of the stress-energy tensor. Recently, Basumatary and Dewri (2021) and Brahma and Dewri (2021) have studied the DE model with a particular form of scale factor in Sen-Dunn's theory and Lyra Geometry of gravitation based on Bianchi type VI_0 and V respectively. The magnetic field significantly describes the universe's energy distribution because it includes highly ionized matter. The existence of a magnetic field on a galactic scale is now a well-known fact, and its significance for several astrophysical phenomena is well recognized, as Zeldovich et al. (1993) pointed out. Due to adiabatic compression in clusters of galaxies, a strong magnetic field may form. The large-scale magnetic fields might potentially be responsible for cosmic anisotropies. In contrast to the scenario when the pressure is retained isotropic, the anisotropic pressure induced by magnetic fields dominates the growth of shear anisotropy and decays slowly [Zweibel & Heiles (1997), Barrow (1997). At the end of an inflationary epoch, such fields can be produced. Melvin (1975) has also revealed that matter was strongly ionized and seamlessly associated with the field during the universe's development.

5.2 Metric and the field equations of $f(R,T)$ gravity

Let us consider the Bianchi type-V space time in the form

$$ds^2 = -dt^2 + A^2dx^2 + e^{-2mx}(B^2dy^2 + C^2dz^2) \tag{5.1}$$

where A, B, C are functions of cosmic time (t) and m is a constant.
The EMT of the matter with electromagnetic field is

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij} + E_{ij} \tag{5.2}$$

Here, ρ and p denote the energy density and thermodynamic pressure of the matter, where (E_{ij}) represents the electromagnetic fields of the source and is given as

$$E_{ij} = \frac{1}{4}\left(F_{j\alpha}F^{i\alpha}g^{\alpha\beta} - \frac{1}{4}g_{ij}F^{\alpha\beta}F_{\alpha\beta}\right) \tag{5.3}$$

such that Maxwell equation satisfies the relation

$$F_{ij,\alpha} + F_{j\alpha,i} + F_{\alpha i,j} = 0 \;\; and \;\; \left[F^{ij}\left(\sqrt{-g}\right)\right]_{,j} = 0 \tag{5.4}$$

In the co-moving coordinate system, it is assumed that the magnetic field is in the direction of the x axis so that F_{23} is the only non-vanishing component of the model, i.e., $F_{23} = K =$ constant. Also $u^i = (0,0,0,1)$ is the four-velocity vector in co-moving coordinate system satisfying the condition $u_i u^i = -1$ The non-vanishing components of the source of E_{ij} concerning the given line element are as follows:

$$E_1^1 = -E_2^2 = -E_3^3 = E_4^4 = \frac{K^2}{8\pi B^2 C^2} \tag{5.5}$$

Then, for the line element (5.1), with respect to eq. (5.2), the EFE (1.21) reduces to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 = -hp + \left(\frac{\rho - p}{2}\right) + \frac{hK^2}{8\pi B^2 C^2} \tag{5.6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{C}\dot{A}}{CA} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 = -hp + \left(\frac{\rho - p}{2}\right) - \frac{hK^2}{8\pi B^2 C^2} \tag{5.7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 = -hp + \left(\frac{\rho - p}{2}\right) - \frac{hK^2}{8\pi B^2 C^2} \tag{5.8}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}-\frac{3m^2}{A^2}-\frac{3}{4}\beta^2=h\rho+\left(\frac{\rho-p}{2}\right)+\frac{hK^2}{8\pi B^2C^2} \tag{5.9}$$

$$2\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0 \tag{5.10}$$

where, $\alpha=\left(\frac{8\pi G-\mu c^2}{\mu c^2}\right)$ is a constant and the others symbols have their usual meaning as in RG.

The energy conservation equation $T^i_{i;j}=0$ takes the form

$$\dot{\rho}+\frac{3}{2}\beta\dot{\beta}+\left[(\rho+p)+\frac{3}{2}\beta^2\right]\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \tag{5.11}$$

5.3 Solutions and the Physical behavior of the model in $f(\widetilde{R},T)$ gravity

Case-I: Presence of magnetic field

In solving the above-filed equations (5.6)-(5.10), the following physical parameters are very important, and these parameters are defined as follows:

The spatial volume and the scale factor are given by

$$V=a^3=ABC \tag{5.12}$$

The generalized mean HP is defined as

$$H=\frac{1}{3}\left(H_1+H_2+H_3\right) \tag{5.13}$$

where, H_1, H_2 and H_3 are already defined in the chapter 1.

