Chapter- 1
Introduction

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Introduction

1.1 Introduction

This chapter is the initial part of our investigation which contains all applicable information for the topic of the thesis that helps the readers to understand the topic very easily. Here, we have highlighted the definition and history of the cosmology, some Principles and laws associated to the topic, cosmological models, the fate of the Universe, different cosmological parameters, related basic terminologies, some observational facts for the dark energy and present accelerated expansion of the Universe, some candidates for dark energy, some cosmological problems, Einstein's field equations, Friedmann-Robertson-Walker (FRW) metric and Friedmann equations. Further, the objectives of the research work are also presented here.

1.2 Definition and history of the Cosmology

Cosmology is a Greek word and its meaning is the beauty of the sky. The stars, star clusters, galaxies, nebulae, pulsars, quasars, cosmic rays and background radiation are the main elements of the Universe. Cosmology is a systematic and scientific investigation of the Universe. Some areas relevant to the Cosmology are Astronomy, Astrophysics, Particle Physics, Plasma Physics, and Quantum Mechanics. Since we are an element of the Universe, therefore Cosmology attracts all of us. From the last few years, cosmology has made lots of advancement.

Cosmology has a very lengthy history and there are many statements and theories proposed during this long era. In the olden times, people believed that God made the heavens and the earth, the earth was null and void, only darkness over the empty space and a mighty wind were blown over the surface of the earth, then God said, let there be light and He said that the 'light' is 'day' and the 'darkness' is 'night'. Moreover, several people believed that, the sky and the earth were the only two components of the Universe. Accordingly, the sky was populated by more and larger entities, and the earth was like a little speck in the measureless Universe. Various opinions have been developed by the cosmologists about the origin and evolution of the Universe.

Greek philosophers Aristotle (384 BC-322 BC) promulgated that the Universe had no beginning, but is and always was. In 1654, Irish Bishop James Ussher (1581-1656), after studying biblical chronology had announced that the Universe was created at 9 A.M on 23rd October, 4004 B.C. as per the calendar. Probably based on this date, the Big-Bang theory state that the Universe is at least 10-15 billion year old. But in the 19th century, Cosmologists confirmed that Bishop Ussher's creation date was wrong. Nobody can give a particular date and time about the formation of the Universe till now. The Cosmologists believed that the Universe was created only after a sudden explosion. English Mathematician, Physicist and Astronomer Sir Isaac Newton (1642-1727), changed the previous thinking of the cosmology. He spent much more of his time on alchemy and biblical dating and he did not agree with the James Ussher's creation date of the Universe. He imagined that the Universe was finite in space. He also realized that gravity is the most important force to understand the large-scale structure of the Universe. He assumed that at the beginning, the universe was sufficiently cold so that only gravitational attraction played a role; at that time there was also no pressure. Greek Mathematician, Astronomer, Geographer and Astrologer Claudius Ptolemy (100 AD-170 AD) proposed that the Universe is geocentric, that is, the earth lies at the centre of the Universe.

Aristotle projected that the spherical earth is enclosed by concentric celestial spheres containing the stars and therefore the Universe is static, has finite extent and exists throughout eternity. The Aristotelian model was acknowledged in Western world for regarding 2 millennia. Supported the Aristotelian model, Ptolemy projected that the Universe revolves around the stationary earth and therefore the planets move on circular orbits whose centre's once more move in a very larger circular orbit with a centre close to the earth. The Ptolemic geocentric theory was the accepted theory till sixteenth century. Numerous philosophers and astronomers particularly Italian astronomer and man of science Galileo Galilei (1564-1642) opposed this theory controversy that if the Universe were geocentric , then the sun and therefore the different heavenly bodies would have to be compelled to orbit the planet on massive ways with tremendous speed that wasn't attainable. Probably, the Greek mathematician and astronomer Aristarchus of Samos (310 BC-230 BC) was the primary to propose a heliocentric model of the Universe inserting the sun at the centre and therefore the earth orbiting it on a circular path whereas it's rotating on its axis. He additionally projected that the stars are fixed and therefore the centre of the sphere containing all the stars is at the sun. Throughout the Middle Ages, the Indian astronomer Aryabhata (476 CE-550 CE) additionally projected a model of the Universe. However he wasn't certain whether or not it had been geocentric or heliocentric. Within the middle of sixteenth century, Polish astronomer Nicolaus Copernicus (1473-1543) revived Aristarchu's theory of heliocentric Universe and argued that astronomical information might be explained higher if the earth revolved on its axis and the earth with the other planets rotated around the sun and therefore the sun were placed at the centre of the Universe. The idea of the earth's rotation on its axis is very older. Greek philosopher and scientist Philolaus (470 BC-385 BC), Greek philosopher and astronomer Hiraclides Ponticus (387BC-312BC), Greek pre-Socratic philosopher and mathematician Pythagoras (ca.570 BCE -ca.496 BCE), German philosopher and astronomer Nicholas of Cusa (1401-1464) and Iranian astronomer Al-Sijzi (945-1020) also proposed several models based on rotation. Experimental data was provided by Persian philosopher Nasir al-Din Tusi (1201-1274) and Uzbekistan astronomer and mathematician Ali Qushi (1403-1474). German astronomer, mathematician and astrologer Johannes Kepler (1571-1630) also introduced heliocentric Universe. Johannes Kepler established the famous laws known as Kepler's laws of planetary motions between 1609 and 1619.In 1687, Sir Issac Newton highlighted in the Principia Mathematic how the heavens move. Using Kepler's laws of planetary motions, Newton proposed his famous law of gravitation and this law was considered as a suitable explanation of the gravitational force between masses for more than two hundred years.

The Theory of Relativity is the foundation of the modern cosmology. The Theory of Relativity is basically two types namely the Special theory of relativity and General Theory of Relativity. Both the relativity belonged to Albert Einstein (1879 - 1955). The Special Theory of Relativity deals with the inertial system i.e. system moving with uniform velocity and has been applied in mechanics, electromagnetism and quantum theory The General Theory of Relativity deals with the non-inertial system i.e. system moving with accelerated velocity and has been applied to study the theory of gravitation and the structure of the Universe.

In 1905, Einstein introduced the Special Theory of Relativity in his noted research paper "*On the Electrodynamics of Moving Bodies*". There are two postulates in the Special Theory of Relativity ……..

i) The laws of physics are the equal for all observers in the same motion relative to one another.

ii) The speed of light in free space is the same for all inertial observers, independent of the relative velocity of the source and the observer.

The first postulate states the covariance of the physical laws in every inertial system. It is a fact drawn from the failure of the Michelson and Morley experiments to determine the velocity of earth relative to ether.

The second postulate says that in vacuum, the velocity of the light is constant and independent of the velocity of the source and the observer. The second postulate contradicts the Galilean transformation. Consequently, Lorentz transformation has been introduced.

In 1915, Einstein published the general theory of relativity in his remarkable research paper "*cosmological consideration on the General Theory of Relativity*". Einstein's General theory of relativity is based on three principles namely

a) Principle of covariance

b) Principle of equivalence

c) Mach's principle

The Principle of Covariance states that all laws of physics are of the same form in all space time co-ordinate systems which are in any kind of relative motion. This principle implies that the laws of physics are to be expressed in covariant equations, that is, in tensor equations which do not make use of any particular co-ordinate system.

The Principle of Equivalence states that in the neighborhood of a point, the gravitational fields produced by the attraction of masses and the fields produced by accelerating a frame of reference (co-ordinate system) are equivalent.

Mach’s principle states that the inertial properties of the material objects are created and determined by the interaction of the object masses with all the other masses in the Universe.

Today, Einstein’s General Theory of Relativity is the most important tool for the cosmologists to describe the nature of the Universe in different ways. Based on these principles Einstein wrote his field equations for the gravity, which become the core of the General Theory of Relativity. German cosmologist Karl Schwarzschild (1873-1916) solved the first non-trivial exact solution of the Einstein’s field equations in 1915 and published it in the month of January 1916. Prior to 1920, it was believed that our galaxy, the Milky Way, made up the whole Universe. American astronomer Harlow Shapley (1885-1972) also supported this concept. During 1922-1923, American astronomer Edwin Powell Hubble (1889-1953) measured the distances of spiral nebulae by Hooker Telescope and found that Andromeda and Triangulum galaxies and many other galaxies are well outside the boundary of our galaxy and concluded that there are a great number of galaxies in the universe with vast tracks of empty space between them. In 1912, after measuring the first Doppler shift of spiral nebula, American astronomer Vesto Melvin Slipher (1875-1969) discovered that almost all such nebulae were moving away from the earth (Slipher, 1913). Later on, in 1924, using Slipher’s data, American astronomer Milton La Salle Humason (1891-1972) obtained velocities and Edwin Hubble measured distances of spiral nebulae and found that the farthest nebulae are receding faster than the nearest nebulae from the earth. In 1917, using the Einstein General Theory of Relativity, Dutch mathematician, physicist, and astronomer William de Sitter formulated a model of an expanding universe. In 1922, Russian cosmologist and mathematician, Alexander Friedmann (1888-1925) obtained an exact solution of Einstein’s field equations which predicts that the red-shift of a galaxy should be directly proportional to its distance from us.

