**Chapter 1**

**Introduction**

Our investigations presented in this thesis consist of nine chapters. The present chapter includes introduction of the research area and draws out the inspiration of the work portrayed in other eight chapters. The physical description of gravity is most important as it is the dominant force controlling the large -scale behavior of our universe. Newton as the first man to profound the fundamental law of gravitation sand was enormously successful in explaining most of the observed macroscopic gravitational phenomenon. With the advent of the special theory of relativity, however, it becomes clear that Newton's law as it could be corrected as it did not incorporate the finite speed with which signals travel. The next significant revolutionary development was Einstein General Theory of Relativity which synthesized Newtonian gravitation with the special theory of relativity. Since Einstein first works in gravitation, numerous m proposed which imply to join into the hypothesis certain desirable elements among those speculations Brans-Dicke hypothesis additionally one of the well known hypothesis.

To attain our objective of highlighting the significant results are presented in this thesis and also, we have examined the work of various authors as far as practicable relevant to our investigations.

**1.1 Theory of General Relativity**

The theory of Relativity has grown in two major subdivisions called the special theory of relativity and general theory of relativity. The two approaches differ significantly in the domain and extent of the fields of physics in which they have found applications. Special relativity, dealing with the inertial frames of reference has found broad applications in mechanics, electromagnetism and quantum theory. General relativity has been applied primarily to study the theory of gravitation and the structure of the universe.

The special theory of relativity deals with inertial system i.e. systems moving with uniform velocity, without the gravitational field. General theory of relativity deals with non-inertial frame of reference i.e. systems moving with accelerated speed. It additionally covers gravitational ﬁeld and postulates of the principle of equivalence.

The general principles i.e. principle of covariance, principle of equivalence and Mach' principle mainly guided Einstein (1915).

The principle of covariance states that the law must be expressible in a form which is independent of the particular space-time co-ordinate chosen or in other words, laws of nature remain invariant with respect to any space-time coordinate system.

Thus the line element of special relativity

 (1.1)

is replaced by

, (1.2)

which is valid in any coordinate system. The fundamental tensor is a symmetric covariant tensor of rank two obeying the transformation law

 (1.3)

where dash quantities belong to the new coordinate system .

The principle of equivalence states that in Newtonian theory the gravitational mass and inertial mass are always equal.

Mach's principle gives the idea that the inertial property of the material objects is created and determined by the interactions of the object masses with all the other mass the universe.

**1.2 Cosmology**

The word Cosmology is derived from a Greek word which means the beauty of the sky. Cosmology is that branch of science that deals with the study of the large-scale structures of the universe which consists of stars, star clusters, and galaxies or the nebulae, pulsars, quasars, cosmic rays and background radiation. Dynamics of the system is the fundamental problem in cosmology. The primary force keeping the solar system, stars, and galaxies together is the gravitational force and other long-range interactions such as electromagnetic force. The most dominant assumption in standard cosmological theory is that of homogeneity and isotropy, often referred to as the cosmological principle. The ordinary cosmological principle means that a typical fundamental observer cannot tell from large scale observations where he is located or in what direction he is looking. Equivalently it implies that on a sufficiently large scale the universe has no preferred position or direction. The perfect cosmological principle states that besides the implications of the ordinary cosmological principle the universe also looks the same to a fundamental observer at all epochs. Physically, this implies that there is no preferred position, preferred direction or preferred epoch in the universe. Thus by invoking the cosmological principle, we, grossly, idealize the universe and model it by a simple macroscopic perfect fluid.

**1.3 Cosmological Models**

The main purpose of Physical Cosmology is to study our universe from a global point of view and to describe the physical processes which have occurred throughout its evolution and which have observable consequences nowadays.

Astronomical observations are not exhaustive, and they accommodate us with advice about the cosmos which is limited, both spatially and temporarily. The models describing the universe and its evolution are then necessarily based on an extrapolation of physical theories whose validity has only been locally tested out. This extrapolation, accepted as the Relativity Principle, states that physical laws are everywhere the same and at all times.

Cosmologists construct mathematical models that they believe represent the universe as a whole concentrating on its large scale features and compare these models with the universe as observed by astronomers. The cosmological principle allows us to describe the most general homogeneous and isotropic space-time by the

Robertson-Walker metric

 (1.4)

where is the curvature index which can take values for the open , flat, closed, universe and is the cosmic scale factor. The coordinates forms a comoving coordinate system in the sense that the cosmic fluid is at rest on the coordinate system. Friedmann (1922) was the first to investigate the evolution of scale factor using Einstein field equations for all three curvatures indices. The present universe is spatially homogeneous and isotropic and therefore can be described by the Friedmann-Robertson-Walker (FRW) model (Partridge and Wilkinson 1967, Ehlers et al. 1968). In any case, there is confirmation for a little measure of anisotropic and a little magnetic ﬁeld over a distant cosmic scale. Although this evidence is not conclusive, the mere possibility of even small anisotropy and a small magnetic field at present would suggest a very significant departure from (FRW) model at early stages of evolution of the universe. For that matter, it is useful to study cosmological models which may be highly anisotropic and may contain dynamically relevant electromagnetic fields. For simplicity, it is helpful to restrict ourselves to models that are spatially homogeneous. So Robertson-Walker models can play a significant role in current modern cosmology.

**1.4 Cosmological Principles**

The Cosmological principles consist of the basic belief that the fundamental laws of science are universal and two other simplifying assumptions about the nature of the universe when considered as a single entity.