Then, from eqs. (5.12) and (5.13), we obtain

$$H=\frac{\dot{a}}{a}=\frac{1}{3}\left(H_1+H_2+H_3\right) \tag{5.14}$$

Integrating eq. (5.10), we get

$$A^2 = BC \tag{5.15}$$

In the EFE (5.6)-(5.10), there are five highly non-linear differential equations with six unknowns, namely A, B, C, p, ρ, β. Thus, in order to establish these six variables' constants, let us consider power law relation

$$C = B^n \tag{5.16}$$

where $n \neq 1$ is a positive constant that preserves the anisotropy of the space time [Thorne (1967)].

Now, we consider a hybrid form of scale factor [Mishra et al. (2017)], which is expressed as

$$a(t) = t^l \xi^{nt} \tag{5.17}$$

Here, l, n, ξ are positive constants such that ξ lies between 2 and 3 ($i.e.$ $2 \leq \xi \leq 3$) where, for $\xi = 2.718$, the eq. (5.17) reduces to hybrid scale factor and it is very important to construct cosmic transit from early age of deceleration late time acceleration [Thorne (1967)].

From eqs. (5.12), (5.15), (5.16) and (5.17) together, we get

$$A = t^l \xi^{nt} \tag{5.18}$$

$$B = \left(t^l \xi^{nt}\right)^{\frac{2}{n+1}} \tag{5.19}$$

and,

$$C = \left(t^l \xi^{nt}\right)^{\frac{2n}{n+1}} \tag{5.20}$$

Then, the line element (5.1) in view of eqs. (5.18)-(5.20) reduces to

$$ds^2 = -dt^2 + \left(t^l \xi^{nt}\right)^2 dx^2 + e^{-2mx}\left[\left(t^l \xi^{nt}\right)^{\frac{4}{n+1}} dy^2 + \left(t^l \xi^{nt}\right)^{\frac{4n}{n+1}} dz^2\right] \tag{5.21}$$

From eq. (5.12), the volume of the model reduces to

$$V = a^3 = \left(t^l \xi^{nt}\right)^3 \tag{5.22}$$

The other dynamical parameters are obtained from eqs. (5.14), (1.33) - (1.35) as follows:

$$H = n \ln \xi + \frac{l}{t} \tag{5.23}$$

$$\theta = 3n \ln \xi + \frac{3l}{t} \tag{5.24}$$

$$\sigma^2 = \left(\frac{n-1}{n+1}\right)^2 \left(n \ln \xi + \frac{l}{t}\right)^2 \tag{5.25}$$

$$\Delta = \frac{2}{3}\left(\frac{n-1}{n+1}\right)^2 \tag{5.26}$$

The DP is obtained from eq. (1.32) as

$$q = -1 + \frac{l}{t^2 \left(n \ln \xi + \frac{l}{t}\right)^2} \tag{5.27}$$

Adding eqs. (5.6)-(5.8) and then applying in eq. (5.9), we get

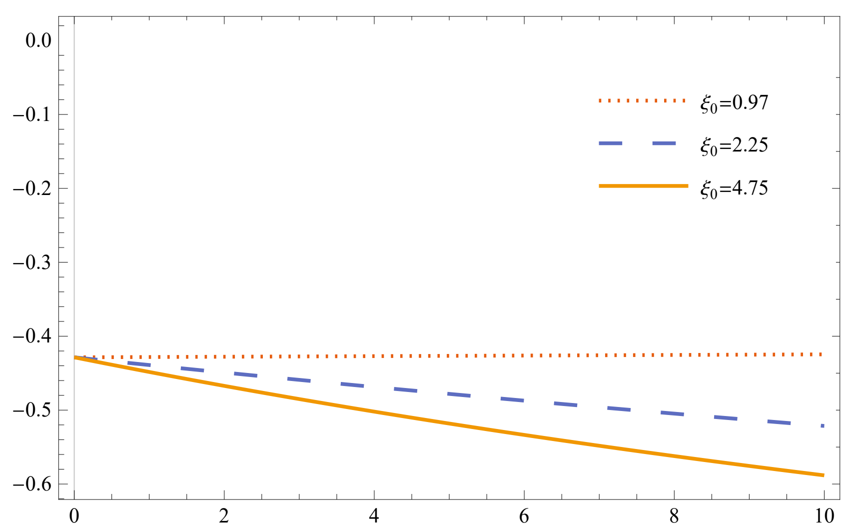

Figure 5.1: Variation of q vs. t, for $l = 1.75, n = 0.02$

$$K_1\rho = -6\left(n\ln\xi + \frac{l}{t}\right)^2 + \frac{2l}{t^2} + \left(\frac{4m^2}{(t^l\xi^{nt})^2}\right) \tag{5.28}$$