In 1927, Belgian astronomer and cosmologist Georges Edouard Lemaitre (1894-1966) derived Friedmann’s equations severally and ended that the recession of the nebulae was because of the growth of the Universe. However the works of Friedmann and Edouard Lemaitre wasn't responded at that point as a result of everyone believed that the Universe was static, even Einstein was additionally positive regarding it. In 1935, their works became known once similar models were derived severally by

American mathematician and scientist H. P. Robertson (1903 –1961) and British scientist A.G.Walker (1909-2001) in response to Hubble's discovery of the uniform growth of the Universe. The image of increasing Universe dates back to solely 1929 once Edwin Hubble supported the observations of Cepheid variable stars in distant galaxies, detected that the galaxies are receding quicker than the nearer ones with velocities proportional to their distances from us. As the growth causes matter and energy to cool down and unfolded with time, it reveals that our Universe should have began from some extent mass abundant hotter and denser than it's these days. The scientific model of the evolution of the Universe, that explains however the current day Universe developed from an especially hot and dense starting, is thought as the big bang theory. The big Bang theory was projected by Georges Edouard Lemaitre in 1927 and was developed by Soviet-American theoretical scientist and astronomer George Gamow (1904-1968), American astronomer Ralph Asher Alpher (1921-2007) and United States scientist Robert Herman (1914-1997) in 1948. According to this theory, the universe originated from a really hot, high density and high pressure state called big bang state. After the big Bang, the Universe had undergone a short period of terribly fast expansion referred to as inflation and because the Universe cooled down, it's been increasing since then. During the primary three hundred thousand years it had been thus hot that matter might exist solely in an exceedingly dense state of plasma. Galaxies began to make by the time the Universe was a couple of billion year old. Stars have shaped in those galaxies, producing significant components that were recycled into later generations of stars. This theory was supported by Hubble's discovery of galactic red shift in 1929 and also the discovery of cosmic microwave background (CMBR) by American scientist Arno Allan Penzias and American astronomer Robert Woodrow Wilson in 1964. The big Bang theory is currently the foremost accepted theory for the origin and evolution of the Universe. But, still, the scientists aren't certain whether or not the Universe started after an explosion from some extent singularity with high density and high pressure or it simply started increasing from a state having some volume, density and pressure. There's another theory, referred to as Steady state theory, regarding the origin, evolution and ultimate fate of the Universe. In 1948, the steady state theory was projected by English astronomer Fred Hoyle (1915-2001). Consistent with this theory, the Universe is static, has steady state, had no starting in time and remains constant for all time to come back. This theory was additionally advocated by Aristotle. Even Einstein thought that the Universe was static and to possess static Universe he introduced a constant, referred to as cosmological constant, in his field equations that was later abandoned by him.

In 1920s and 1930s, most of the cosmologists favored steady state Universe. In 1981, another vital development happened in cosmology, when A.H. Guth and E. J. Weinberg (Guth & Weinberg, 1983) projected inflationary model of the early Universe. Guth inferred that if the growth of the very early Universe is accelerated in such some way that the scale factor of the Universe grows by an element throughout a brief period just when the Big Bang, then, the ‘flatness’ and ‘ horizon’ issues of the big Bang theory might be avoided. Till the late 1990s, the astronomers thought that the expansion of the Universe was decelerated because of the attraction of the masses in it. But, in 1998, two independent groups of astronomers, one guided by A. G. Riess and B. P. Schmidt (Riess et al., 1998) and also the other by S. Perlmutter (Perlmutter et al., 1998) proved that the growth of the Universe wasn't decelerating, rather it had been accelerating and therefore the expansion history of our Universe over the past 5-6 billion years. Riess et al. (1998) within the High Red-shift supernova Search Team analyzed 16 distant and 34 close supernovae and Perlmutter et al. (1998) within the expanding Cosmology Project Team analyzed 18 close supernovae and 42 high red shift supernovae. Their experimental information powerfully indicates that the current Universe is increasing with acceleration and therefore the present rate of expansion is more than the rate of expansion of the Universe five billion years past.

Later on, it was established by a good number of cosmological observations such as Wilkinson Microwave Anisotropy Probe (WMAP), Large Scale Structure (LSS), Baryon Acoustic Oscillations (BAO), Cosmic Microwave Background (CMB) and their cross-relations. But, the cosmologists are unable to discover the actual cause of this acceleration. Most of the cosmologists suggest that an energy with negative pressure known as dark energy is the actual cause of this acceleration. But the nature of the dark energy is still unknown. There are many candidates for the dark energy proposed by the researchers. The cosmological constant is the simplest candidate for the dark energy which is introduced by Albert Einstein in his field equations. Moreover, Quintessence, Phantom, Tachyon, K-essence, Dilaton, Polytropic gas, Chaplygin gas, Holographic dark energy, Holographic Ricci dark energy, Agegraphic dark energy, New Agegraphic dark energy, Wet dark fluid, Viscous fluid, Quintom etc. are also considered as candidates for the dark energy.

Some researchers recommend that the acceleration of the Universe is because of the repulsive gravitational interaction of antimatter (Benoit–Levy & Chardin 2012; Hajdukovic, 2012) or deviation of attraction laws from General Theory of Relativity. Many modified theories of gravity like f(R) theory of gravity, Scalar-tensor theory, Brans-Dicke theory, etc. have additionally been projected within the literature. To analyze the late-time cosmic acceleration, f (T) theory of gravity, a modification of Teleparallel Equivalent of General Theory of Relativity is additionally thought-about within the literature. There's additionally an exotic matter in our Universe, dubbed dark matter, that emits no electromagnetic radiation however are often inferred to exist from gravitational effects on visible matter. In 1933, the existence of dark matter was 1st acknowledged by Swiss American astronomer Fritz Zwicky (Zwicky, 1933) once he compared the dispersion velocities of galaxies with the determined mass within the Coma cluster. Consistent with information offered from the observations by Planck team (Ade et al., 2014a &2014b), the Universe is concerning 13.799 billion years old. The age of the Universe was firm by mapping fluctuations in temperature within the cosmic microwave background radiation (CMBR). The Universe contains principally dark energy, dark matter and normal matter. The Universe additionally contains a much bit of electromagnetic radiation and antimatter.

1.3 Principles and laws

1.3.1 Cosmological Principles

Cosmological principles were developed by the astronomer Edward Arthur Milne. There are two basic assumptions employed in constructing Cosmological models of the Universe. These are

(i) The Universe is homogenized and isotropic, that is, it's the same everyplace on cosmological scale and therefore the same in each direction seen from any location.

(ii) The same laws of physics, that are valid on the earth, are valid for the complete Universe and for all time.

1.3.2 Hubble's law

In 1929, Edwin Hubble discovered that distant galaxies are receding from us with a speed v given by

$$v = Hd \tag{1.1}$$

That is understood as Hubble's law, wherever d the right distance of the galaxy from us and$H$ is that the constant of proportion referred to as Hubble constant.

1.4 Cosmological Models

Cosmological models are mathematical as well as astrophysical justification about the origin, evolution, geometry, contents, behavior and ultimate fate of the Universe. The cosmological models are based on direct experiments and observations. Most of the cosmological models are based on the Einstein's General Theory of Relativity, field equations and Cosmological Principles. In the General Theory of Relativity, gravity has a geometric property of the space-time and the field equations indicate the relations between the curvature of the space-time and the energy-momentum. So, the solutions of the field equations describe the evolution of the Universe. The simplest and well known cosmological model is the Lambda Cold Dark Matter (ΛCDM) model. It is based on the General Theory of Relativity and the Cosmological Principle. In this model, the Universe contains cosmological constant Λ which deals with dark energy and cold dark matter. It is considered as the 'Standard cosmological model' of the Universe. It provides the existence and structure of the cosmic microwave background (CMB), large scale structures, abundance of Hydrogen, Helium and Lithium and the accelerating expansion of the Universe. Most of the cosmological models are based on this model.

1.5 The fate of the Universe

The ultimate fate of the Universe depends on its mean densityρ and critical densityρ_c. It is an interesting topic in physical cosmology.

Big Crunch: If$\rho > \rho_c$, then by the big bang theory, the Universe would expand endlessly until it would achieve its highest size. Afterward, it would start to collapse due to the attraction of the masses inside it. As a result, the smaller stars would fall into the bigger stars due to this attraction and accordingly, the bigger stars would rise bigger and bigger. After that, these bigger stars would start to merge and the Universe would slowly arrive near to a hot dense state like Big Bang state. This phenomenon is known as Big Crunch. But this situation may happen after 20 billion years (Davies, 1997; Wang et al., 2004; Bergman, 2003).

Big Bounce: Immediately after the Big Crunch, the Universe would rebirth with one more Big Bang and would start to expand. This incident is known as Big Bounce (Brandenberger & Peter, 2017).

Cyclic Universe: After the Big Bounce, the universe would expand first and after that end as another Big Crunch followed by another Big Bang and the process would carry on without end. The Big Crunch of the earlier Universe would be the Big Bang of the next Universe and the Universe might contain countless cycles of finite universes. This type of Universe is called cyclic Universe (Steinhardt & Turok, 2001a & 2001b).