(I) The Homogeneity of Space

It is the belief that over the largest distance scales, matter and energy are distributed approximately uniformly. There is no favored area in space

(II) The Isotropy of Space

It states that there is no preferred direction in space.

So, the hypothesis that the universe is isotropic and homogeneous is known as the cosmological principle.

**1.5 Cosmological constant**

The cosmological constant is an extra term in Einstein equations of general relativity which physically represents the possibility that there are density and pressure associated with empty space. It describes the energy density of the vacuum and a significantly important contributor for the dynamical history of the universe. Ordinary matter can clump together or disperse as it evolves, the energy density in a cosmological constant is a property of space-time itself, and under ordinary circumstances is the same everywhere. The presence this vacuum energy term can significantly affect cosmological theories.

Modern cosmology is mostly built upon Einstein's theory of general relativity in which the universe can be pictured as a finite entity with no boundaries rather like the surface of a ball. Einstein theory does not imply that the universe must necessarily be finite as one of the perceived problems with a finite universe is that with only attracting masses in the universe there seemed to be way to build a model so that the universe was not continually a model so that the universe was not continuously contracting under its gravity. In order to solve his problem Einstein invented the cosmological constant, which acted like negative mass uniformly distributed throughout space acting in opposition to matter and so keeping the universe static.

Einstein included the term in the equations of general relativity because his equations did not allow for a stationary universe. Gravity would cause a universe which was initially a dynamical equilibrium to begin to contract. To counteract the concentration, Einstein added the cosmological constant. However, soon after Einstein developed his theory, observations by Edwin Hubble indicated that the cosmos is not at equilibrium, but rather in expanding. Moreover, adding the cosmological constant to Einstein's equations did not lead to a universe at equilibrium because the equilibrium is unstable if the universe expands slightly then the expansion releases vacuum energy, which causes more expansion yet. Likewise, a universe which contract slightly will continue contracting, There sorts of small contractions are inevitable, due to the uneven distribution of matter throughout the universe.

**1.6 Hubble's Law**

Hubble discovered that the galaxies recede from the earth with a velocity proportional to their distance i.e. the recession velocity is proportional to the distance.

Recessional velocity=(Hubble's constant) times distance.

i.e. (1.5)

where V is the observed velocity of the galaxy away from us, usually in km/sec.

H is Hubble's constant in km/sec/MPc.

D is the distance of galaxies in MPc.

The basic entity in this law is, however, the Hubble's constant $H$ which has to be first calibrated accurately before the law can be used. The calibration of H, however, contains some inherent uncertainties in it.

One has to find by independent methods the distances to galaxies for which the red shifts are significant; The recessional speed must largely supersede the random speed of the galaxy. For the purpose, the Virgo Cluster of galaxies has so far been considered the most suitably situated. It contains a large number of bright galaxies, and it distances , the photometric method of distances measurement is applicable on the one hand and, on the other hand, red-shifts are significant. But unfortunately, the random velocities of the individual member galaxies of the Virgo Cluster about the center of mean recessional motion are of the same order as the mean recessional motion itself.

From the above relation, we know that the Hubble Parameter or Hubble constant H defines the rate of cosmic expansion. The recession velocity V of an object situated at a distance D was given by . Also it is the logarithmic derivative of the scale factor i.e. .

Since the mean recessional speed of the cluster is computed from the motions of these individual members, themselves having a significant random motion, substantial uncertainty may be introduced in the computation of the mean recessional speed.

When this rate is used to compute the value of , that value should be accepted with reservation have devotedly worked for many decades for the correct evaluation of .

In 1929 Hubble estimated the value of the expansion factor, now called the Hubble constant to about . For many decades the controversy rests between two groups of astronomers. Alan Sandage and his coauthors claim by their observation that , Also, Vaucauleurs and his coauthors argue that the value should be around 10km/sec/Mpc. But many outsiders thought the geometric mean of their value was a good compromise. The controversy persists while authors often work with some intermediate value of in the sixties and seventies lots of work had been done with . Considering various aspects of the problem and inherent uncertainties in the determination. A. Dressler has suggested that should be a better fair value. Many researchers are however currently working with the value . But from the latest source the Hubble space, Telescope key team came up with the answer

 (1.6)

And finally, WMAP came up with

 (1.7)

where million light years.

**1.7 Energy-momentum tensors**

(a) Perfect fluid: An ideal fluid is a frictionless, homogeneous and incompressible fluid which is not capable of sustaining any tangential stress or action in the form of shear but the normal force acts between the adjacent layers of fluid. The pressure at every point of a perfect fluid is equal in all directions, whether the fluid is at rest or in motion. It can be completely characterized by its rest frame energy stresses, viscosity and heat conduction.

 For a perfect fluid whose characteristics are the possessions of a proper density, a four-velocity and a scalar pressure , and then the complete energy momentum tensor is given by

 (1.8)

where is the mass density, is the four-velocity of the individual particles and is the pressure.

(b) Viscous fluid: A viscous fluid resists movement or the movement of an object through the fluid. The energy-momentum tensor for the viscous fluid is

 (1.9)

with four velocity vector, isotropic pressure

and fluid density and are the coefficients of shear and bulk viscosity respectively.

