Chapter-5

Reconstruction of the Holographic and New Agegraphic Dark Energy Models for the Polytropic Gas

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5.1 Introduction

There are several cosmological observations and observational evidence suggesting an accelerated expanding Universe.(Perlmutter, 2003; Riess, et al., 2004 & 2007; Caldwell & Doran, 2004; Koivisto & Mota, 2006; Daniel et al., 2008; Spergel et al., 2003; Tegmark et al., 2004). Dark energy (DE) is liable for this expansion (Overduin & Cooperstok, 1998; Sahni, &Starobinsky, 2009). But the character of the DE is still researchable. The earliest candidate for the dark energy is cosmological constant (Weinberg, 1989). In addition to the cosmological constant, the researchers have suggested two different dynamic dark energy models (i) "scalar-field models" such as Quintessence (Wetterich, 1988; Ratra & Peebles, 1988; Barreiro et al., 2000; Capozziello et al., 2002). Phantom (Caldwell, 2002; Caldwell et al., 2003; Carroll et al., 2003;Nojiri & Odintsov, 2003), K-essence (Chiba et al., 2000; Armendariz-Picon et al., 1999 & 2001), Tachyon (Padmanabhan, 2002; Sen, 2002; Bagla et al., 2003), Dilaton (Gasperini et al., 2002; Piazza & Tsujikawa, 2004; Arkani-Hamed et al., 2004) etc. (ii) The "interacting models" such as Holographic (Horava & Minic, 2000; Thomas, 2002; Jamil et al., 2009),Agegraphic (Cai, 2007; Wei & Cai, 2008) etc. The Polytropic gas is a model for understanding the existence of dark energy (Das & Singh, 2020a). Pressure is a function of energy density in the Polytropic gas DE model (Mukhopadhyay et al., 2008; Malekjani, 2013).

The energy density for Holographic (Taji & Malekjani, 2013; Das & Singh, 2020a) is

$$\rho_\Lambda = 3c^2 M_P^2 L^{-2} \tag{5.1}$$

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Where c, M_P, L are dimensionless constant, reduced Planck mass and size of the Universe respectively.

The Agegraphic dark energy model is based on the Karolyhazy relation (Karolyhazy, 1966)

$$\delta t = \beta t_P^{\frac{2}{3}} t^{\frac{1}{3}} \tag{5.2}$$

Where β is a dimensionless constant and $t_P = l_P = \frac{1}{M_P}$ with t_P, l_P and M_P being the reduced Planck time, length and mass respectively. There are two types of Agegraphic models such as Original Agegraphic dark energy (OADE) and New Agegraphic dark energy (NADE) models. The OADE model is projected by R.G.Cai to clarify the accelerated expansion of the Universe, based on the uncertainty relation of quantum mechanics and the gravitational effect in general theory of relativity. But the OADE model has some difficulties. Because it cannot justify the matter-dominated era (Cai, 2007). This encouraged H. Wei and R.G. Cai (Wei & Cai, 2008) to intend the NADE model, where the time scale is chosen to be the conformal time.

5.2 Polytropic gas model

The Polytropic gas EOS (Karami et al., 2009; Das & Singh, 2020a) is

$$P_\Lambda = K\rho_\Lambda^{1+\frac{1}{n}} \tag{5.3}$$

The pressure, energy density, Polytropic constant and index are denoted by P_Λ,ρ_Λ,K, and n respectively (Setare & Darabi, 2013; Das & Singh, 2020a).

The equation of dark energy conservation is

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + P_\Lambda) = 0 \tag{5.4}$$

Solving (5.1) & (5.3) and integrating we have

$$\rho_\Lambda = \left[Ba^{3/n} - K\right]^{-n} \tag{5.5}$$

Here B is a positive constant of integration and a (t) is the Universe's time scale factor (Karami et al., 2009; Das & Singh, 2020a).

Also the pressure is

$$P_\Lambda = K\left[Ba^{3/n} - K\right]^{-n-1} \quad (5.6)$$

The EOS parameter of the Polytropic gas is

$$\omega_\Lambda = \frac{P_\Lambda}{\rho_\Lambda} = -1 + \frac{Ba^{3/n}}{Ba^{3/n} - K} \quad (5.7)$$

When $K > Ba^{3/n}$, then (5.6), implies $\omega_\Lambda < -1$, a Phantom field. It shows that the Universe expands with acceleration in the context of Polytropic gas.

In the scalar field φ , the pressure and energy density are

$$P_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \quad (5.8)$$

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \quad (5.9)$$

Also the scalar potential and the kinetic energy terms for the Polytropic gas are

$$V(\varphi) = \frac{\frac{B}{2}a^{3/n} - K}{\left(Ba^{3/n} - K\right)^{n+1}} \quad (5.10)$$

$$\dot{\varphi}^2 = \frac{Ba^{3/n}}{\left(Ba^{3/n} - K\right)^{n+1}} \quad (5.11)$$

If $> Ba^{3/n}$, then (5.11) implies $\dot{\varphi}^2 < 0$ and it shows a Phantom scalar field φ.