Big Rip: If the Universe expands endlessly without limit, then the Universe would arrive at a position where all material objects of the Universe starting with the stars, galaxies and eventually all forms, no matter how small, would be torn apart and disappear. This position is known as Big Rip. It is irreversible (Caldwell et al., 2003; Devlin, 2015).

Big Freeze: If$\rho \leq \rho_c$, then the expansion of the Universe would delay for some period, but never stop. For the collision, the masses of the Universe would gather and the temperature of the Universe would slowly reduce to absolute zero. This situation is known as Big Freeze (Griswold, 2012).

Heat Death: If protons were unstable, then because of proton decompose, the baryonic matter would vanish and only radiation and black holes would stay. At last the black holes would dissolve leaving radiation only. This event is known as Heat Death (Adams & Laughlin, 1997).

1.6 Cosmological parameters

The cosmological parameters describe the physical state of the Universe. There are many cosmological parameters used in cosmology. Some of them are discussed below.

1.6.1 Scale factor of the Universe

In cosmology, the scale factor of the Universe is a dimensionless parameter. It is a function of cosmic time$t$ and denoted by $a(t)$. It describes how the size of the Universe is changing with respect to its size at the present time. It is the ratio of the proper distance between two heavenly objects at some time$t$ to the proper distance between the two heavenly objects at the present time t_0. Thus, if $d(t)$ is the proper distance between two heavenly objects at some time$t$ and $d(0)$is their distance at the present timet_0 then, the scale factor of the Universe is given by

$$a(t) = \frac{d(t)}{d(0)} \tag{1.2}$$

Again, if z is the red-shift of a light ray originally emitted by a distant heavenly object at the present time t , then

$$a(t) = \frac{1}{1+z} \text{(Mukhanov, 2005)} \tag{1.3}$$

The scale factors for different eras are obtained by solving the Friedmann equations in the Friedmann–Lemaître–Robertson–Walker metric. They are

(i) $a(t) \alpha t^{\frac{1}{2}}$, for radiation dominated era,

(ii) $a(t) \alpha t^{\frac{2}{3}}$, for matter dominated era

(iii) $a(t) \alpha \exp(Ht)$, for dark energy dominated era, where $H = \sqrt{\Lambda/3}$

1.6.2 Hubble parameter

From Hubble's law, we have $v = Hd$, where v is the velocity of recession of a galaxy at a distance d from us and $H$is a constant known as Hubble constant or Hubble parameter. The Hubble parameter $H$is a function of the cosmic time t. Again, from the definition of scale factor of the Universe, the Hubble parameter$H$ can be written as

$$H = \frac{\dot{a}}{a} = \frac{\dot{d}}{d} \tag{1.4}$$

The Hubble parameter$H$ is used to describe the expansion of the Universe.

1.6.3 Equation of state parameter

If P is the pressure and ρ is the energy density of a perfect fluid then its equation of state (EoS) parameter(ω) is

$$\omega = \frac{P}{\rho} \tag{1.5}$$

Where ω may be a constant or a function of the cosmic time t. It plays a significant role to describe the evolution and the ultimate fate of the Universe.

If $\omega = 1$ then the Universe is stiff fluid dominated

If $\omega = \frac{1}{3}$ then the Universe is radiation dominated

If $\omega = 0$ then the Universe is dust dominated

If $\omega = -1$ then the Universe is vacuum energy dominated (negative pressure)

If $\omega = -\frac{1}{3}$ then the Universe is dark energy dominated (accelerated expansion)

If $-1 < \omega < -\frac{1}{3}$ then the Universe is quintessence dominated

If $\omega < -1$ then the Universe is phantom energy dominated and it expands exponentially to reach Big Rip.

1.6.4 Density parameter

If ρ is the actual density and ρ_c is the critical density of the Universe at time t, then the density parameter(Ω) is defined by

$$\Omega = \frac{\rho}{\rho_c} = \frac{8\pi G\rho}{3H^2} \tag{1.6}$$

From the Friedmann's first equation, we have

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} \tag{1.7}$$

This gives

$$\Omega - 1 = \frac{K}{(aH)^2}, \tag{1.8}$$

Where $K = -1,0,1$

For $\Omega > 1$ or $K = 1$, Universe would be closed,

For $\Omega = 1$ or $K = 0$, Universe would be flat,

For $\Omega < 1$ or $K = -1$, Universe would be open.

If Ω_m, Ω_Λ and Ω_K are the density parameters for the matter, dark energy and curvature respectively, then we have

$\Omega_m = \frac{\rho_m}{3H^2}$, $\Omega_\Lambda = \frac{\rho_\Lambda}{3H^2}$ and $\Omega_k = -\frac{K}{(aH)^2}$ so that

$$\Omega_m + \Omega_\Lambda + \Omega_K = 1 \tag{1.9}$$

Again from the observational data (Riess et al., 2004; Spergel et al., 2003), we have $\Omega_m \approx 0.3$ and $\Omega_\Lambda \approx 0.7$

1.6.5 Deceleration parameter

The deceleration parameter is a dimensionless measure of the acceleration of the expansion of the Universe, which is denoted by q and is defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{1.10}$$

Here a is the scale factor of the Universe and the dots indicating the derivatives with respect to cosmic time.

The expansion of the Universe is accelerating if $\ddot{a} > 0$ or$q < 0$ and decelerating if $\ddot{a} < 0$ or$q > 0$.

Again from the Hubble parameter $H = \frac{\dot{a}}{a}$, we have

$$\text{Hence } \frac{d}{dt}\left(\frac{1}{H}\right) = -\frac{\dot{H}}{H^2} = \frac{\dot{a}^2 - a\ddot{a}}{\dot{a}^2} = 1 - \frac{a\ddot{a}}{\dot{a}^2} \tag{1.11}$$

Therefore q can be expressed in terms of H as

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -\frac{\dot{H}}{H^2} - 1 \tag{1.12}$$

1.6.6 Expansion scalar

The expansion scalar is denoted by Θ and is defined by

$$\Theta = 3H = 3\frac{\dot{a}}{a} \tag{1.13}$$

This measures the relative rate of expansion or contraction of the Universe.

1.6.7 Anisotropy parameter

The anisotropy parameter is denoted by A_m and is defined by

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\frac{(H_i - H)^2}{H} \tag{1.14}$$

Where H_i (i = 1, 2, 3) are directional Hubble parameters.

If$A_m = 0$, then the Universe becomes isotropic

1.6.8 Shear scalar

The shear scalar is denoted by σ and is defined by

$$\sigma^2 = \frac{1}{2}\left[\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\Theta^2\right] = \frac{3}{2}A_m H^2 = \frac{1}{6}A_m\Theta^2 \tag{1.15}$$

If $\sigma = 0$, then the Universe becomes isotropic.

1.7 Basic Terminologies

The word ‘Terminology’ is a universal word for the group of particular words related to a fixed area and their studies. In this section, we have studied some specific terms that are direct related to the topic of the thesis.

1.7.1 Baryonic matter

Baryonic matters are composed of baryons. The baryons are basically protons and neutrons. After the initial inflation and at the starting of the Universe, when the Universe cooled down adequately, baryons were formed. Planets, stars, neutron stars, comets, asteroids, black holes are examples of baryonic matter. Baryonic matter makes up about 5% of the total content of the Universe (Persic & Salucci, 1992; Copi et al., 1995).

1.7.2 Non baryonic matter

A hypothetical form of matter not containing baryon is called non baryonic mater. Non-baryonic matter has been suggested as an element of the dark matter in the Universe. It had no role in the creation of the elements in the early Universe. But, it has gravitational effect like baryonic matter.

1.7.3 Dark matter

Dark matter could be a hypothetic matter and it's totally different from the normal matter. It doesn't emit, reflect or absorb light or the other electromagnetic wave. Thus the matter doesn't act with the electromagnetic force; on the opposite hand it interacts with the normal matter gravitationally. So, dark matter is also composed of baryonic matter. However, most of the cosmologists believed that the dark matter is dominated by non-baryonic matter.

Moreover, the galaxies are rotating in such a speed that the gravity generated by the tiny visible masses wouldn't are enough to stay them unbroken. They might are torn apart way back. This encourages the researchers to suppose that there's something additional masses that don't seem to be visible, however liable for the additional gravity that's required by the galaxies to exist. This additional invisible mass is dubbed as ‘dark matter’.

The existence of the dark matter was 1st discovered by the Dutch astronomer Jan Hendrik Oort (1900-1992) in 1932 and confirmed by Fritz Zwicky (Zwicky, 1937). In 1933, Fritz Zwicky studied a cluster of galaxies referred to as Coma cluster. whereas Zwicky estimated the mass of the cluster by observing motions of the galaxies close to its edge and scrutiny the same to the estimated mass supported the entire brightness and therefore the variety of galaxies in Coma cluster, He found that the cluster had concerning 400 times additional mass than its visible mass. So, He argued that there was some non-luminous matter that provided the additional mass and therefore the gravity needed to carry the cluster along. He dubbed this unseen mass as ‘dark matter’. Throughout 1960 to 1970, American astronomers Vera C. Rubin and W. Kent Jr. Ford established the existence of the dark matter by using galaxy rotation curves (Rubin & Ford, 1970). They found that the majority of the galaxies contain dark matter concerning six times the visible mass. One of the evidences of the existence of the dark matter is the Gravitational lensing. According to the Albert Einstein's General Theory of Relativity, the light travels in a straight line and it gets bending when it passes through a massive object. As a result, a massive object acts as a lens and therefore, the gravitational lensing can be used to measure the total mass of a galaxy or a cluster of galaxies. This represents the existence of dark matter. Today, all the cosmologists agree to the existence of the dark matter.