Here, is the volume expansion of fluid lines;

and is the projection tensor and is the shear tensor given by

 (1.10)

(c) Electromagnetic field: An electromagnetic field is a physical field produced by electrically charged objects. It affects the behavior of charged objects in the vicinity of the field. The electromagnetic field extends indefinitely throughout space and describes the electromagnetic interaction.

If we consider a flowing field of charged matter which is described by a proper density , a four-velocity and current density vector . Then for this field energy-momentum tensor is given by

 (1.11)

**1.8 Dark energy and Dark matter**

Dark matter does not interact with the electromagnetic force like common matter which implies that it doesn't retain or reﬂect or emit light, making it extremely hard to spot. Indeed, specialists have been able to infer the presence of dark matter just from the gravitational effect it appears to have on the visible matter.

Dark energy seems, by all accounts, to be connected with the vacuum in space. It is disseminated uniformly all through the universe, in space as well as in time. The even appropriation implies that dark energy does not have any local gravitational effects, yet rather a worldwide impact on the universe all in all which prompts to a repulsive force, which tends to accelerate the expansion of the universe. The rate of expansion and its acceleration can be measured by observations based on the Hubble law.perceptions in view of the Hubble law. These estimations, together with other scientiﬁc information, have conﬁrmed the presence of dark energy.

The present matter-vitality thickness of the Universe is near its basic estimation of which is credited to Dark Energy, to Cold Dark Matter and just is the normal matter of baryonic nature.

**1.9 Robertson-Walker Metric**

For simplicity of the mathematical description of the cosmological problem, we make the natural assumption that matter is distributed homogeneous and the large-scale distribution of extragalactic nebula clusters in space is roughly isotropic. Also, we desire that the geometry of space be determined by the matter distributions. This general requirement is known as Mach's principle. By Mach's principle, we require that the geometry of the three-dimensional space be homogeneous like the matter distribution. Therefore, we start with a purely geometrical study of the form of a metric which describes four -dimensional subspaces of similar geometry. The following mathematical hypothesis is our starting points:

(I) There exists a global time coordinate which serves as the of a Gaussian co-ordinate system.

(II) The 3-dimensional spaces belonging to varying constant values of this time-coordinate are local isotropic.

(III) Any other points in a three-space belonging to a given fixed time are equivalent.

Let denote the proper time in the co-moving frame. By setting , we have the line element in the form,

 (1.12)

The component of the metric tensor is thus unity i.e.

 (1.13)

Further, a particle at rest in a co-moving frame, that is, one with must satisfy the free-falling equation of motion.

The equation of a test particle is

 (1.14)

for any affine parameter via the relation , where is chosen as constant along the particle trajectory in such a way that, for massless particle it vanishes.

By choosing the affine parameter p the proper time, and using equation (1.14), we have

 (1.15)

And therefore,

 (1.16)

From equations (1.14) and (1.17), the non-trivial part is

 (1.17)

The validity of the two results (1.14) and (1.17) depend only on the choice of the co-moving frame.

the most general form of the metric in spherical polar co-ordinates for isotropic universe is

 (1.18)

From equations (1.14), (1.18) and (1.19), we have

 (1.19)

Equation (1.14) does not uniquely fix the time variable and remains unchanged, under the transformations

 (1.20)

where ) is an arbitrary function of spatial co-ordinates.

The off diagonal term from equation (1.19) can be eliminated, by choosing

 (1.21)

against the freedom of co-ordinate transformation. The line element after dropping the primes comes out to be

 (1.22)

whereand are unknown functions.

By assuming the universe to be homogeneous i.e. the line element retains its form under diosplacement of spatial coordinates. Let us consider an infinitesimal spatial displacement

,i=1,2,3 (1.23)

where is infinitesimal parameter. Homogenity means that the form of the metric is preserved under the transformation (1.24) that

; (1.24)

The transformation of equatioon (1.24) is not affected by the results . Under the transformation of covariant tensors, we have from equation (1.24)

 (1.25)

and from the Taylor's series expansion we get

 (1.26)

From equation (1.24), (1.25) and (1.26), we obtain

 (1.27)

We have three equations from(1.28) by taking and

 (1.28)

 (1.29)

and

 (1.30)

Equation (1.29) requires that be a function of r but not time . Hence must be factorized as

 (1.31)

The same argument when applied to (1.30) and (1.31) leads to

 (1.32)

Now, our $r$ co-ordinate has not been completely fixed as yet, since (1.23), (1.32) and (1.33) are unchanged under a transformation of type with arbitrary. We can use to set

 (1.33)

This means that we have chosen our co-ordinate such that the surface area of a sphere with radius is equal to In other words, at a suitably chosen time (when ), the surface area is . The variable and evidently have the usual meaning of angle variables in polar co-ordinates. Thus our spatial variables are now completely specified. From equations (1.29) and (1.31), we have

 (1.34)

From equations (1.30), (1.31) and (1.32), we have

 (1.35)

where is any arbitrary constant.

If we consider the translation to be along axis this law is . This gives , then equation (1.35) gives the solution

 (1.36)

where is a constant.

From equations (1.23), (1.32), (1.33), (1.35) and (1.37), we can write the form as

 (1.37)

In the above, is an unknown function of time. At one instant of time, the three-dimensional space is isotropic and homogeneous. In other words, can be defined as "cosmic standard time" such that at every instant of time the universe looks isotropic and homogeneous.

