Abstract

In the contemporary world, every part of our environment or society faces a lot of interconnected items. The concept of networking will come into the light from such types of interconnected items. In mathematical literature, we study the networks by using the concept of graph theory. The famous Konigberg's Bridge Problem (1736) is one of the fundamental pillars of development, in the field of networks. The modern theory of networks was very much guided and influenced by Frank Harary [17], who is recognized as the father of modern graph theory.

A **Graph** G consists of a finite nonempty set V of p points together with a prescribed set X of q unordered pairs of distinct points of V. The set V is the set of vertices and X is called the set of edges of the graph G. An edge (u, v) is said to join the vertices u and v and is denoted by uv. Thus, uv and vu mean exactly the same edge.

In the present civilization, no one can deny the importance of graph theory and its application to various aspects of our daily life situation.

Though graph theory has wide application in our daily life it describes some binary relations only. Thus, in some situations e.g., Family Relationship [15], Optimal Routing [15], Projective Geometries [15], etc. where graph theory is not always sufficient for modelling problems or data which involve relations of order higher than binary. Looking at the applicability of graph theory in diverse fields of knowledge, attempts were made to generalize it so that its new version can be applied to those fields where the results of graph theory cannot be applied directly.

One such attempt by C. Berge [8, 9] to generalize a graph into a hypergraph. A hypergraph is defined as follows:

Let $X = \{v_1, v_2, v_3, \ldots., v_n\}$ be a finite set. A **Hypergraph** on X is a family $H = \{E_1, E_2, E_3, \ldots., E_m\}$ of subsets of X such that $E_i \neq \varphi$, $i = 1,2,3, \ldots., m$ and $\bigcup_{i=1}^{m} E_i = X$.

In 1994, the second attempt was made by E. Sampathkumar [15] to propose a new kind of generalization of graph originally named graphoid was renamed later on as semigraph by B. D. Acharya [2] was structurally designed to apply to problems as demanded by situations in nature that involve relations more than binary ones. Here he attempts a generalization of graphs in the following way:

Semigraph is a, pair (V, X) where V is a non-empty set of elements called vertices and X is a set of *n*-tuples called edges of distinct vertices for various $n \geq 2$ such that two edges have at most one vertex in common and considered the edges $(u_1, u_2, u_3, \ldots, u_n)$ and $(u_n, u_{n-1}, \ldots, u_2, u_1)$ are exactly the same edge.

A compromise between the concept of graph and hypergraph is semigraph. Semigraphs are a kind of generalization in which all mathematical discrete structures are studied. The strength of semigraph as a mathematical model and its application is more powerful than the graph model and this was realised when the concept of semigraph was used for DNA splicing by S. J. Bharathi *et al*. [48, 49].

According to Huckel's the sum of the energies of all the electrons in a molecule is called the total π-electron energy [21]. In the year 1978, Ivan Gutman [22] studied this electron energy in graph theory and introduced graph energy. An extensive study has been done on graph energy and various energies like distance energy, color energy and minimum covering energy, etc. have been studied over the past few years.

G. Indulal *et al*. [19] study on distance energy of graphs of diameter 2 and obtain bounds for distance spectral radius and their distance energy. A vertex labeled graph *G* can be uniquely represented by a matrix called *L*-matrix, which was introduced by E. Sampathkumar *et al*. [13] in the year 2013 and obtained its characteristics. They also show that the *L*-matrix of a colored graph with usual coloring is the adjacency matrix of a signed graph on the same vertex set of *G*.

In the year 2010 C. Adiga *et al*. [6] introduced the concept of color energy of a graph and compute the color energy of a few families of graphs with the minimum number of colors on its vertices. They also established an upper bound and lower bound for color energy and compute energies of the complement of colored graphs of a few families of graphs. Further, P. B. Joshi *et al*., [34, 35] obtained some new bounds for the color energy of graphs in terms of Zagreb index, Laplacian energy, and signless Laplacian energy. Also established the relationship between color energy and the energy of a graph.

C. Adiga *et al*., [5] introduce a new kind of graph energy called minimum covering energy in the year 2012. They computed the energies of some well-known graphs and found that it depends both on the structure of graph *G* and on its particular minimum cover *C*. Further, minimum covering distance energy and minimum covering color energy of graphs were studied by M. R. R. Kanna *et al*., [29, 30] and computed the energies for star graph, complete graph, crown graph, bipartite graph, and cocktail graphs. Upper and lower bounds for the energies are also established.