Dark matter helps in the structure formation of the Universe. The Structure formation started after the Big Bang when density perturbations collapsed to form stars, galaxies and galaxy clusters. Since, the ordinary matter is affected by the radiation, its density perturbations would not get enough time to condense to form any structure. Therefore, if there had been only ordinary matter in the Universe, then there would have been no stars or galaxies. But the dark matter protected this peculiar situation. As it is not affected by radiation, their density perturbation grow faster and helps structure formation and later the collapsing ordinary matter speeds up the structure formation process. There are three types of dark matters.

Hot dark matter: Hot dark matter (HDM) is a theoretical form of dark matter and its particles move with the ultra-relativistic velocity over 0.95c close to the velocity of light. HDM has no capable to support the structure formation. HDM may be dominated by neutrinos because neutrinos have small mass less than that of an electron and interact with ordinary matter only through gravity and weak force.

Warm dark matter: Warm dark matter (WDM) is a hypothesized form of dark matter and its particles move with relativistic velocity between 0.1c and 0.95c. It is heavier than HDM. WDM has no possibilities to the structure formation because it can’t be bound to galaxies due to its high speed. The neutrinos and gravitinos are the candidates of WDM.

Cold dark matter: Cold dark matter (CDM) is hypothesized form of dark matter and its particles travel with sub-relativistic or non-relativistic velocity, generally less than 0.1c. It is heaviest out of three dark matters. The Axions and the massive compact halo objects (MACHOS) such as black holes, neutron stars, white dwarfs, etc. are the possible candidates for the CDM. Most of the cosmologists suggest that the CDM helps in structure formation.

1.7.4 Dark energy

In cosmology, Dark energy is an interesting form of energy which is responsible for the accelerated expansion of the Universe. In 1998, two independent teams of astronomers, one guided by A. G. Riess and B. P. Schmidt (Riess et al., 1998) and the other by S. Perlmutter (Perlmutter et al., 1998) first proved that the Universe expands with acceleration and it is due to an unknown energy with negative pressure called dark energy. Later on, it was confirmed by a number of astrophysical and cosmological observations. According to the Planck Collaboration (Ade et al., 2014b), the total mass-energy of the Universe contains 68.3% dark energy. Therefore, determining the actual behavior of the dark energy becomes one of the most challenging problems in the Modern Cosmology.

1.7.5 Entropy

Entropy is related to the thermo dynamics. The word 'entropy' was first used by a German physicist and mathematician Rudolf Claucius (1822-1888) and its concepts was developed by French mechanical engineer Sadi Carnot(1796-1832), English physicist and mathematician James Prescott Joules (1818-1889), Irish-Scottish mathematical physicist and engineer William Thomson(1824-1907) and so on. It is a thermo dynamical quantity which represents the amount of the thermal energy of a given system. It is also defined as the measure of the level (degree) of disorder in a changing system in which energy can be transferred only in the direction from ordered state to disordered state. Since, the Universe was born and evolves in a most chaotic state so it is thought that the Universe will end when its entropy ceases to exist. The entropy plays an important role in the study of thermodynamics.

1.7.6 Thermodynamics and its laws

Thermodynamics is a division of the Physics containing the heat and temperature and their connection with the energy, work and radiation and. The laws of thermodynamics sum up the physical quantities such as energy, temperature and entropy. There are three laws of thermodynamics

1st law of thermodynamics: Energy cannot be created and destroyed but it is transferred from one kind to a distinct.

2nd law of thermodynamics: The entropy of any isolated system forever will increase.

3rd law of thermodynamics: The entropy of a system tends to a constant if its temperature tends to absolute zero.

1.7.7 Viscosity

Viscosity is a property of a fluid due to which it offers some resistance to sliding motion of a particle pass another particle. It is a measure of the resistance of the fluid for flow. Viscosity is also referred as the internal friction of a fluid. Almost all known fluids have this property.

A fluid is said to be perfect or non-viscous if it has no viscosity, that is, it has no resistance to flow. Again, a fluid is said to be real or viscous if it has some viscosity, that is, it has some resistance to flow.

1.7.8 Lagrangian

Generally, a Lagrangian is denoted by L and it is defined by

$$L = T - V \tag{1.16}$$

Here T and V are respectively its kinetic energy and potential energy.

For a system of particles, Lagrangian is defined by

$$L = \sum(T_i - V_i) \tag{1.17}$$

Here T_i and V_i are respectively the kinetic energy and potential energy of a particle of the system.

Lagrangian can be used in formation of the equation of motion (Euler's equation) and in the formulation of momentum.

Again, if we define a quantity $\mathcal{L}$ such that $L = \int \mathcal{L} dV$, then $\mathcal{L}$ is called Lagrangian density or simply Lagrangian, If φ is a single real scalar, then the Lagrangian is given by

$$\mathcal{L} = \frac{1}{2}\partial^{\mu}\varphi\partial_{\mu}\varphi - V(\varphi) \tag{1.18}$$

And for homogeneous model it becomes

$$\mathcal{L} = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \tag{1.19}$$

1.8 Observational facts for the dark energy and present accelerated expansion of the Universe

There are many observational facts for the dark energy and present accelerated expansion of the Universe. In this section, we have discussed about some well known observational facts specially Supernovae Type Ia (SNe Ia), Cosmic microwave background radiation (CMBR), Baryon Acoustic Oscillation (BAO), Wilkinson Microwave Anisotropy Probe (WMAP).

1.8.1 Supernovae Type Ia (SNeIa)

Supernovae (SNe) are very luminous and energetic stellar explosions that can last for several weeks. They are categorized mainly in two types such as supernovae Type I and supernovae Type II based on the existence of the hydrogen in their spectra. There are different subtypes for each category (Filippenko, 1997).

Type I (SNe I): not contains hydrogen lines.

Type Ia (SNe Ia): contains a line of silicon.

Type Ib (SNe Ib): contains a line of helium.

Type Ic (SNe Ic): neither contains a line of helium nor contains a line of silicon.

Type II (SNe II): contains hydrogen lines.

Here, we have discussed about the supernovae Type Ia (SNe Ia).

It is believed that the SNe Ia may occur anywhere and in any type of galaxy, on the other hand the other supernovae such as Type Ib, Type Ic and Type II may occur mainly in the populations of massive stars. SNe Ia is expected to come from the thermonuclear explosion of a carbon-oxygen white dwarf which reaches the limit of the Chandrashekhar mass ($M_{ch}\sim$ 1.4 M⊙) through the matter accretion from a companion star. This SNe Ia is considered as standard candles to study the expansion history of the Universe.

In 1997, S. Perlmutter et al. discovered 7 SNe Ia at redshifts $0.35 < z < 0.65$ and constructed the Hubble diagram using their light curves. The results obtained by them were consistent with the standard decelerating Friedmann cosmology (Perlmutter et al., 1997). But after one year, S. Perlmutter et al. again updated their results using another measurement of a very high redshifts SNe Ia at $z \sim 0.83$ (Perlmutter et al., 1998). These results changed the scenario of the standard decelerating Friedmann cosmology and it was ruled out at 99% confidence level. In the same year, these results were confirmed independently by A. G. Riess et al. (Riess et al., 1998). They discovered 16 SNe Ia at redshifts $0.16 < z < 0.62$ and the constructed Hubble diagram indicated the accelerated expansion scenario of the Universe at 99% confidence level. Later on, J. L. Tonry et al. added another eight newly discovered SNe Ia at redshift $0.3 < z < 1.2$ with the previously acquired dataset (Tonry et al., 2003). Their results supported the previous observations on rapid expansion of the Universe and also gave the first hint of decelerated expansion of the Universe at high redshift, $z \geq 0.6$. Thus the Universe had a transition from the decelerating to the accelerating expansion stage in the past. In 2004, it was A. G. Riess et al. also confirmed the first reliable proof of this transition. They also discovered 16 new high redshift SNe Ia using the Hubble Space Telescope (HST) and included them with the available dataset. Later on, they re-analyzed all the available dataset in a uniform manner and also constructed a strong and reliable dataset consisting of 157 points. This dataset was acknowledged as Gold dataset. With help of this dataset and applying a simple model of the expansion history, they accurately constrained the transition between the two epochs (from decelerating to accelerating expansion) at $z = 0.46 \pm 0.13$ (Riess et al., 2004).