**1.10 Brans-Dicke Theory of Gravitation:**

Brans and Dicke (B-D) formulated a theory of gravitation (Brans and Dicke 1961), in which besides a gravitational part, a dynamical scalar field is introduced to incorporate for variable gravitational constant and Mach's principle in Einstein's theory. It can be considered as a natural extension of Einstein's general theory of relativity. The simplest case of the scalar tensor theory (Brans 1997), is defined by a constant coupling parameter $\omega$ and a scalar field $\phi$. In B-D theory, the gravitational constant varies inversely of a time dependent scalar field which couples to gravity with a coupling parameter $\omega$. One important property of this theory is that it gives expanding solutions (Mathiazhagan and Johri 1984, La and Steinhardt 1989) for scalar field and scale factor which are compatible with solar system observations (Perlmutter et al.1999, Riess et al.1998, Garnavich et al. 1998). The solar system observations (Bertotti et al. 2003) also impose lower bounds on . General relativity is recovered when goes to infinity (Weinberg 1972) and from timing experiments using the Viking space probe (Reasenberg et al. 1979), must exceed 500. This constraint ruled out many of extended inflation (Weinberg 1989, La and Steinhardt,1989) and provides a succession of improved models of extended inflation (Holman et al. 1990, 1991, Barrow and Maeda 1990, Steinhardt and Accetta 1990). Furthermore, all important features of the evolution of the universe such as: inflation (Mathiazhagan and Johri 1984), early and late time behavior of universe (Shogin and Hervik 2014), cosmic acceleration and structure formation (Banerjee and Pavon 2001), quintessence and coincidence problem (Sen and Seshadri 2003), self -interacting potential and cosmic acceleration (Errahmani and Ouali 2006), high energy description of dark energy in an approximate B-D (Weinberg 1989) could be explained successfully in the B-D formalism. For a large value of the parameter, B-D theory gives the correct amount of inflation and early and late time behaviors, while small and negative values explain cosmic acceleration, structure formation and coincidence problem. The dark energy, qualified as responsible for the cosmic acceleration determines the feature of a future evolution of the universe. The investigations on the nature of the dark energy lead to various candidates. Among them, the most popular one, the cosmological constant (Padmanabhan 2003, Mak et al. 2002, Caldwell et al. 2003), the dynamical scalar field like quintessence (Bertolami and Martins 2000, Caldwell et al. 2002,2003,2005, Tsujikawa and Sami 2004) or like phantom field (Bento et al. 1991, Cline et al. 2004, Nesseris and Perivolaropoulos 2007, Huang et al. 2008). Astronomical observations strongly indicates that the universe is undergoing a phase of accelerated expansion. The present day accelerated expansion of universe is naturally addressed within the Brans-Dicke theory with inverse of Hubble Scale and power law temporal behavior of scale factor. B-D Scalar-tensor theory of gravitation is quite important in view of the fact that scalar field play a vital role in inflationary cosmology. Many cosmological models (Barrow et al. 1993, Bento et al. 1992, Kolitch and Eardley 1995, Sahoo and Singh 2002, 2003, Luke 1976 ) can be successfully explained by using the B-D theory and it's extended versions. The solutions of the B-D field equations with the Robertson -Walker line element have been obtained by (Luke and Szamosi 1972) using a self consistent numerical method. They have obtained a lower bound in by taking in the field equations of Brans-Dicke (Dicke 1964) and concluded the presently available data cannot discriminate between different theories. Morganstern (1971) has obtained similar conclusion on the basis of the observed values of matter density, the Hubble's constant, the deceleration parameter and the ages of different objects in the universe.

The main objective of this thesis is to discuss cosmological models in the frame work of Brans-Dicke theory of gravitation considering Robertson-Walker metric in different scenario. Besides studying perfect and viscous fluids in different problems by considering various equation of state interacting with electromagnetic field and B-D field our objective is to compare our solutions with the present day observational findings.

**1.11 Mathematical formulation of Brans-Dicke theory**

The main feature of the Brans-Dicke theory is the inclusion of a variable gravitational constant varying inversely with a long-range scalar . Therefore, the mathematical formulation of Brans-Dicke theory should include this variable gravitational constant in a very natural wary Brans-Dicke proposed the development of the theory as a useful generalization of the conventional Einstein theory. This involves the modification of the variational principle of Einstein theory.

 (1.38)

where is the Riemannian curvature scalar, is the Lagrangian of matter and is the gravitational constant. The B-D field equations are obtained by dividing both sides of the equation (1.38) by and introducing another Lagrangian of the scalar field. Thus the variational principle for the B-D theory is presented by

 (1.39)

where is coupling constant which is undetermined. It much be in general of the other of magnitude unity.

Here it is to be mentioned that it is closely related to one of the Jordan (1955) type of variational principle. In Brans-Dicke theory we have restrictions that normal Lagrangian density of matter is a function of matter variable and only, and not a function of and the energy-momentum tensor must have a vanishing covariant divergence,

 (1.40)

where

 (1.41)

The condition of varying covariant divergence suggests a geodesic motion of a test particle in Brans-Dicke theory. The field equations for this theory are derived by varying the components of metric tensor and its first derivatives in the equation (1.39) as

 (1.42)

The wave equation for is obtained by varying and in the equation (1.39) and is given by

 (1.43)

Here the covariant de' Alembertian operator is defined to be covariant divergence of ,

 (1.44)

From equation (1.44) it is evident that and the lagrangian density for the scalar field serve as the source term for the generation of waves. The combined effect of these two terms can be replaced by the trace of the energy momentum tensor by contracting the relation given by (1.42)

 (1.45)

Then equation (1.42) can be reads as

 (1.46)

Thus equation (1.42) and (1.46) constitute the field equations of the B-D scalar-tensor theory.