where $(K_1 = h\gamma - h - 1 + \gamma)$ is constant. We choose the parameters $\gamma = -0.96$ and $h = 0.02$ to find the deterministic solution and draw up the model's behaviour. These values are crucial in determining the model's behavior when utilizing the hybrid form of scale factor. In addition, another condition of the equation of state $p = \gamma\rho$ is considered to derive the negative pressure (DE) of the model, and by applying this condition; we obtain the equation of DE in our model based on a hybrid form of scale factor as

$$K_1 p = -6\gamma\left(n\ln\xi + \frac{l}{t}\right)^2 + 2\gamma\frac{l}{t^2} + \left(\frac{4m^2\gamma}{(t^l\xi^{nt})^2}\right) \tag{5.29}$$

Adding eqs. (5.6) - (5.8), we obtain the displacement field vector as

$$\begin{aligned}(1-\gamma)\frac{3}{4}\beta^2 = &-3\left(n\ln\xi + \frac{l}{t}\right)^2 + \frac{2l}{t^2} - 3\gamma\left(n\ln\xi + \frac{l}{t}\right)^2 + \frac{m^2(1+3\gamma)}{(t^l\xi^{nt})^2} + \frac{K_2h(1-\gamma)}{(t^l\xi^{nt})^4} \\ &- \frac{3K_3}{K_1}\left(n\ln\xi + \frac{l}{t}\right)^2 - \frac{K_3}{K_1}\left(\frac{n}{\xi} - \frac{l}{t^2}\right) + \frac{2K_3}{K_1}\frac{m^2}{(t^l\xi^{nt})^2} \\ &- (1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi + \frac{l}{t}\right)^2\end{aligned} \tag{5.30}$$

where $K_2 = \frac{K^2}{24\pi}$ and $K_3 = (1+\gamma)(1-\gamma)$ are both positive constant and depict the variation of graph, by considering the choice of $K_1 = -5.88$, $K_2 = 0.16$ and $K_3 = 0.078$, being $K = 3.5$

From eqs. (5.28) and (5.29), the trace $(T = \rho - 3p)$ and the Riemannian curvature of $f(\widetilde{R}, T)$ gravity and the function of Ricci Scalar tensor with electromagnetic field based on Lyra geometry are obtained as

$$T = \left(\frac{1-3\gamma}{K_1}\right)\left[-6\left(n\ln\xi + \frac{l}{t}\right)^2 + \frac{2l}{t^2} + \frac{4m^2}{(t^l\xi^{nt})^2}\right] \tag{5.31}$$

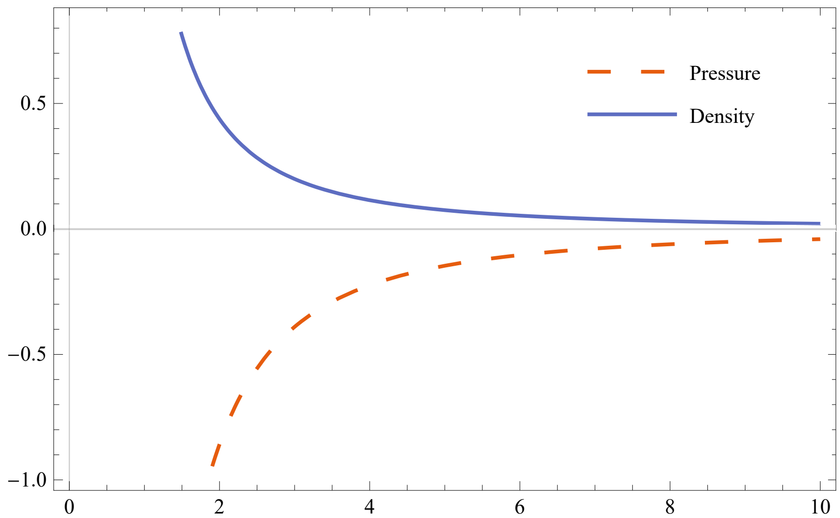

Figure 5.2: Variation of p and ρ vs. t for $n = 0.02, l = 1.75, \xi = 2.25, h = 2, \gamma = -1.96, m = 0.09$