5.3 Holographic dark energy model

Energy density for Holographic dark energy (Huang and Li, 2004; Das & Singh, 2020a) is

$$\rho_\Lambda = 3c^2 M_P^2 R_h^{-2} \quad (5.12)$$

The proper size for future event horizon is

$$R_h = a(t)\int_t^\infty \frac{dt'}{a(t')} = \int_a^\infty \frac{da'}{H'a'^2} \quad (5.13)$$

The fractional energy density is

$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}$, where $\rho_{cr} = 3M_P^2 H^2$ is the critical energy density.

Equation (5.12) implies

$$HR_h = \frac{c}{\sqrt{\Omega_\Lambda}} \tag{5.14}$$

Now equation (5.13) implies

$$\dot{R}_h = HR_h - 1 = \frac{c}{\sqrt{\Omega_\Lambda}} - 1 \tag{5.15}$$

By means of the equation (5.12), with time, the rate of change of the Holographic dark energy is

$$\frac{d\rho_\Lambda}{dt} = -6c^2 M_P^2 R_h^{-3} \dot{R}_h = -2H\left(1 - \frac{\sqrt{\Omega_\Lambda}}{c}\right)\rho_\Lambda \tag{5.16}$$

The energy density equation for the conservation of energy momentum tensor (Das & Singh, 2020a) is

$$\frac{d}{da}(a^3\rho_\Lambda) = -3a^2 P_\Lambda$$

Moreover

$$P_\Lambda = -\frac{1}{3}\frac{d\rho_\Lambda}{dlna} - \rho_\Lambda \tag{5.17}$$

The Holographic EoS (Das & Singh, 2020a) is

$$\omega_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} = -\frac{dln\rho_\Lambda}{dlna} - 1 = -\frac{1}{3}\left(1 + \frac{2\sqrt{\Omega_\Lambda}}{c}\right) \tag{5.18}$$

Where $dlna = Hdt$ is applied.

Equation (5.18) implies $\omega_\Lambda \cong -1$ if any energy dominated and $\omega_\Lambda = -\frac{1}{3}\left(1 + \frac{2}{c}\right)$ if dark energy dominated. If c=1, then it functions almost as the constant of cosmology and so the Universe extends with acceleration.

5.4 Reconstruction of the Holographic Polytropic gas model

Now comparing equations (5.5), (5.12) and applying (5.14), we get

$$3M_p^2 H^2 \Omega_\Lambda = \left[Ba^{3/n} - K\right]^{-n} \tag{5.19}$$

And

$$K = Ba^{3/n} - \left(3M_p^2 H^2 \Omega_\Lambda\right)^{-\frac{1}{n}} \tag{5.20}$$

Again comparing the equations (5.7), (5.18) and using the equation (5.20), we get

$$K = \left(3M_p^2H^2\Omega_\Lambda\right)^{-\frac{1}{n}}\left(-\frac{1}{3}-\frac{2}{3}\frac{\sqrt{\Omega_\Lambda}}{c}\right) \tag{5.21}$$

From (5.21) & (5.20) we get

$$B = \left(3M_p^2H^2\Omega_\Lambda a^3\right)^{-\frac{1}{n}}\left(-\frac{2}{3}-\frac{2}{3}\frac{\sqrt{\Omega_\Lambda}}{c}\right) \tag{5.22}$$

From (5.21) & (5.22), we can reconstruct the scalar potential (5.10) and the kinetic energy term (5.11) as mentioned (Das & Singh, 2020a)

$$\dot{\varphi}^2 = M_p^2H^2\left(2\Omega_\Lambda - 2\frac{(\Omega_\Lambda)^{3/2}}{c}\right) \tag{5.23}$$

$$V(\varphi) = M_p^2H^2\left(2\Omega_\Lambda + 2\frac{(\Omega_\Lambda)^{3/2}}{c}\right) \tag{5.24}$$

This is the scalar potential for the Polytropic gas due to evolution of New Agegraphic DE.

The another form of the equation (5.23) is obtained by differentiating with respective to $x = lna$ and using $\dot{\varphi} = \varphi' H$ as

$$\varphi' = M_p\left(2\Omega_\Lambda - 2\frac{(\Omega_\Lambda)^{3/2}}{c}\right)^{\frac{1}{2}} \tag{5.25}$$

In conclusion, the scalar field's evolutionary is

$$\varphi(a) - \varphi(0) = \int_0^{lna} M_p\left(2\Omega_\Lambda - 2\frac{(\Omega_\Lambda)^{3/2}}{c}\right)^{\frac{1}{2}} dx \tag{5.26}$$

Here we take $lna_0 = 0$ at current time. Thus for the evolution of Holographic DE, the resulting scalar field model for Polytropic gas has reformed.