**1.12Work related with Brans-Dicke Theory of gravitation**

Our research into the very early universe is focused on the theoretical and observational consequences of cosmological models which are presently a very active area of research. Morgeanstern (1970) obtained solutions for B-D cosmological equations for flat Friedmann type expanding the universe with . Nayak (1975) investigated the static-charged dust distribution in the B-D theory and shown that the ratio found from charge density to mass density is related to the precise interaction . The result implies the existence of a finite electron in the B-D theory of gravitation through a statically-charged dust distribution. Nayak and Tiwari 1976 obtained a class of exact plane symmetric interior solution top the B-D field equations for a physically reasonable equation of state. They also got a class of exact solutions to the source free B-D Maxwell field equations.Nayak and Tiwari 1978 had shown that the solution of the Brans-Dicke field equations corresponding to a static spherically symmetric charged dust distribution could be reduced to solving a generalized Schrodinger equation in one dimension.

Chauvet and Obregon (1979) derived a set of exact cosmological solutions for the Jordan-Brans-Dicke theory of gravitation. Batista (1980) studied the homogeneous solutions of Brans-Dicke for generalizing to a non-homogeneous space-time. But equations obtained did not give new solutions. Uehera and Kim 1982 studied the B-D equations with cosmological constant and obtained exact solutions for the spatially flat R-W metric in the matter dominated the universe.

Cerveró and Estévez (1983) studied general vacuum solutions for a cosmological R/W metric in the Brans-Dicke theory. Chauvet (1983) studied the general vacuum-universe solutions in Brans-Dicke cosmology for the non-flat space of the JBD cosmology in a parametric form through a rescaling of the scalar field Lorentz (1983,1984) in a series of papers derived a new method to obtain exact vacuum solutions for the class of spatially homogeneous space-time of the Friedmann-Robertson-Walker and Bianchi-Kantoswki Sach models. He has extended this work to get some perfect fluid solutions in B-D theory. Bertolami (1986) has studied about the time-dependent cosmological term by considering the B-D theory with a scalar field. Bertolami (1986) studied Time-dependent cosmological term in Brans-Dicke theory to find solutions of the field equations in a homogeneous and isotropic space-time. Weinberg E. J. (1989) studied some cosmological models with extended inflation scenario. Tarachand and Usham (1989) investigated a spherically symmetric perfect fluid distributions in Brans-Dicke theory along with electromagnetic field. Tarachand and Ratnaprabha (1989) studied exact solutions in B-D theory involving particle creation by considering the R-W line element. Berman et al. (1989) studied flat and nonflat static models with Brans-Dicke theory and R-W metric including the case which considers Bertolami time cosmological term. Berman (1989) studied inflationary phase in B-D theory with a cosmological constant. Bernmman and Som(1990) studied about the Brans-Dicke models with a time-dependent cosmological term and presented more general solutions than those submitted by Bertolami. Berman (1990) studied Brans-Dicke cosmology with a time-dependent cosmological term. Berman (1991) considered Cosmological models with a variable cosmological term of the form . Beesham (1993) found cosmological models with a variable cosmological term and bulk viscous models Recently, flat variable-$\Lambda$ cosmological models have been discussed by Berman. It is shown that these models are equivalent to perfect fluid models with a bulk viscosity which had been presented previously. Ahmadi-Azar and Riazi (1995) studied some cosmological solutions of Brans-Dicke theory with cosmological constant term. They introduced a category of actual cosmological solutions of Brans-Dicke (B-D) equations with constant in ﬂat Robertson-Walker metric. Banerjee and Beesham (1996) found precise solutions for the spatially ﬂat Robertson-Walker cosmological model in Brans-Dicke scalar-tensor theory within the presence of a causal viscous fluid. Ibochouba and Ali (1996) studied the dynamics of the particle creation within the relativistic viscous ﬂuid in Robertson-Walker universe by Brans-Dicke theory. Beesham (1996) mentioned Robertson-Walker cosmological models with a bulk viscosity in B-D theory that can be relevant to the inﬂationary situation. Ram and Singh (1997) studied bulk viscosity in early Brans-Dicke cosmological models. Banerjee and Sen (1997) had shown that the Brans-Dicke theory of gravity does not always go over to general relativity in the limit. It is also shown, by order of magnitude estimate, that one can get general relativity as a limit of Brans-Dicke theory when , only if the trace of the energy-momentum tensor is non-zero describing all fields other than the Brans-Dicke scalar field. Ibotombi and Biren (1999) investigated R-W universe filled with perfect fluid in B-D theory with gravitational field. Bertolami and Martins (2000) studied nonminimal coupling and quintessence. Sen and Sen (2001) considered the possibility of having a late time acceleration in Brans-Dicke cosmology, suggested by recent supernova observation. Banerjee and Pavón (2001) studied Cosmic acceleration without quintessence and argued that the present day accelerated expansion may be explained by Brans-Dicke theory without resorting to a cosmological constant or quintessence matter. Rao et al.(2009) studied string cosmological model with Brans-Dicke ﬁeld. With the help of a positive cosmological constant El-Nabulsi (2010) investigated dark energy and phantom energy with Brans-Dicke scalar field El- Nabulsi (2010) discussed the accelerated expansion of an anisotropic Brans–Dicke cosmological model. Bahrehbakhsh et al. (2011) examined FRW cosmology from five-dimensional vacuum Brans–Dicke theory. By considering variable and . Jamil et al. (2011) discussed Friedman-Robertson-Walker cosmological model. Interaction of B-D field and Chaplygin gas within an accelerating universe was investigatyed by Singha et al. (2011) . Chakraborty et al. (2011) discussed Brans-Dicke theory in anisotropic models with a viscous fluid. Arık et al. (2011) presented an exact cosmological solution of generalized B-D theory with the complex scalar field. Rao et al. (2012) discussed a dark energy cosmological model in a scalar-tensor theory of gravitation. Yousef Bisabr et al. (2012) investigated cosmic evolution of the universe considering B–D theory as well as modified B-D theory. Singh et al. (2012) presented Friedman Robertson-Walker models with particle creation in B-D theory. Considered LRS Bianchi type I Kucukakca et al. (2012) investigated cosmological models in Brans-Dicke theory. Sharif et al. (2012) discussed cosmic acceleration and Brans-Dicke theory. Rao et al. (2012) discussed a dark energy cosmological model which interacts with B-D scalar field Bianchi type-I metric. Rai et al. (2012) reviewed an anisotropic homogeneous cosmological model in a modified Brans-Dicke Cosmology. Farajollahi et al. (2012) analysed stability of holographic dark energy with Brans-Dicke theory. Reddy et al. (2012) examined field of a charged particle using scalar-tensor theory of gravitation. Pradhan et al.(2012) discussed a new class of Bianchi Type-I cosmological models in Scalar-Tensor Theory of Gravitation and Late Time Acceleration. Chaubey (2012) considered bulk viscous cosmological models with particle creation in B-D theory considering Bianchi type-V. Rai et al. (2013) discussed cylindrically Symmetric and Plane Symmetric Vacuum Cosmological Models in Brans–Dicke Theory. Reddy et al. (2013) discussed Kaluza-Klein Universe with Cosmic Strings and Bulk Viscosity in a Scalar-Tensor Theory of Gravitation. By using axially symmetric space-time metric and strange quark matter Rao et al. (2013) studied Brans–Dicke cosmological model universe. For different forms of the scale factor Pasqua and Chattopadhyay (2013) studied new agegraphic Dark Energy model in chameleon Brans-Dicke cosmology. Reddy et al. (2014) considered Bianchi type-V bulk viscous string cosmological model in Saez-Ballester scalar-tensor theory of gravitation. Sagar et al. (2014) discussed some bulk viscous string cosmological model in B-D theory by considering Bianchi type-III metric. Mishra and Chand (2016) studied cosmological models with bilinear deceleration parameter . Rasouli and Moniz (2016) investigated some exact cosmological solutions in modified Brans–Dicke theory. Chand et al.(2016) studied cosmological models in B-D theory with variable deceleration parameter as well as a dynamical cosmological term with FRW metric. Rai et al. ( 2016) obtained a plane symmetric cosmological universe filled with viscous fluid in B-D theory. By considering a non-minimal coupling between the matter and scalar field Singh et al. (2016) studied bouncing cosmological models. Recently, Fabris et al. (2016) introduced some views on classical and quantum B-D theory in a new manner.