In the year 2012 paper due to C.M. Deshpande and Y. S. Gaidhani [10] gives a new definition for the adjacency matrix of a semigraph and states necessary and sufficient conditions for a matrix to be semigraphical. They also defined the spectrum of a semigraph and studied its spectral properties. After that, Y. S. Gaidhani, C. M. Deshpande, and B. P. Athawale [65] again introduced an adjacency matrix that represents a semigraph uniquely and develop an algorithm to construct the semigraph from a given square if semigraphical is given, in the year 2017.

Recently, Y. S. Gaidhani *et al*. [64] introduced the energy of semigraph in 2019. In their paper, they studied the energy of semigraphs in two ways, one, the matrix energy, as a summation of singular values of the adjacency matrix of a semigraph, and the other, polynomial energy, as the energy of the characteristic polynomial of the adjacency matrix and obtained some bounds for matrix energy and show that matrix energy is never a square root of an odd integer and polynomial energy cannot be an odd integer. Also, investigate the matrix energy of a partial semigraph and change in the matrix energy due to edge deletion.

The concept of energy of semigraph has vast applications in the field of chemistry, physics, computer science, biotechnology, social science, etc. Thus, we are motivated by the above-mentioned works, to research the nature of matrices in connection with semigraphs and their energy.

However, despite a considerable number of useful results, semigraphs and their energy are still in their nascent stage and many areas of graph theory, topological graph theory, and algebraic graph theory are yet to find analogies in semigraphs and ramifications if successfully formulated. So, this study focused on finding out the energies of different matrices, in the field of semigraph.

The objective of the presents study will be directed towards this end and to be specific, our primary focus will be on the following points:

To introduce the concept of

i. Adjacency matrix of *e*-signed and *v*-signed semigraphs.
ii. Adjacency matrix of *ve*-signed semigraphs and their properties.
iii. Distance matrix and distance energy of a semigraph.
iv. Minimum covering distance energy of a semigraph.
v. Color matrix and its energy of a semigraph.
vi. Minimum covering energy of a semigraph.
vii. Minimum covering color energy of a semigraph.

The nature of the investigation of the proposed work is theoretical and analytical unless demanded otherwise by unforeseen circumstantial factors.

This thesis makes effort to introduce and analysed some important graph theoretic concepts like adjacency matrix, distance matrix, and energy, color matrix and energy, minimum covering matrix and energy, minimum covering color energy, minimum covering distance energy along with their implications in the context of semigraph and obtained some new results. The thesis consists of six chapters. The organization of the thesis is as follows.

In the **first chapter,** a general introduction regarding graphs and their generalizations is given. A literature survey on recent development through works of almost all well-known researchers working on graph and semigraph after E. Sampathkumar up to date has been included in this chapter which justified the relevance and importance of the problem under consideration.

In the **second chapter,** we mentioned the basic concepts and some results which are required for our study.

In the **third chapter,** we introduced the adjacency matrix for *e*-signed, *v*-signed, and *ve*-signed semigraph. And discuss them with suitable examples and derived some properties of adjacency matrix associated with the respectively signed semigraphs. A necessary and sufficient condition for a matrix to be *e*-signed, *v*-signed, or *ve*-signed semigraph is successfully established. A part of the work has been published in [50].

In the **fourth chapter**, another important concept, the distance matrix energy of graph is generalized into semigraph. In this chapter, we discuss on distance matrix and the energy of a semigraph. The eigenvalues of the distance matrix are called *D*-eigenvalues. The energy of the distance matrix of a semigraph i.e. *D*-energy is defined as the sum of the absolute values of *D*-eigenvalues. we investigate some properties and bounds for the distance spectral radius and *D*-energy of semigraph of diameter 2. Further, the minimum covering distance energy of a semigraph and its properties are also obtained. A part of the work has been published in [51]

A **coloring** of a semigraph G is an assignment of colors to its vertices such that not all vertices in the same edge are colored the same. In the **fifth chapter**, we introduce the concept of color matrix and energy of semigraphs. We defined the color energy of a semigraph as the sum of the absolute values of the eigenvalues of the color matrix. Obtained some properties, and bounds on color energy of semigraphs.

In the **sixth chapter,** we introduce the concept of minimum covering matrix and minimum covering energy of a semigraph G. The minimum covering energy is the summation of singular values of the minimum covering matrix. Upper and lower bounds for the energy are obtained, and also derive some relationship between minimum covering energy and energy $E(G)$ of semigraph. Further, the minimum covering color energy of semigraphs and their properties are also discussed.

In the last part of the thesis, we mentioned a brief conclusion and future aspects of our work.