5.5 New Agegraphic model

The energy density for the New Agegraphic DE (Sheykhi, 2009; Das & Singh, 2020a) is

$$\rho_\Lambda = \frac{3n^2M_P^2}{\eta^2} \tag{5.27}$$

Here $M_P = (8\pi G)^{-\frac{1}{2}}$, n is a numerical parameter and η is considered as the conformal age of the Universe defined by

$$\eta = \int \frac{dt}{a} = \int \frac{da}{Ha^2} \tag{5.28}$$

Here a is the scale factor of the Universe $H = \frac{\dot{a}}{a}$ is the Hubble parameter and the dot denotes the derivatives with respective to cosmic time.

The corresponding fractional energy density is

$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}$, where $\rho_{cr} = 3M_P^2H^2$ is the critical energy density of the Universe. So from the equation (5.27), we have

$$\rho_\Lambda = 3M_P^2H^2\Omega_\Lambda \tag{5.29}$$

$$\Omega_\Lambda = \frac{n^2}{\eta^2H^2} \tag{5.30}$$

And

$$\eta = \frac{n}{H\sqrt{\Omega_\Lambda}} \tag{5.31}$$

It gives the conformal time of the Universe in terms of the fractional energy density of dark energy.

Differentiating (5.27) with respect to cosmic time and using $\dot{\eta} = \frac{1}{a}$, in (5.31) we get

$$\dot{\rho_\Lambda} = -\frac{2H\sqrt{\Omega_\Lambda}}{na}\rho_\Lambda \tag{5.32}$$

For the New Agegraphic DE, We may find the EoS parameter (Sheykhi, 2009) as

$$\omega_\Lambda = -1 + \frac{2}{3n}\frac{\sqrt{\Omega_\Lambda}}{a} \tag{5.33}$$

When $a \to \infty$ and $\Omega_\Lambda \to 1$ then $\omega_\Lambda \to -1$ in the late time. At the present time i.e. if $a = 1$, $\Omega_\Lambda = 1$ then $\omega_\Lambda < -1$ for $n < 0$ and $\omega_\Lambda > -1$ for $n > 0$

5.6 Reconstruction of the New Agegraphic Polytropic gas model

The equations (5.5) and (5.29), implies

$$3M_P^2H^2\Omega_\Lambda = \left[Ba^{3/n} - K\right]^{-n} \tag{5.34}$$

And

$$K = Ba^{\frac{3}{n}} - (3M_P^2H^2\Omega_\Lambda)^{-\frac{1}{n}} \tag{5.35}$$

Again from the equations (5.7), (5.33) and (5.35)

$$K = (3M_P^2H^2\Omega_\Lambda)^{-\frac{1}{n}}\left(-1 + \frac{2\sqrt{\Omega_\Lambda}}{3na}\right) \tag{5.36}$$

Using (5.36) in (5.35), we get

$$B = (3M_P^2H^2\Omega_\Lambda a^3)^{-\frac{1}{n}}\left(\frac{2\sqrt{\Omega_\Lambda}}{3na}\right) \tag{5.37}$$

The scalar potential (5.10) and the kinetic energy term (5.11) can be rewritten by means of the equations (5.36) and (5.37) as

$$\dot{\varphi}^2 = (2M_P^2H^2)\left(\frac{(\Omega_\Lambda)^{\frac{3}{2}}}{na}\right) \tag{5.38}$$

And

$$V(\varphi) = 3M_P^2H^2\Omega_\Lambda\left(1 - \frac{2\sqrt{\Omega_\Lambda}}{3na}\right) \tag{5.39}$$

This is the scalar potential for the Polytropic gas due to evolution of New Agegraphic DE.

The another form of the equation (5.38) is obtained by differentiating with respective to $x = lna$ and using $\dot{\varphi} = \varphi' H$ as

$$\varphi' = M_P\left(\frac{2(\Omega_\Lambda)^{\frac{3}{2}}}{na}\right)^{\frac{1}{2}} \tag{5.40}$$

Finally, the evolutionary form of scalar field is obtained as

$$\varphi(a) - \varphi(0) = \int_0^{\ln a} M_P\left(\frac{2(\Omega_\Lambda)^{\frac{3}{2}}}{na}\right)^{\frac{1}{2}} dx \tag{5.41}$$

For the present time we take $a_0 = 1$. Thus for the evolution of New Agegraphic DE, the resulting scalar field model for Polytropic gas has reformed. This indicates that the Universe expands with acceleration.

5.7 Conclusion

In this chapter, we have studied the Polytropic gas DE model with the Holographic and New Agegraphic DE models within the flat FRW Universe. If $K > Ba^{3/n}$, then a

Polytropic gas model of Phantom activities that indicates an accelerated expanding Universe is obtained. For $c = 1$, Holographic DE model functions nearly as a cosmological constant. Also for $a \to \infty$, New Agegraphic DE model roughly acts as a cosmological constant. In addition, we have projected a correspondence for the scalar fields of the Polytropic gas with the Holographic and New Agegraphic DE models. Due to the evolution of the Holographic and New Agegraphic DE, these correspondences describe a Universe of accelerating expansion. The potential and dynamics of the Polytropic gas scalar fields have been reformed.