**Chapter 2**

**Interaction of Gravitational field and Brans-Dicke field in R/W universe containing Dark Energy like fluid**

**2.1 Introduction**

Since many forms of dark energy are always accompanied and inter-related with a scalar field we are motivated to see whether the Brans-Dicke scalar field can manifest some form of dark energy and what roles it can play in causing the accelerated expansion of the universe. We are also motivated to investigate different interesting forms of model universes containing Brans-Dicke field interacting with gravitational field, and specially their inter-relation with dark energy in the evolution of our universe. And from our study we get the evidence for the existence of dark energy, in one form or the other, in almost all the model universes obtained by us under different conditions, during the periods of their evolution, which gives the testimony to the present accelerated expansion of the universe. One peculiarity of some of the models we obtain is the existence of two forms of dark energy simultaneously in such models, one from cosmological constant and other due to Brans-Dicke scalar field. In one case there is possibility of our model universe to collapse and become a black hole. Interesting enough, yet in another case, one of our models is facing the fate of a Big Rip. And one of the model universes we obtain seems to behave like a cyclic model of the universe.

**2.2 Solutions of Field Equations**

Here, we consider the spherically symmetric Robertson-Walker metric

 (2.1)

where is the curvature index which can take values .

The action of the Brans-Dicke (B-D) theory of gravity is

 (2.2)

where represents the curvature scalar; is the determinant of ; is a scalar field; is a dimensionless coupling constant; is the Lagrangian of the ordinary matter component.

The Einstein field equations in the most general form are given by

 (2.3)

where

 (2.4)

where is the trace of is the cosmological constant, is Ricci-tensor, is metric tensor and is the partial differentiation with respect to coordinate.

The energy-momentum tensor for the perfect fluid distribution is

 (2.5)

with is four velocity vector satisfying , is the pressure and is the energy density.

Here a comma or semicolon followed by a subscript denotes partial differentiation or a covariant differentiation respectively. Also the velocity of light is assumed as unity.