There are many Supernovae surveys already done by the cosmologists. Some of such surveys with different red shift are ESSENCE Supernova Survey (Miknaitis et al., 2007), Supernova Legacy Survey (Astier et al., 2006), the SDSS-II supernova survey (Frieman et al., 2008) etc. The Compilation of the SNeIa from different surveys is extremely helpful for the cosmological studies. The Union 2.1 is one example of such compilation that provides the distance modulus to 580 SNeIa (Suzuki et al., 2011). The Constraints on Ω_m and ω as obtained by the Supernova Cosmology project from combination of different data sets including the Union compilation is shown in the figure-1.1

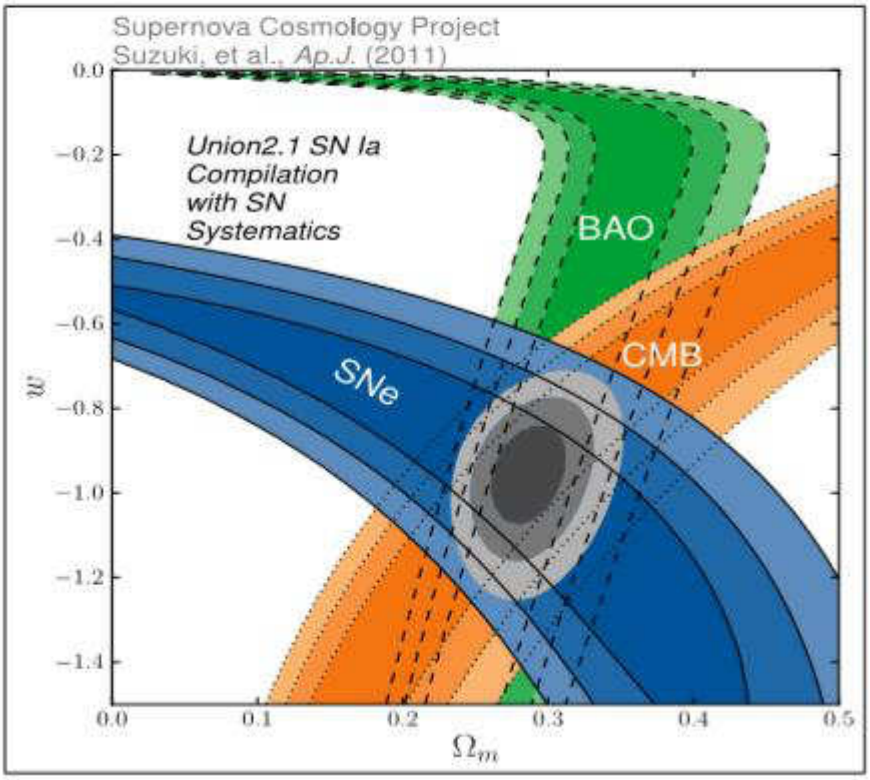

Figure-1.1: 68%, 95%, and 99.7% constraints onΩ_m and ωobtained from CMB (orange), BAO (green), and the Union Compilation (blue). Figure taken from the Union compilation (Suzuki et al., 2011)

1.8.2 Cosmic Microwave Background (CMB) Radiation

In the very early stages and prior to the formation of matter, the Universe was dominated by the radiation, mostly photons and neutrons. However, the radiation epoch ended after 47,000 years after the Big Bang, a fraction of it has been remaining. This remainder of the radiation in the form of the microwave band is recognized as cosmic microwave background (CMB) radiation. The existence of the CMB radiation was first predicted in 1948by American physicist Ralph Alpher along with Robert Herman and George Gamow and it was discovered by two radio astronomers Arno Allan Penzias and Robert Woodrow Wilson in 1964 (Penzias & Wilson, 1965). The accelerated expansion of the universe was confirmed by the independent evidence of the dark energy from measurements of CMB anisotropy (Jaffe et al., 2001& Pryke et al., 2002).

Various researchers have focused their contribution in the field of cosmological problems including the study on the dark energy and the CMB radiation (de Bernardis, P. et al., 2000; Hanany, S. et al., 2000; Melchiorri et al., 2000; Netterfield et al., 2002).

In figure-1.2, we show the all-sky picture of the Universe created from nine years of WMAP data.

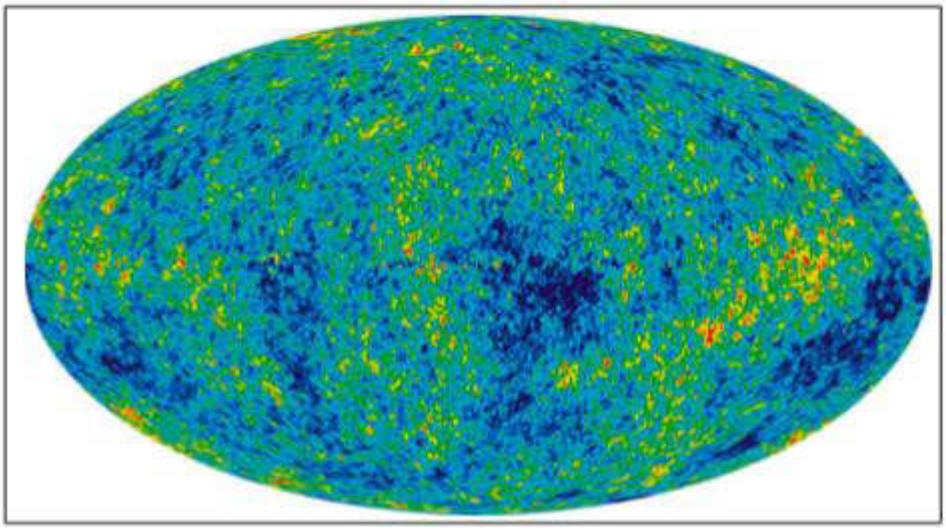

Figure-1.2: The picture of the temperature fluctuations of the 13.77 billion year ago with different colors related to the seeds that grew to become the galaxies. This picture indicates a temperature range of ± 200 micro Kelvin (image courtesy: NASA/WMAP Science Team)

1.8.3 Baryon Acoustic Oscillation (BAO)

Prior to recombination and decoupling, the Universe was dominated by the hot plasma composed of photons and baryons which were tightly coupled by means of Thomson scattering. The opposing forces of the radiation pressure and the gravity formed oscillations in this plasma. Spherical density perturbations in this plasma propagated outwards as acoustic oscillations. At the time of recombination, the Universe took the neutral position and the pressure on the baryons also vanished. The baryon wall was stalled and the photons begun propagating freely. Today, these photons are suggested as the cosmic microwave background radiation (CMBR).After recombination, baryons are decoupling from radiation and the oscillations are frozen (Bassett & Hlozek, 2009). This phenomenon is known as baryon acoustic oscillations (BAO).

BAO has an important role in cosmology and it is used as a 'standard ruler' for the length scale measurement. The length of this standard ruler is given by the utmost distance of the acoustic waves which could travel within the primordial plasma before the plasma cooled to the stage wherever it became neutral atoms and that stopped the expansion of the plasma density waves by 'freezing' them. The length of the ruler may be calculated by seeing at the large scale structure of the matter using cosmological surveys (Eisenstein et al., 2005).The BAO measurements help the cosmologists to know more about the nature of dark energy that indicates the accelerated expansion of the Universe by constraining cosmological parameters.

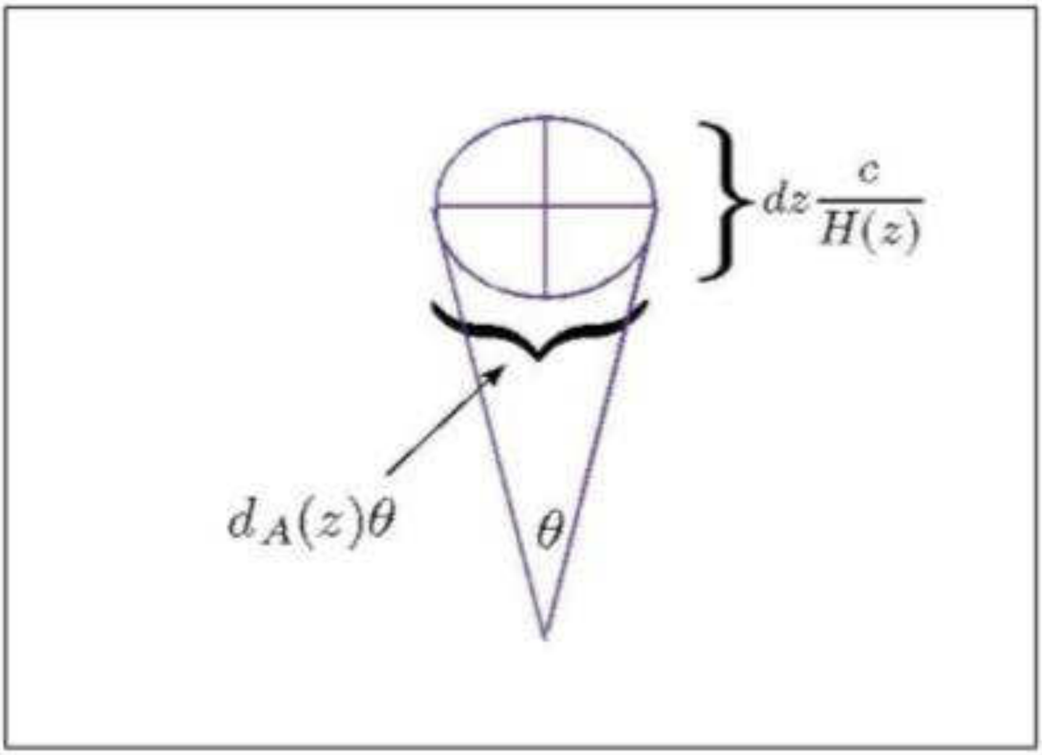

Figure-1.3: Diagram presentation of the radial length ($cdz/H(z)$)and the transverse size $dA(z)\theta$ of an object.For BAO, theoretically it can determine the diameter, and hence determine $dA(z)$ and $H(z)$ separately. [Image courtesy Bassett and Hlozek (2009)]

There are many cosmological surveys related to BAO. Some of them are 6dF Galaxy Survey (Beutler et al., 2011), Wiggle Z (Blake et al., 2011), BOSS (Anderson, et al., 2012) etc.