$$R = \frac{6m^2}{(t^l \xi^{nt})^2} - 6\left(n \ln \xi + \frac{l}{t}\right)^2 - 6\left(\frac{n}{\xi} - \frac{l}{t^2}\right) - 8K_4\left(n \ln \xi + \frac{l}{t}\right)^2 \tag{5.32}$$

and,

$$\begin{aligned}
\widetilde{R} = &\frac{6m^2}{(t^l \xi^{nt})^2} - 6\left(n \ln \xi + \frac{l}{t}\right)^2 - 6\left(\frac{n}{\xi} - \frac{l}{t^2}\right) - 8K_4\left(n \ln \xi + \frac{l}{t}\right)^2 + 9W_1\left(n \ln \xi + \frac{l}{t}\right) \\
&+ W_2K_5\left[\frac{3l}{t^2}\left(n \ln \xi + \frac{l}{t^2}\right)\left(1 + 2\gamma + \frac{2K_3}{K_1}\right) + \frac{2l}{t^2}(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n \ln \xi + \frac{l}{t^2}\right)\right] \\
&- W_2K_5\left[\left(2 + \frac{K_3}{K_1}\right)\frac{2l}{t^3} - \frac{2m^2(1+3\gamma)}{(t^l \xi^{nt})^2}\left(n \ln \xi + \frac{l}{t}\right) + \frac{4hK_2(1-\gamma)}{(t^l \xi^{nt})^4}\left(n \ln \xi + \frac{l}{t}\right)\right] \\
&- W_2K_5\left[\frac{4m^2K_3}{K_1(t^l \xi^{nt})^2}\left(n \ln \xi + \frac{l}{t}\right) + \left(\frac{2}{1-\gamma}\right)\left(-3\left(1 + \gamma + \frac{K_3}{K_1}\right)\left(n \ln \xi + \frac{l}{t}\right)^2\right)\right] \\
&+ K_5\left[\frac{2l}{t^2} + \frac{m^2(1+3\gamma)}{(t^l \xi^{nt})^2} - \frac{hK_2(1-\gamma)}{(t^l \xi^{nt})^4} - \frac{K_3}{K_1}\left(\frac{n}{\xi} - \frac{l}{t^2}\right) + \frac{2K_3}{K_1}\frac{m^2}{(t^l \xi^{nt})^2}\right] \\
&- 2\left(\frac{n-1}{n+1}\right)^2\left(n \ln \xi + \frac{l}{t^2}\right)^2
\end{aligned} \tag{5.33}$$

where $K_4 = \frac{n^2+n+1}{n^2+2n+1}$ and $K_5 = \frac{2}{1-\gamma}$ are positive constant and keeping $n = 0.02$ to describe the behaviour of the model, provided $n \neq 1$. W_1 and W_2 are given by

$$W_1(t) = \frac{2}{\sqrt{3(1-\gamma)}}[-3\left(1+\gamma+\frac{K_3}{K_1}\right)\left(n\ln\xi+\frac{l}{t}\right)^2+\frac{2l}{t^2}+\frac{m^2(1+3\gamma)}{(t^l\xi^{nt})^2}-\frac{hK_2(1-\gamma)}{(t^l\xi^{nt})^4}$$
$$-\frac{K_3}{K_1}\left(\frac{n}{\xi}-\frac{l}{t^2}\right)+\frac{2K_3}{K_1}\frac{m^2}{(t^l\xi^{nt})^2}-(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi+\frac{l}{t^2}\right)^2]^{\frac{1}{2}} \tag{5.34}$$

and,

$$W_2(t) = \frac{\sqrt{3(1-\gamma)}}{2}[-3\left(1+\gamma+\frac{K_3}{K_1}\right)\left(n\ln\xi+\frac{l}{t}\right)^2+\frac{2l}{t^2}+\frac{m^2(1+3\gamma)}{(t^l\xi^{nt})^2}-\frac{hK_2(1-\gamma)}{(t^l\xi^{nt})^4}$$
$$-\frac{K_3}{K_1}\left(\frac{n}{\xi}-\frac{l}{t^2}\right)+\frac{2K_3}{K_1}\frac{m^2}{(t^l\xi^{nt})^2}-(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi+\frac{l}{t^2}\right)^2]^{\frac{-1}{2}} \tag{5.35}$$