For the metric (2.1) surviving field equations are

 (2.6)

 (2.7)

 (2.8)

 (2.9)

 (2.10)

 (2.11)

where a dot and dash denotes differentiation with respect to time and .

Equations (2.6) and (2.7) gives

 (2.12)

From equation (2.12), we get

 (2.13)

Integrating equation (2.13), we get

 (2.14)

where and are functions of time.

Integrating equation (2.10), we get

 (2.15)

Equation (2.12) gives

 (2.16)

Using equation (2.15) in equation (2.16), we get

 (2.17)

From which it is obvious that is a function of only, i.e. in equation (15) gives

 (2.18)

Comparing equation (2.14) and (2.15), we get

From equation (2.14), we get

 (2.19)

Equations (2.17) and (2.18) gives

 (2.20)

Integrating, we get

 (2.21)

where is integration constant.

Using equations (2.19) and (2.20) in equations (2.6), (2.7), (2.9) and (2.11), we get

 (2.22)

 (2.23)

and

 (2.24)

Equations (2.22) and (2.23) gives

 (2.25)

Equations (2.24) and (2.25) gives

 (2.26)

Case I: When

In this case, equations (2.22), (2.23) and (2.26) reduces to

 (2.27)

 (2.28)

 (2.29)

Equation (2.29) leads to

, when (2.30)

, when (2.31)

, when (2.32)

where is arbitrary constant of integration.

Case I(a): When , we get

 (2.33)

From equation (2.21), we get

 (2.34)

From equation (2.19), we get

 (2.35)

The gravitational variable is given by

 (2.36)

Equation (2.27) and (2.28) gives

 (2.37)

and,

 (2.38)

Hubble's parameter is

 (2.39)

Scalar expansion is

 (2.40)

In this model universe, it is seen that the gravitational variable has a tendency to increase the pressure and decrease the density of the fluid whereas the Brans-Dicke scalar field has the tendency to decrease the pressure and increase the density of this universe. This model has a singularity at .

Case I(b): When , we get

 (2.41)



Figure 2.1: Graph of according to (2.41)

From equation (2.21), we get

 (2.42)

From equation (2.19), we get

 (2.43)

which is both function of and .

When , both and tends to . And when and , the B-D scalar tends to .

Therefore, we conclude that for the B-D scalar is an increasing function of both and .

The gravitational variable is

 (2.44)

which shows that gravitational variable decreases as and increases and tends to zero when either or .

Equations (2.27) and (2.28) gives

 (2.45)

and

 (2.46)

Hubble's parameter is

 (2.47)

Scalar expansion is

 (2.48)

For this model universe, the scalar field helps in the expansion of the universe. Also the expansion factor R increases with time thus making testimony to the expansion of the universe. Here in this type of model universe it is seen that pressure is negative and the equation of state . Thus this universe seems to be a universe containing dark energy due to cosmological constant . Again, here the scalar field also contributes to the expansion of this universe. Thus some part of the dark energy contained may be taken as quintessence form of dark energy

which is in agreement with the present day observational data, as according to the present observations, equation of state .

Case I(c): When , we get

 (2.49)

and

 (2.50)

From equation (2.13), we get

 (2.51)

which is both functions of and .

When , .

And either or , the B-D scalar tends to infinity.

The gravitational variable is given by

 (2.52)

which shows that gravitational variable decreases as and increases and tends to zero as or t.

Equations (2.27) and (2.28) gives

 (2.53)

and

 (2.54)

Hubble's parameter is

 (2.55)

Scalar expansion is

 (2.56)

Again for the solution in this case, it is found that the Brans-Dicke scalar field is singular at the origin. However, on the other hand, at the origin, the gravitational force is very strong. As time increases, the pressure decreases whereas the density increases. Thus there is possibility that the model universe in this case contracts gradually and at some stage the density will be very high, thereby it is possible that the universe becomes a black hole in course of time. Or, on the other hand, here the equation of state is whereas the pressure is negative. This implies that our model universe is an expanding universe containing dark energy due to the cosmological constant which is in agreement with the present observational data,

namely, .

Case II: When and

From equation (2.29), we get

 (2.57)

Integrating, we get

 (2.58)

where and are integration constants.

From equation (2.21), we get

 (2.59)

Case II(a): When

From equation (2.58) and (2.59), we get

 (2.60)

and

 (2.61)

From equation (2.19), we get

 (2.62)

which is function of both and .

The gravitational variable is given by

 (2.63)

where .

From equations (2.60), (2.61), (2.62) and (2.63), we see that the reality condition for and is and .

Equations (2.27) and (2.28) gives

 (2.64)

and

 (2.65)

which is function of both and . The reality condition is same as above.

Hubble's parameter is

 (2.66)

Scalar expansion is

 (2.67)



Figure 2.2: Graph of according to (2.67)

For this model universe, it is seen that at time given by there may be a gravitational collapse. Since, in this case, the energy density is negative there is possibility that this universe contains phantom form of dark energy. But there is a doubt in this case as here the

pressure is zero and this universe is closed, since dark energy is assumed to help in the accelerated expansion of the universe. Thus, when and , the problem reduces to the case of dust distribution.

Case II(b):When

From equations (2.58) and (2.59), we get

 (2.68)

and

 (2.69)



Figure 2.3: Graph of according to (2.68)

From equation (2.19), we get

 (2.70)

which is function of both and .

When , the radius of universe tends to infinity.