1.8.4 Wilkinson Microwave Anisotropy Probe (WMAP)

The Wilkinson Microwave anisotropy Probe (WMAP) was a notable spacecraft working from 2001 to 2010 to calculate the temperature variations across the sky inside the cosmic microwave background (CMB) – the heat remaining from the big Bang. This mission was developed during a joint partnership between the National Aeronautics and Space Administration (NASA) Goddard Space Flight Center and Princeton University guided by academician Charles L. Bennett of Johns Hopkins University. The WMAP space vehicle was launched on June 30, 2001 from Florida. The WMAP mission succeeded the Cosmic Background explorer (COBE) space mission and it was the Medium-class Explorers (MIDEX) spacecraft during the National Aeronautics and Space Administration Explorers program. The most results of the mission are contained within the numerous oval maps of the CBR temperature variations. These oval pictures point out the temperature sharing obtained by the WMAP team during the mission. The WMAP's measurements help the cosmologists to establish the present standard Model of Cosmology. From the 9 years of observations of the WMAP satellite, we've cleared several things. The present Universe is of 13.77 billion years old, and consists of 4.6% atoms, 24%, cold dark matter and 71% dark energy (Bennett et al., 2013; Hinshaw et al., 2013).The WMAP spacecraft diagram is shown in figure-1.4

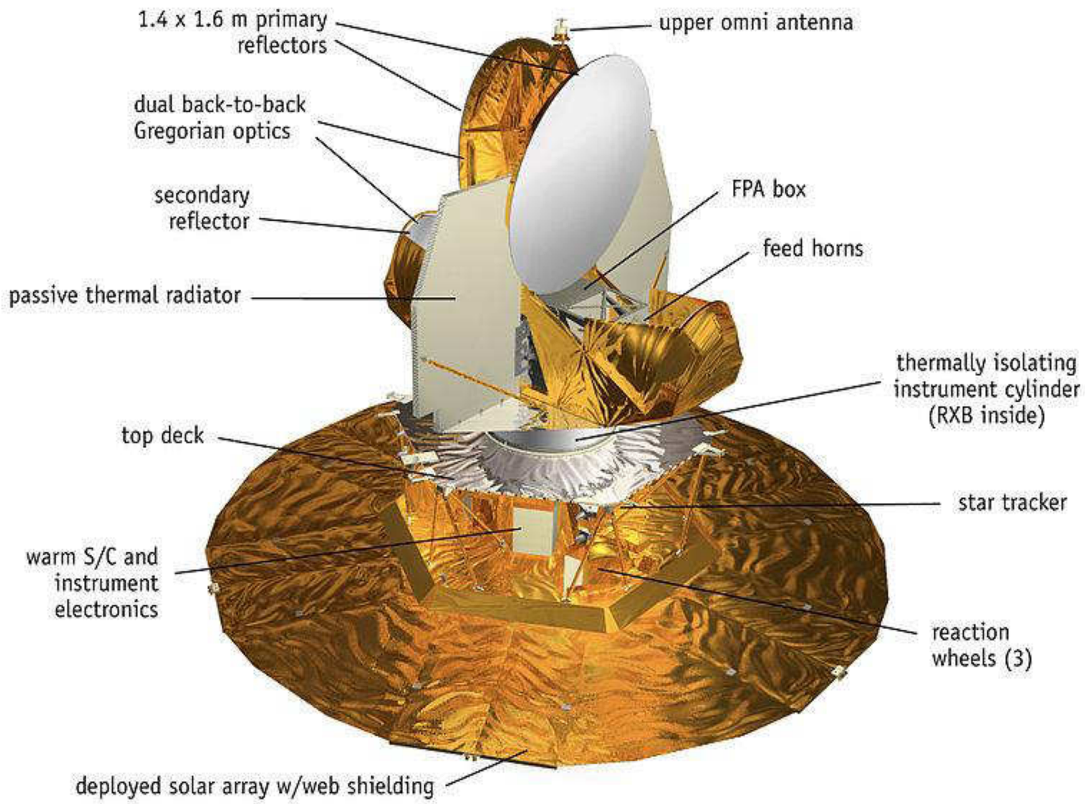

Figure-1.4: Image of the WMAP spacecraft diagram with major elements

Credit: NASA: http://map.gsfc.nasa.gov/m_ig/990115/990115L.jpg

1.8.5 Chandra X-ray Observatory

The Chandra X-ray Observatory (CXO) is a Flagship-class space telescope launched on 23rdJuly 1999 by the National Aeronautics and Space Administration. This telescope is recognized in the name of Nobel Prize victorious Indian-American astrophysicist Subrahmanyan Chandrasekhar. It is invented to determine the X-ray discharge from extremely hot regions of the Universe mainly from the exploded stars, clusters of galaxies and matters in the region of black holes that help cosmologists to know the structure and evolution of the Universe.

The image of the Chandra X-ray Observatory with major elements is shown in figure-1.5

Figure- 1.5: Image of the Chandra X-ray Observatory

Credit: NASA/CXC/NGST

1.9 Some candidates of Dark energy

There are lots of candidates for the dark energy proposed by the researchers to explain the nature of the dark energy that represent the expansion history of the Universe. Some of them have discussed below.

1.9.1 Cosmological constant

In cosmology, the cosmological constant represents the energy density of space, or vacuum energy and it is denoted by the Greek capital letter lambda Λ. It was firstly introduced by Albert Einstein in 1917 as a term in his field equations of the General Relativity to describe 'static Universe', because at that time it was believed that the Universe was static. But, in 1929, Edwin Hubble discovered that the Universe is expanding. During 1930s-1990s, most of the cosmologists proposed the cosmological constant to be equal to zero. It became the main point of discussion for the cosmologists when, in 1998, it has been confirmed from the observations of distant supernovae of Type Ia (SNe Ia) that the Universe is expanding with acceleration. This gives the possibility of a positive nonzero value for the cosmological constant. The cosmological constant is considered to be the simplest candidate for the dark energy. But, due to its non-evolving nature, the cosmological constant faces some problems. Firstly, its equation of state (EoS) parameter is $\omega = \frac{P}{\rho} = -1$ but, most of the observations indicates that ω is close to -1 not necessarily to -1. Secondly, cosmological constant is to be fine-tuned to have the observed small value (Weinberg, 1989).

The present observed value of Λ is

$$\Lambda = 1.1056 \times 10^{-52} m^{-2}$$

$$= 2.888 \times 10^{-122} \quad \text{in Planck units}$$

$$= 4.33 \times 10^{-66} eV^{2} \quad \text{in natural units.}$$

1.9.2 Scalar Fields

A scalar field on a region is a real or complex valued function defined at each point of the region. This region may be a subspace of the Euclidean or Minkowski space. The scalar fields are widely used in the particle physics and in the string theory. These can be formulated in such a way that they can act as a candidate for the dark energy. There are many scalar field models have been proposed by the researchers to explain the accelerated expansion of the Universe. Some of the popular scalar fields are Quintessence, K-essence, Tachyon, Phantom, Quintom, Dilaton etc.

Quintessence:

P. Ratra and L. Peeble initially proposed the concept of Quintessence (Ratra & Peebles, 1988).

The action for quintessence scalarφ is given by (Barreiro, T. et al., 2000)

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{ij}\partial_i\varphi\partial_j\varphi - V(\varphi)\right] \tag{1.20}$$

Here $V(\varphi)$ is the quintessence potential

The energy momentum tensor T_{ij} for the quintessence scalar φ is

$$T_{ij} = \partial_i\varphi\partial_j\varphi - g_{ij}\left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi + V(\varphi)\right] \tag{1.21}$$

Again the energy density ρ_φ and pressure P_φ for the quintessence scalar φ are

$$\rho_\varphi = -T_0^0 = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \tag{1.22}$$

$$P_\varphi = T_i^i = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \tag{1.23}$$

Hence, the EoS parameter ω_φ for the quintessence is given by

$$\omega_\varphi = \frac{P_\varphi}{\rho_\varphi} = \frac{\dot{\varphi}^2 - 2V(\varphi)}{\dot{\varphi}^2 + 2V(\varphi)} \tag{1.24}$$

Where ω_φ varies with time and always lies between $-1$and 1. But, for the accelerating expansion, $\omega_\varphi < -\frac{1}{3}$ if $\dot{\varphi}^2 < V(\varphi)$. The difference between cosmological constant and Quintessence is that cosmological constant does not change with time, but Quintessence is dynamic and its density and EoS parameter change in space and time.