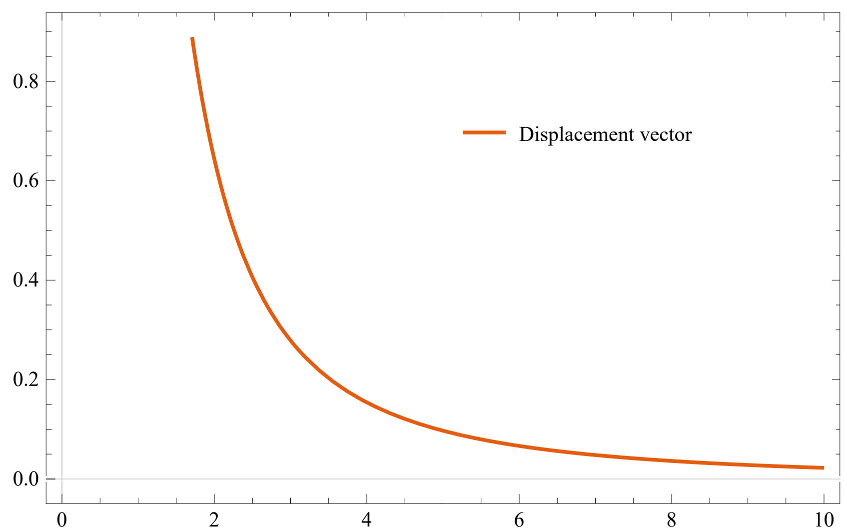

Figure 5.3: Variation of β^2 vs. t, for $n = 0.02, l = 1.75, \xi = 2.25, h = 2, \gamma = -1.96, m = 0.09$

Here, we are interested in obtaining the behavior of the DE model using the frame of Harko et al. (2011) as $f(\widetilde{R},T) = \mu\widetilde{R}+\mu T$ which leads from the Eqs. (5.31) and (5.33), given by

$$\frac{1}{\mu}f(\widetilde{R},T)=\frac{6m^2}{(t^l\xi^{nt})^2}-6\left(n\ln\xi+\frac{l}{t}\right)^2-6\left(\frac{n}{\xi}-\frac{l}{t^2}\right)-8K_4\left(n\ln\xi+\frac{l}{t}\right)^2+9W_1\left(n\ln\xi+\frac{l}{t}\right)$$
$$+W_2K_5\left[\frac{3l}{t^2}\left(n\ln\xi+\frac{l}{t^2}\right)\left(1+2\gamma+\frac{2K_3}{K_1}\right)+\frac{2l}{t^2}(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi+\frac{l}{t^2}\right)\right]$$
$$-W_2K_5\left[\left(2+\frac{K_3}{K_1}\right)\frac{2l}{t^3}-\frac{2m^2(1+3\gamma)}{(t^l\xi^{nt})^2}\left(n\ln\xi+\frac{l}{t}\right)+\frac{4hK_2(1-\gamma)}{(t^l\xi^{nt})^4}\left(n\ln\xi+\frac{l}{t}\right)\right]$$
$$-W_2K_5\left[\frac{4m^2K_3}{K_1(t^l\xi^{nt})^2}\left(n\ln\xi+\frac{l}{t}\right)+\left(\frac{2}{1-\gamma}\right)\left(-3\left(1+\gamma+\frac{K_3}{K_1}\right)\left(n\ln\xi+\frac{l}{t}\right)^2\right)\right]$$
$$+K_5\left[\frac{2l}{t^2}+\frac{m^2(1+3\gamma)}{(t^l\xi^{nt})^2}-\frac{hK_2(1-\gamma)}{(t^l\xi^{nt})^4}-\frac{K_3}{K_1}\left(\frac{n}{\xi}-\frac{l}{t^2}\right)+\frac{2K_3}{K_1}\frac{m^2}{(t^l\xi^{nt})^2}\right]$$
$$-2\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi+\frac{l}{t^2}\right)^2$$
$$+\left(\frac{1-3\gamma}{K_1}\right)\left[-6\left(n\ln\xi+\frac{l}{t}\right)^2+\frac{2l}{t^2}+\frac{4m^2}{(t^l\xi^{nt})^2}\right] \tag{5.36}$$