And the B-D scalar tends to infinity either when or .

The gravitational variable is

 (2.71)

From equation (2.71), we see that the gravitational variable decreases when and increase and tends to zero when or .

Equations (2.27) and (2.28) gives

 (2.72)

and

 (2.73)

which is real where .

Hubble's parameter is

 (2.74)

Scalar expansion is

 (2.75)

In the solution for this case, it is obtained that as time increases the radius of our (model) universe increases, that is our universe is an expanding one which is the sign of being a realistic one. But here it is seen that this universe expands initially at a high rate and gradually the expansion slows down until it stops at infinitely large time preparing for contraction. In this model universe the Brans- Dicke field has its influence in the area given by , and is inversely proportional to the gravitational potential due to . Thus, when and , the problem reduces to the case of dust distribution.

Case II(c):When .

From equations (2.58) and (2.59), we get

 (2.76)

and

 (2.77)

From equation (2.76), we know that that radius of the universe tends to infinity when t tends to infinity.

From equation (2.13), we get

 (2.78)

which is function of both and .

When either or , the B-D scalar tends to infinity.

The gravitational variable is

 (2.79)

which show that the gravitational variable is decreases when and increase and tends to zero when either or .

Equations (2.27) and (2.28) gives

 (2.80)

And

 (2.81)

Hubble's parameter is

 (2.82)

Scalar expansion is

 (2.83)

From equation (2.81), we see that decreases where is fixed and increases and increases when increases and decreases.

Regarding our model universe in this case, we have seen, from the expressions of and , that the scalar field has a tendency to increase the radius of the universe, thereby helping in the expansion of the universe. The density of this universe is also seen to decrease with time which is the sign of a realistic universe. The expansion factor here is found to increase with time, thereby implying our universe to be an expanding one which is in testimony with the present universe. Thus, when and , the problem reduces to the case of dust distribution.

Case III: When and

Since is function of . So, we consider the only case . Then, equation (2.26) reduces to

 (2.84)

Integrating, we get

 (2.85)

where and are arbitrary constant of integration.

From equation (2.21), we get

 (2.86)

If , the radius of the universe increases as t increases and tends to infinity as tends to infinity.

From equation (2.13), we get

 (2.87)

which is function of both and .

If , the B-D scalar tends to infinity either when or .

The gravitational variable is given by

 (2.88)

If decreases as r and t increase and tends to zero when either or .

Equations (2.22) and (2.23) gives

 (2.89)

and

 (2.90)

Hubble's parameter is

 (2.91)

Scalar expansion is

 (2.92)

Regarding the solution obtained in this case, the gravitational variable G is found to vary inversely with the scalar field \phi. Thus in this case the Brans-Dicke scalar field has a tendency to decrease the gravitational potential. For this universe it is seen that the equation of state ,namely, . Thus the dark energy contained in this universe may be taken as the k-essence form of energy. Here we see that for the k-essence energy, with , the scalar field grows in the future. And since the k-essence fields are similarly uniform on small scale, the abundance of k-essence energy within a bound object grows with time, thereby expecting a growing influence on the internal dynamics. Ultimately, there is possibility that the repulsive k-essence energy will overcome the forces holding this model together and rips apart this universe in a Big rip. Thus, when and , the problem reduces to the case of dust distribution.

Case IV: When and

Since is function of only. Therefore, we consider the only case

Then, equation (2.26) reduces to

 (2.93)

Integrating, we get

 (2.94)

and

 (2.95)

If the radius of the universe tends to infinity as tends to infinity.

From equation (2.13), we get

 (2.96)

which is function of both and .

When either or , the B-D scalar tends to infinity.

The gravitational variable is

 (2.97)

which is function of both and .

From equation (2.97), we see that the gravitational variable decreases when and increase and tends to zero when either or t.

Equations (2.22) and (2.23) gives

 (2.98)

and

 (2.99)

Hubble's parameter is

 (2.100)

Scalar expansion is

 (2.101)

In this model universe, the scalar field is seen to have a tendency to increase the expansion of the universe, thereby flattening the universe. Here, also the Brans -Dicke field has a tendency to decrease the gravitational potential. And the gravitational variable G tends to decrease the pressure and the density of the universe. Since here, as , it is found that as well as , there is possibility of a bounce at some point of time, thereby remembering us of this universe to behave as a cyclic universe. If , then this model universe will have an accelerated expansion instigated by the negative pressure. And in this model the vacuum energy due to the cosmological constant may be taken as the dark energy part causing the accelerated expansion of the universe.

**2.3 Conclusion**

The universes we have investigated are found to behave in different ways and to show different manifestations under different conditions. Some of them show signs containing cosmological constant form and quintessence form of dark energy, whereas some others seem to contain fluids behaving like phantom and k-essence forms of dark energy, which can explain the present day accelerated expansion of the universe. Thus the model universes we obtain in these cases may be taken as realistic models of our universe, and many more unknown properties of the universe and of dark energy may be realized and known from further studies of these models, which we will perform and report elsewhere afterwards. Further, one model of ours seems to undergo a gravitational collapse leading to a black hole; whereas another model surprisingly seems to face the fate of a Big Rip. Another new finding in some of our models is that they contain simultaneously two forms of dark energy, one due to cosmological constant and another due to Brans-Dicke scalar field. And, interestingly enough, one of our models seems to behave like a universe obeying the newly proposed cyclic theory of the universe.