K-essence:

K-essence is a short form of Kinetic Quintessence. Firstly, K-essence was known as K-inflation and it was used to explain the early time inflation of the Universe at high energies (Armendariz-Picon et al., 1999). This development was first applied to the dark energy by T. Chiba et al. (Chiba et al., 2000). Shortly, it was applied to the dark energy to explain the late time inflation and was called K-essence (Armendariz-Picon et al., 2001). The K-essence is driven by scalar field with a non-canonical kinetic energy.

The action for K-essence is given by (Armendariz-Picon et al., 1999).

$$S = \int d^4x\sqrt{-g}P(\varphi, X) \tag{1.25}$$

Where the Lagrangian $P(\varphi, X)$ represent the pressure density and $X = -\frac{1}{2}(\nabla\varphi)^2$

The energy density and pressure for the K-essence are given by

$$\rho = f(\varphi)(-X + 3X^2) \tag{1.26}$$

$$P = f(\varphi)(-X + X^2) \tag{1.27}$$

Hence, the EoS parameter ω_φ for the K-essence is given by

$$\omega_\varphi = \frac{1-X}{1-3X} \tag{1.28}$$

Thus, ω_φ does not vary if X is a constant. If $\frac{1}{2} < X < \frac{2}{3}$, then, $-1 < \omega_\varphi < -\frac{1}{3}$ and K-essence behaves as dark energy giving rise to accelerated expansion.

Again, if $X = \frac{1}{2}$, then $\omega_\varphi = -1$ and K-essence behaves like cosmological constant. Also, if $\frac{1}{3} < X < \frac{1}{2}$, then $\omega_\varphi = -1$ and K-essence behaves like Phantom dark energy driving the Universe to Big Rip.

The difference between Quintessence and K-essence is that Quintessence depends on potential energy of the field to explain the accelerated expansion of the Universe, whereas K-essence depends on modified form of kinetic energy to do the same.

Tachyon:

Generally, Tachyon or Tachyonic field is a scalar field with an imaginary mass. In 1967, Gerald Feinberg first introduced the term 'Tachyon' when he studied quantum field with imaginary mass (Feinberg, 1967) and later on, it plays a vital role in astrophysics (Kutasov, D. et al., 2000; Sen, 2002; Gibbons, 2002).

The action of the Tachyon scalar φ (Gibbons, 2002) is as follows

$$S = \int d^4x\sqrt{-g}\left[-V(\varphi)\sqrt{1 - g^{ij}\partial_i\varphi\partial_j\varphi}\right] \tag{1.29}$$

Here $V(\varphi)$is the Tachyon potential.

The energy density ρ_T and pressure P_T for the Tachyon scalar field are represented by

$$\rho_T = \frac{V(\varphi)}{\sqrt{1-\dot{\varphi}^2}} \tag{1.30}$$

$$P_T = -V(\varphi)\sqrt{1 - \dot{\varphi}^2} \tag{1.31}$$

Hence, the EoS parameter ω_T for the Tachyon scalar field is given by

$$\omega_T = \frac{p_T}{\rho_T} = \dot{\varphi}^2 - 1 \tag{1.32}$$

If $\dot{\varphi}^2 < \frac{2}{3}$, then $\omega_T < -\frac{1}{3}$ and Tachyon behaves as dark energy for accelerated expansion of the Universe**.**

Phantom Field:

The scalar fields for the candidates of dark energy discussed above was restricted to the EoS parameter $\omega \geq -1$. But the recent cosmological data's suggest that the EoS parameterω may be < -1 (Caldwell, 2002; Caldwell et al., 2003).The theoretical form of dark energy satisfying $\omega < -1$ is considered as a Phantom dark energy. A scalar field $\varphi$with negative kinetic energy providing explanation about the Phantom dark energy is known as Phantom field.

The action for the Phantom field is given by (Caldwell, 2002)

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}(\nabla\varphi)^2 - V(\varphi)\right] \tag{1.33}$$

The energy density and pressure for the field of Phantom (Malekjani, 2013) are

$$\rho = -\frac{1}{2}\dot{\varphi}^2 + V(\varphi) \tag{1.34}$$

$$P = -\frac{1}{2}\dot{\varphi}^2 - V(\varphi) \tag{1.35}$$

The EoS parameter (ω) for the Phantom field (Malekjani, 2013) is

$$\omega = \frac{P}{\rho} = \frac{\dot{\varphi}^2+2V(\varphi)}{\dot{\varphi}^2-2V(\varphi)} \tag{1.36}$$

Thus, $\omega < -1$, if $\dot{\varphi}^2 < 2V(\varphi)$and Phantom field behaves as dark energy for super accelerated expansion of the Universe and the Big Rip singularity may be avoided if the potential has a maximum.

Quintom:

When the Phantom field faces a trouble for the quantum instabilities, then, a new dynamical dark energy model, dubbed Quintom, is proposed. It is a model of two scalar fields, one of them has a canonical kinetic energy term and the other has a negative kinetic energy term. In the Quintom situation, the EoS parameter crosses the boundary $\omega < -1$) throughout the evolution. In this situation, the quantum instability trouble can be ignored because ω does not remain less than -1 forever. It remains less than -1 for a few time and then crosses to the other side. After a few time, it once more crosses to the Phantom side (Hu, 2005).

Dilaton:

Edmund J. Copeland et al. have demonstrated that the Phantom field with negative kinetic energy has a quantum instability problem (Copeland et al., 2006). In order to avoid this problem, so called Dilaton model is proposed which is later on used as a dark energy model. Also, it is believed that this model avoids the quantum instabilities with respect to the Phantom field. This model is found from the lower limit of string theory (Piazza & Tsujikawa, 2004).

The pressure and energy density for the Dilaton scalar field are respectively given by (Gasperini et al., 2002; Piazza &Tsujikawa, 2004; Copeland et al., 2006).

$$P_\varphi = -\chi + ce^{\lambda\varphi}\chi^2 \tag{1.37}$$

$$\rho_\varphi = -\chi + 3ce^{\lambda\varphi}\chi^2 \tag{1.38}$$

Here $c$and λ are the positive constants and $\chi = \frac{\dot{\varphi}^2}{2}$

For the Dilaton scalar field, the EoS parameter is

$$\omega_d = \frac{P_\varphi}{\rho_\varphi} = \frac{-1+ce^{\lambda\varphi}\chi}{-1+3ce^{\lambda\varphi}\chi} \tag{1.39}$$

The kinematic term with the negative coefficient in the Einstein frame indicates a Phantom like behavior for the Dilaton scalar field.

1.9.3 Holographic Dark Energy

The Holographic dark energy model is derived from the Holographic principle. The Holographic principle was first established by a Dutch theoretical physicist Gerard t' Hooft using black hole physics('t Hooft, 1993)and it had been extended by an American physicist Leonard Susskind within the context of string theory(Susskind, 1995).Consistent with his principle, the maximum number of degrees of freedom in a volume is proportional to the surface area.

In 1998, a new version of this principle for the cosmology is planned by W. Fischler and L. Susskind that states that the attraction entropy inside the closed surface must not be greater than the particle entropy going through the light-like surface of the entropy (Fischler & Susskind, 1998). The upper limit of the entropy of the Universe can then be obtained by applying this theory (Majumder, 2013). Backed the theory of quantum field, Cohen et al. suggested that the short distance cut-off is connected to the long distance cut-off due to the limit imposed by the creation of a black hole (Cohen et al., 1999). Alternatively, if the quantum zero-point energy density (ρ_Λ) is due to a short distance cut-off, the cumulative energy in the area of size L does not exceed the mass of the black hole.

Thus by M. Li (Li, 2004), we have

$$L^3\rho_\Lambda \leq LM_P^2 \tag{1.40}$$

Here M_P is the reduced Planck mass given by $M_P^{-2} = 8\pi G$ with G as the universal gravitational constant. If we think the total Universe, then this zero-point energy density associated with this Holographic principle will be expressed as dark energy called Holographic dark energy.

If the largest L that saturates the above inequality, then the Holographic dark energy density is given by (Li, 2004)

$$\rho_\Lambda = 3n^2M_P^2L^{-2} \tag{1.41}$$

Here $3n^2$is a constant introduced for convenience.

If we take the current size of the Universe as$L = H^{-1}$, where H being the Hubble parameter, then the Holographic dark energy density becomes

$$\rho_\Lambda = 3n^2M_P^2H^2 \tag{1.42}$$

It doesn't indicate the accelerated increasing Universe under Friedmann-Robertson Walker (FRW) space-time, as a result of the EoS parameter for it zero.

Also, M. Li (Li, 2004) thought-about L as the size of the future event horizon given by

$$R_h = a\int_t^{\infty}\frac{dt}{a} = \int_a^{\infty}\frac{da}{Ha^2} \tag{1.43}$$

In this case, the EoS parameter ω_Λ less than $-\frac{1}{3}$ and hence the accelerated increasing Universe is possible.

