Chapter-6

Modified Polytropic $f(T)$ gravity model

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6.1 Introduction

Several cosmological research and discoveries, indicates that the present Universe is expanding with acceleration. Negative pressure energy, called dark energy (DE), is responsible for this expansion in the standard cosmology of Friedman Lemaitre Robertson Walker (FLRW) (Overduin & Cooperstok, 1998; Sahni & Starobinsky, 2009). The action of the DE is still uncertain and researchers have suggested different ideas in this area. Around 68.3 percent of the Universe's current energy is stored in DE. The first and easiest candidate for dark energy is the cosmological constant (Weinberg, 1989). There are several dark energy models, in addition to the cosmological constant, that have been proposed to describe cosmic acceleration. Polytropic gas is one of the models of dark energy that describes the Universe's cosmic acceleration (Mukhopadhyay et al., 2008; Karami et al., 2009; Christensen-Dalsgard, 2004; Das & Singh, 2020a). An interesting alternative to General Relativity is the so-called $f(T)$ gravity, which has received considerable attention as a possible explanation of the late time acceleration of the Universe (Ferraro & Fiorini, 2007; Bengochea & Ferraro, 2009; Linder, 2010). It is based on the old idea of the "teleparallel" equivalence of the General Relativity (Hayashi & Shirafuji, 1979). With the Polytropic gas dark energy, we have reconstructed the $f(T)$ gravity model in this chapter. In the background of Polytropic gas, we have found the $f(T)$ gravity model's state equation to describe the acceleration of the Universe.

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6.2 The basic equation of the $f(T)$ gravity model

The action of the $f(T)$ gravity is given by (Bengochea & Ferraro, 2009; Linder, 2010; Das & Basak, 2018e)

$$S = \frac{1}{2k^2}\int d^4 x\sqrt{-g}[f(T) + L_m] \tag{6.1}$$

Where $k^2 = 8\pi G, T$ is the torsion scalar, $f(T)$ is the general differentiable function of the torsion T and $L_m$is the Lagrangian density of the matter inside the Universe

In the case of $f(T)$ gravity, the modified Friedmann equation for the spatially flat FRW Universe is given by (Karami & Abdolmaleki, 2012)

$$H^2 = \frac{1}{3}(\rho_m + \rho_T) \tag{6.2}$$

$$2\dot{H} + 3H^2 = -(p_m + p_T) \tag{6.3}$$

Where

$$\rho_T = \frac{1}{2}(2Tf_T - f - T) \tag{6.4}$$

$$p_T = -\frac{1}{2}\left[-8\dot{H}f_{TT} + \left(2T - 4\dot{H}\right)f_T - f + 4\dot{H} - T\right] \tag{6.5}$$

$$T = -6H^2 \tag{6.6}$$

Here $H = \frac{\dot{a}}{a}$ is the Hubble parameter, ρ_m and p_m denotes the energy density and pressure of matter, ρ_T and p_T are the torsion contributions to the energy density and pressure. Also $f_T$and f_{TT} denotes one and two times derivatives with respect to the torsion scalar T(Karami & Abdolmaleki, 2012).

The energy conservation laws are

$$\dot{\rho}_T + 3H(\rho_T + p_T) = 0 \tag{6.7}$$

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0 \tag{6.8}$$

Using (6.4) & (6.5), the equation of state (EOS) due to torsion contribution is defined by

$$\omega_T = -1 + \frac{4\dot{H}(2Tf_{TT}+f_T-1)}{2Tf_T-f-T} \tag{6.9}$$

The scale factor $a(t)$ is represented by (Nojiri & Odintsov, 2006)

$$a(t) = a_o(t_s - t)^{-h}, t \leq t_s, h > 0 \tag{6.10}$$

From (6.5) & (6.9) one can write

$$H = \frac{h}{t_s-t} \tag{6.11}$$

$$\dot{H} = \frac{H^2}{h} = \frac{h}{(t_s-t)^2} \tag{6.12}$$

$$T = -6\left(\frac{h}{t_s-t}\right)^2 \tag{6.13}$$

From the equations (6.10) & (6.11), the scale factor $a(t)$ can be rewritten as

$$a(t) = a_o\left(\frac{H}{h}\right)^h \tag{6.14}$$

From the equation (6.12), we see that $\dot{H} > 0$ which represents a supper accelerated FRW Universe with a Big Rip singularity at $t = t_s$.

Also from the equation (6.13), we see that

$$\text{When} \quad t = t_s, T = -\infty \tag{6.15}$$

6.3 Polytropic$f(T)$ gravity model

The Polytropic gas equation of state (EOS) is given by (Das & Singh, 2020a; Karami et al., 2009)

$$P_\Lambda = K\rho_\Lambda^{1+\frac{1}{n}} \tag{6.16}$$

The pressure, energy density, Polytropic constant and index are denoted by P_Λ,ρ_Λ,K, and n respectively (Das & Basak, 2018e; Setare & Darabi,2013).

Solving (1.6) & (6.7) and integrating we get

$$\rho_\Lambda = \left[-K + Ba^{3/n}\right]^{-n} \tag{6.17}$$

Here B is a positive constant of integration and a(t) is the Universe's time scale factor (Karami et al., 2009).

From (6.14) and (6.17) one can obtain

$$\rho_\Lambda = \left[-K + \alpha(-6H^2)^{\frac{3h}{2n}}\right]^{-n} \tag{6.18}$$

$$\text{Where } \alpha = B a_0^{\frac{3}{n}} (-6H^2)^{\frac{-3h}{2n}} \tag{6.19}$$

Equating (6.4) and (6.18) we get

$$(-12H^2 f_T - f + 6H^2) - \left[-K + \alpha(-6H^2)^{\frac{3h}{2n}}\right]^{-n} = 0 \tag{6.20}$$

Using the solution of (6.20) into (6.9), the EOS of parameter of torsion contribution is given by (Karami & Abdolmaleki, 2012).

$$\omega_T = -1 - \frac{1}{\frac{K}{B}\left[a_0\left(\frac{H}{h}\right)^h\right]^{\frac{-3}{n}} - 1} \quad , \text{h>0} \tag{6.21}$$

Equation (6.12) can be rewritten as

$$\frac{\dot{H}}{H} = \frac{H}{h} \tag{6.22}$$

Using (6.22) in (6.21) we get

$$\omega_T = -1 - \frac{1}{\frac{K}{B}\left[a_0\left(\frac{\dot{H}}{H}\right)^h\right]^{\frac{-3}{n}} - 1} \quad , \text{h>0} \tag{6.23}$$

When $\frac{K}{B}\left[a_0\left(\frac{\dot{H}}{H}\right)^h\right]^{\frac{-3}{n}} > 1$then from (6.23) we see that $\omega_T < -1$which represents a Phantom like accelerated Universe.

6.4 Conclusion

We see from above discussion that the equation of state of the $f(T)$ gravity model in the context of Polytropic gas represent a Phantom like accelerated Universe.