Case-II: In absence of magnetic field, i.e. $F_{23}=K=0$ In this case, the model's pressure, density and the displacement vector are obtained as

$$K_1p=-6\gamma\left(n\ln\xi+\frac{l}{t}\right)^2+\frac{2l\gamma}{t^2}+\frac{4m^2\gamma}{(t^l\xi^{nt})^2} \tag{5.37}$$

$$K_1\rho=-6\left(n\ln\xi+\frac{l}{t}\right)^2+\frac{2l}{t^2}+\frac{4m^2}{(t^l\xi^{nt})^2} \tag{5.38}$$

and,

$$(1-\gamma)\frac{3}{4}\beta^2=-3\left(n\ln\xi+\frac{l}{t}\right)^2+\frac{2l}{t^2}-3\gamma\left(n\ln\xi+\frac{l}{t}\right)^2+\frac{m^2(1+3\gamma)}{(t^l\xi^{nt})^2}-\frac{3K_3}{K_1}\left(n\ln\xi+\frac{l}{t}\right)^2$$
$$-\frac{K_3}{K_1}\left(\frac{n}{\xi}-\frac{l}{t^2}\right)+\frac{2K_3}{K_1}\frac{m^2}{(t^l\xi^{nt})^2}-(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi+\frac{l}{t}\right)^2 \tag{5.39}$$

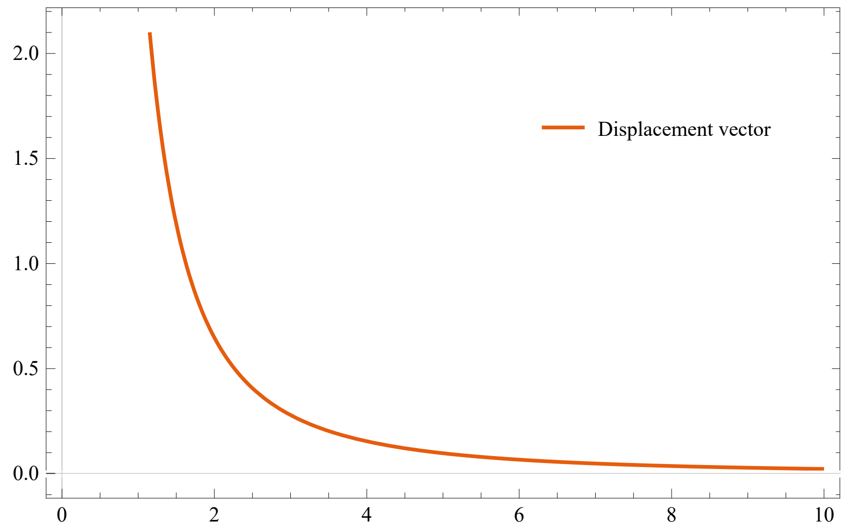

Figure 5.4: Variation of β^2 vs. t, in absence of magnetic field, for $n = 0.02, l = 1.75, \xi = 2.25, h = 2, \gamma = -1.96, m = 0.09$

In the same way, the trace, function of Ricci Scalar Tensor, Riemannian curvature, and the $f(\widetilde{R}, T)$ gravity are obtained as follows:

$$T = \left(\frac{1-3\gamma}{K_1}\right)\left[-6\left(n\ln\xi + \frac{l}{t}\right)^2 + \frac{2l}{t^2} + \frac{4m^2}{(t^l\xi^{nt})^2}\right] \tag{5.40}$$

$$R = \frac{6m^2}{(t^l\xi^{nt})^2} - 6\left(n\ln\xi + \frac{l}{t}\right)^2 - 6\left(\frac{n}{\xi} - \frac{l}{t^2}\right) - 8K_4\left(n\ln\xi + \frac{l}{t}\right)^2 \tag{5.41}$$