1.9.4 Agegraphic Dark Energy

The Holographic dark energy models face some problems. Firstly, it does not indicate the accelerated expanding Universe. Secondly, it arise some causality problems. To remove the problems, R.G. Cai (Cai, 2007) has introduced a new dark energy model, dubbed Agegraphic dark energy (ADE), where the time scale is chosen as the age of the Universe. The energy density ρ for ADE is defined by

$$\rho = \frac{3n^2M_P^2}{T^2} \tag{1.44}$$

Here the age of the Universe given by

$$T = \int_0^{a}\frac{da}{Ha} \tag{1.45}$$

Here a being the scale factor of the Universe.

In the Agegraphic DE model, the space time and matter field fluctuations are responsible for the dark energy.

1.9.5 New Agegraphic dark Energy

The Agegraphic dark energy model proposed by R.G. Cai (Cai, 2007) has some problems such as it cannot cover the matter dominated era. In order to solve these problems, H. Wei and R.G. Cai proposed a new model called the New Agegraphic dark energy (NADE) model wherever the time scale is preferred to be the conformal time as a replacement for of the Universe age (Sheykhi, 2009; Wei & Cai, 2008).

The energy density ρ for NADE is defined by

$$\rho = \frac{3n^2M_P^2}{\eta^2} \tag{1.46}$$

Here the conformal time η is defined by

$$\eta = \int\frac{dt}{a} \tag{1.47}$$

The NADE model proves the observational data from the Supernovae of Type Ia (SNe Ia), the cosmic microwave background and the large scale structure. Moreover, this model can solve the cosmic coincidence problem.

1.9.6 Polytropic Gas

There are many dark energy models proposed by the cosmologists to explain the nature of the dark energy as well as the late time cosmic acceleration of the Universe. The most popular one amongst these models is the Polytropic. Pressure is a function of energy density in this model (Das & Basak, 2018a; Nojiri et al., 2005; Mukhopadhyay et al., 2008; Karami et al., 2009).

The equation of state (EoS) for the Polytropic gas is derived from the equation of Lane–Emden and is given by (Das & Singh, 2020a; Karami et al., 2009)

$$P_\Lambda = K\rho_\Lambda^{1+\frac{1}{n}} \tag{1.48}$$

The pressure, energy density, Polytropic constant and index are denoted by P_Λ,ρ_Λ,K, and n respectively (Das & Basak, 2018a; Setare & Darabi, 2013).

The energy density and pressure for the Polytropic gas are

$$\rho_\Lambda = \left[Ba^{3/n} - K\right]^{-n} \tag{1.49}$$

$$P_\Lambda = K\left[Ba^{3/n} - K\right]^{-n-1} \tag{1.50}$$

Here B is a positive integration constant and $a(t)$ is the Universe's time scale factor.

The EOS parameter for the dark energy model of the Polytropic gas is thus obtained as

$$\omega_\Lambda = \frac{P_\Lambda}{\rho_\Lambda} = -1 + \frac{Ba^{3/n}}{Ba^{3/n} - K} \tag{1.51}$$

If $> Ba^{3/n}$, then $\omega_\Lambda < -1$, so a Universe dominated by the Phantom field corresponds to the accelerated expansion of the Universe (Das & Singh, 2020a; Karami et al., 2009).

1.10 Some cosmological problems

There is a pair of problems with the standard Big Bang model and the cosmological constant. In this section, we have studied about these problems.

1.10.1 Flatness Problem

The first problem of the standard Big Bang model is the Flatness Problem---why is the Universe so flat? The flatness problem can be understood with the help of the energy density parameter of the Universe. The energy density parameter (Ω) is a ratio of the actual density(ρ) and critical density (ρ_{cr}) of the Universe. Therefore

$$\Omega = \frac{\rho}{\rho_{cr}} \tag{1.52}$$

With $\Omega = 1, > 1$ or < 1 for the spatial curvature $k = 0,1 \text{or} -1$ respectively

If $\Omega = 1$ for zero curvature ($k = 0$), then the Universe is flat. But, after deviating from the non- zero curvature ($k = \pm 1$) for billions of years, Ω is still close to unity, accordingly the Universe is very close to flat. This condition is known as 'Flatness problem'.

1.10.2 Horizon Problem

The second problem of the standard Big Bang model is the horizon problem---why does the Universe look the same in all directions? Generally, the particle horizon divides the space into two regions. One is where the information can be sent to the observer from a particle and the other is where the information cannot be sent to the observer from any particle. Now, if there are two observers with the distinct horizons having no causal connection, then it is actually not possible for the Universe to build up homogeneity. But, the observations of the cosmic microwave background radiation (CMBR) indicate that the Universe is nearly homogenous. This horizon related problem is known as 'Horizon problem'.

1.10.3 Cosmological Constant Problems

The cosmological constant Λ introduced by Albert Einstein in his field equations as a candidate for the dark energy faces two big problems known as 'Cosmological constant problems'. These two problems are namely the fine tuning problem and the cosmic coincidence problem.

The fine tuning problem

The first problem faced by the cosmological constant Λ after the discovery of the accelerated expansion of the Universe in 1998 is its fine tuning. We know that the cosmological constant is roughly equal to the square of the present value of the Hubble parameter H_0(Copeland et al., 2006). Therefore,

$$\Lambda \approx {H_0}^2 = (2.13h \times 10^{-42} GeV)^2 \tag{1.53}$$

The corresponding critical energy density ρ_Λ is

$$\rho_\Lambda = \frac{\Lambda M_P^2}{8\pi} \approx 10^{-47} GeV^4 \tag{1.54}$$

Here $M_P = 1.22 \times 10^{19} GeV$, $h = 0.70$

Again, the vacuum energy density ρ_{Vac} is

$$\rho_{Vac} = \frac{M_P^4}{16\pi^2} \approx 10^{74} GeV^4 \tag{1.55}$$

Thus the vacuum energy density ρ_{Vac} is about10^{121} orders of magnitude larger than the observed value ρ_Λ (Copeland et al., 2006). This problem is well-known as ‘fine tuning problem’.

Cosmic Coincidence Problem

According to the Planck Collaboration, the dark matter, dark energy and ordinary matter contribute about 68.3 %, 26.8 % and 4.9 % respectively of the total content of the Universe (Ade et al., 2014b). But, the present energy density of the dark matter and the dark energy are of the same order, that is $\frac{\rho_{de}}{\rho_{dm}} \approx O(1)$. This is an extraordinary coincidence, because at a time, the dark matter energy density and dark energy density are different as well as they have the same order. This type of situation is known as ‘cosmic coincidence problem’.

1.11 Einstein’s Field Equation

The most important equations in cosmology are the Einstein’s field equations introduced by Albert Einstein in 1915. These equations are as follows

$$R^{mn} - \frac{1}{2} R g^{mn} = -\frac{8\pi G}{c^4} T^{mn} \tag{1.56}$$

The negative sign is inserted for later convenience, $T^{mn}$is a tensor containing the energy density and pressure of the matter, $R^{mn}$is the Ricci tensor, R is the trace of Riccitensor, G is the gravitational constant and $c$is the speed of light. The left hand side of Einstein's Field equations represents geometry and right hand side represents matter. Matter in the space-time tells space-time to curve and curvature in the space-time tells matter how to move.

Einstein modified his field equation in 1917. The modified Einstein field equations are

$$R^{mn} - \frac{1}{2} R g^{mn} + \Lambda g^{mn} = -\frac{8\pi G}{c^4} T^{mn} \tag{1.57}$$

Here Λ is the cosmological constant.

1.12 Friedmann -Robertson-Walker (FRW) Metric

The present Universe is homogeneous and isotropic as per the cosmological principle. The most general metric satisfying the cosmological principle is the Friedmann Robertson-Walker (FRW) Metric (Weinberg, 1972; Olive & Peacock, 2012).The Friedman-Robertson-Walker (FRW) metric also known as Friedmann Lemaitre-Robertson-Walker (FLRW) because this line element was developed independently by Alexander Friedmann, George Lemaitre, Howard P. Robertson and Arthur Geoffrey Walker. It is an exact solution of the Einstein's field equations of General Relativity.

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1-kr^2} - r^2 (d\theta^2 + sin\theta d\emptyset^2) \right] \tag{1.58}$$

Where $r, \theta, \emptyset$ are commoving spatial coordinates, t is the time and $a(t)$ is the scale factor of the expansion conveniently normalized to unity at present time. The values of the curvature constant k becomes 1, 0 or -1 for a closed, flat or open Universe respectively.

1.13 Friedmann Equations

For the Friedmann-Lemaître-Robertson-Walker metric and a perfect fluid with a given mass density and pressure, Alexander Friedmann first derived the Friedmann equations from the Einstein field equation of gravitation in 1992.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{1.59}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3} \tag{1.60}$$

Here ρ is the whole energy density of the universe, P is that the pressure and Λ is that the cosmological constant.

1.14 Objectives of the Research Work

(i) To create a correspondence between the dark energy model of Polytropic gas and other dynamical models of dark energy.

(ii) To reconstruct different dynamical dark energy models according to the behavior of the Polytropic gas dark energy models.

(iii) To study the Big Rip problems to know the fate of future Universe.

(iv) To discuss the cosmic and thermodynamic behavior of the Polytropic gas as a candidate for dark energy.

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