and,

$$\widetilde{R}=\frac{6m^2}{(t^l\xi^{nt})^2}-6\left(n\ln\xi+\frac{l}{t}\right)^2-6\left(\frac{n}{\xi}-\frac{l}{t^2}\right)-8K_4\left(n\ln\xi+\frac{l}{t}\right)^2+9W_4\left(n\ln\xi+\frac{l}{t}\right)$$
$$+W_3K_5\left[\frac{3l}{t^2}\left(n\ln\xi+\frac{l}{t^2}\right)\left(1+2\gamma+\frac{2K_3}{K_1}\right)+\frac{2l}{t^2}(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi+\frac{l}{t^2}\right)\right]$$
$$-W_3K_5\left[\left(2+\frac{K_3}{K_1}\right)\frac{2l}{t^3}-\frac{2m^2(1+3\gamma)}{(t^l\xi^{nt})^2}\left(n\ln\xi+\frac{l}{t}\right)-\frac{4m^2K_3}{K_1(t^l\xi^{nt})^2}\left(n\ln\xi+\frac{l}{t}\right)\right]$$
$$+K_5\left[-3\left(1+\gamma+\frac{K_3}{K_1}\right)\left(n\ln\xi+\frac{l}{t}\right)^2+\frac{2l}{t^2}+\frac{m^2(1+3\gamma)}{(t^l\xi^{nt})^2}-\frac{K_3}{K_1}\left(\frac{n}{\xi}-\frac{l}{t^2}\right)\right]$$
$$+K_5\left[\frac{2K_3}{K_1}\frac{m^2}{(t^l\xi^{nt})^2}-(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi+\frac{l}{t^2}\right)^2\right]$$
$$+\left(\frac{1-3\gamma}{K_1}\right)\left[-6\left(n\ln\xi+\frac{l}{t}\right)^2+\frac{2l}{t^2}+\frac{4m^2}{(t^l\xi^{nt})^2}\right] \tag{5.42}$$

$$\frac{1}{\mu}f(\widetilde{R},T)=\frac{6m^2}{(t^l\xi^{nt})^2}-6\left(n\ln\xi+\frac{l}{t}\right)^2-6\left(\frac{n}{\xi}-\frac{l}{t^2}\right)-8K_4\left(n\ln\xi+\frac{l}{t}\right)^2$$
$$+9W_4\left(n\ln\xi+\frac{l}{t}\right)+W_3K_5\left[\frac{3l}{t^2}\left(n\ln\xi+\frac{l}{t^2}\right)\left(1+2\gamma+\frac{2K_3}{K_1}\right)\right]$$
$$+W_3K_5\left[\frac{2l}{t^2}(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi+\frac{l}{t^2}\right)-\left(2+\frac{K_3}{K_1}\right)\frac{2l}{t^3}\right]$$
$$-W_3K_5\left[\frac{2m^2(1+3\gamma)}{(t^l\xi^{nt})^2}\left(n\ln\xi+\frac{l}{t}\right)-\frac{4m^2K_3}{K_1(t^l\xi^{nt})^2}\left(n\ln\xi+\frac{l}{t}\right)\right]$$
$$+K_5\left[-3\left(1+\gamma+\frac{K_3}{K_1}\right)\left(n\ln\xi+\frac{l}{t}\right)^2+\frac{2l}{t^2}+\frac{m^2(1+3\gamma)}{(t^l\xi^{nt})^2}\right]$$
$$-K_5\left[\frac{K_3}{K_1}\left(\frac{n}{\xi}-\frac{l}{t^2}\right)+\frac{2K_3}{K_1}\frac{m^2}{(t^l\xi^{nt})^2}-(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi+\frac{l}{t^2}\right)^2\right]$$
$$+\left(\frac{1-3\gamma}{K_1}\right)\left[-6\left(n\ln\xi+\frac{l}{t}\right)^2+\frac{2l}{t^2}+\frac{4m^2}{(t^l\xi^{nt})^2}\right] \tag{5.43}$$

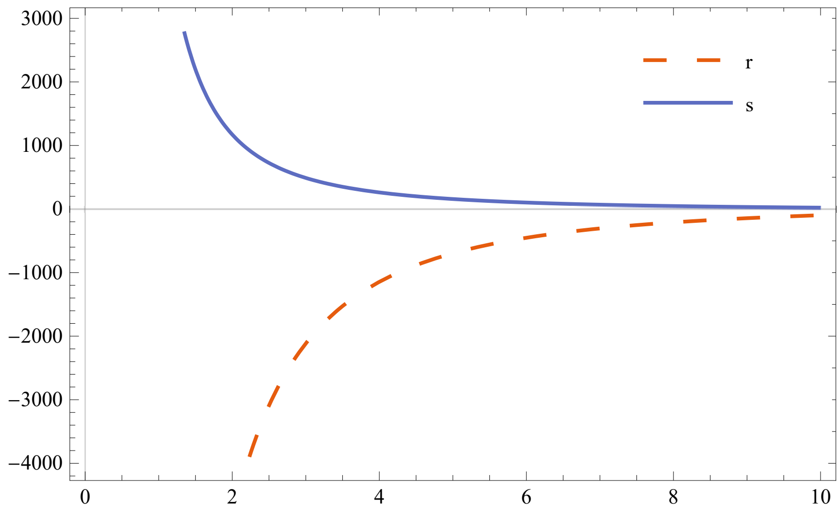

Figure 5.5: Variation of r and s vs. t, for $n = 0.02, l = 1.75, \xi = 2.25$

$$W_3(t) = \frac{2}{\sqrt{3(1-\gamma)}}[-3\left(1+\gamma+\frac{K_3}{K_1}\right)\left(n\ln\xi+\frac{l}{t}\right)^2+\frac{2l}{t^2}+\frac{m^2(1+3\gamma)}{(t^l\xi^{nt})^2}-\frac{K_3}{K_1}\left(\frac{n}{\xi}-\frac{l}{t^2}\right)$$
$$+\frac{2K_3}{K_1}\frac{m^2}{(t^l\xi^{nt})^2}-(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi+\frac{l}{t^2}\right)^2]^{\frac{1}{2}} \tag{5.44}$$

and,

$$W_4(t) = \frac{\sqrt{3(1-\gamma)}}{2}[-3\left(1+\gamma+\frac{K_3}{K_1}\right)\left(n\ln\xi+\frac{l}{t}\right)^2+\frac{2l}{t^2}+\frac{m^2(1+3\gamma)}{(t^l\xi^{nt})^2}-\frac{K_3}{K_1}\left(\frac{n}{\xi}-\frac{l}{t^2}\right)$$
$$+\frac{2K_3}{K_1}\frac{m^2}{(t^l\xi^{nt})^2}-(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi+\frac{l}{t^2}\right)^2]^{\frac{-1}{2}} \tag{5.45}$$

The statefinder parameter are defined as

$$r = \frac{\dddot{a}}{a} = 1+3\frac{\dot{H}}{H^2}+\frac{\ddot{H}}{H^3} \tag{5.46}$$

and,

$$s = \frac{r-1}{3(q-\frac{1}{2})} \tag{5.47}$$

which are obtained from (5.18), (5.23) and (5.27) as follows:

$$r = 1 + \frac{3}{n\xi(\ln\xi)^2} - \frac{3l}{n^2t^2(\ln\xi)^2} + \frac{2l}{t^3(n\ln\xi + \frac{l}{t})^3} \tag{5.48}$$

and,

$$s = \frac{\frac{3}{n\xi(\ln\xi)^2} - \frac{3l}{n^2t^2(\ln\xi)^2} + \frac{2l}{t^3(n\ln\xi+\frac{l}{t})^3}}{3\left[-\frac{3}{2} + \frac{\frac{l}{t^2}}{(n\ln\xi+\frac{l}{t})^2}\right]} \tag{5.49}$$

5.4 Conclusion

In this study, the solutions of the models are obtained by using a hybrid form of scale factor in Lyra based $f(\widetilde{R},T)$ gravity. The magnetic field source is along the x axis, $F_{23} \neq 0$. It is found that the absence of an electromagnetic field does not affect much in the present model. We have obtained the pressure, density, and displacement vector in both circumstances. Also, at the initial moment $t = 0$, all the physical and kinematical parameters like p, ρ, R, β, $\widetilde{R}$, T and $f(\widetilde{R},T)$ tends to infinity. As a result, the model begins from an initial singularity with infinite pressure and density. However, when $t \to \infty$, p, ρ, R, β, $\widetilde{R}$, T and $f(\widetilde{R},T)$, all the parameters tend to a constant value. From the eq. (5.22), it is seen that the universe approaches an infinitely large volume as time increases in both scenarios of the $f(\widetilde{R},T)$ gravity model. Fig. 5.1 shows behavior of DP to be $q < 0$ for $t \to \infty$. Fig. 5.2 shows the pressure orientation and the energy density in positive and negative directions in each case with an increase in time (t). As per these observations of p and q, the present model has a presence of DE with the accelerated universe expansion with an electromagnetic field, and this result fits the recent observational data like LSS, CMB, SNe Ia. It is clear from Fig. 5.4 that the displacement field vector positively gradually decreases with an increase in cosmic time t. From Fig. 5.5, it is concluded that the ΛCDM model does not evolve in this model. Eqs. (5.25) and (5.26) indicate that this model of$f(\widetilde{R},T)$ gravity is not free from shear scalar, and the behavior of the model is anisotropic provided $n \neq 1$.