Chapter-7

Bianchi Type-I Polytropic gas dark energy models in Cosmology

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7.1 Introduction

Most cosmological observations demonstrate that with acceleration, our Universe expands. This expansion is due to a new force of negative pressure, called dark energy (DE) (Overduin & Cooperstok, 1998; Sahni, & Starobinsky, 2009). The action of the DE is still uncertain and researchers have suggested different ideas in this area. The easiest candidate for dark energy is the cosmological constant (Weinberg, 1989). To describe cosmic acceleration, there are several dark energy models that have been suggested. Polytropic gas is one of the models of dark energy that describes the Universe's cosmic acceleration (Mukhopadhyay et al., 2008; Karami et al., 2009; Christensen-Dalsgard, 2004; Das & Basak, 2018a). It is evident from the various observational data that our Universe is homogeneous and isotropic; however, no physical proof disputes the chances of an anisotropic Universe. In reality, there are several scientific reasons that promote the presence of the Universe's anisotropic phase entering the isotropic phase (Hinshaw et al., 2003, 2007 & 2009). In the early phase of the evolution of the Universe, anisotropy plays a critical rule and is therefore considered most relevant to research homogeneous and anisotropic cosmological models. The Bianchi type models are mainly spatially homogeneous and usually anisotropic (Das & Basak, 2018d). K.S. Adhav et al. studied various Bianchi type models (Adhav.et al., 2011, 2013, 2014, 2016 &2017). There are also some researchers including S.D. Katore and D.V. Kapse (Katore & Kapse, 2018), A. Pradhan and A. K.

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Vishwakarma (Pradhan & Vishwakarma, 2002), M.A. Rahman and M. Ansari (Rahman & Ansari, 2014) worked in this field.

In this part, we analyzed the cosmological model of Bianchi type-1 with the Polytropic gas and solved the field equation as long as the scalar expansion (θ) is proportional to the shear scalar (σ). Many physical and cosmological features of this model have been discussed.

7.2 Metric and field equation

Consider the type-I LRS Bianchi metric as

$$ds^2 = dt^2 - A(dx^2 + dy^2) - Bdz^2 \tag{7.1}$$

With A and B, the metric functions of cosmic time $'t'$ (Das & Basak, 2018d)

The field equations of Einstein are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \tag{7.2}$$

The Ricci tensor, Ricci scalar and the energy momentum tensor are denoted by R_{ij}, R and T_{ij}

The equation for the energy conservation is

$$T^{ij}_{;j} = 0 \tag{7.3}$$

Where $T^{ij}_{;j} = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^j}\left(T^{ij}\sqrt{-g}\right) + T^{jk}\Gamma^i_{jk}$

Again, the bulk viscous fluid's energy momentum tensor is

$$T_{ij} = (\rho_\Lambda + P_\Lambda)u_i u_j + P_\Lambda g_{ij} \tag{7.4}$$

Such that ρ_Λ, P_Λ and u^i are respectively energy density, pressure and four velocity vector that satisfying $g_{ij}u^i u^j = 1$

We have claimed in this model, that the Universe is contained by the Polytropic gas.

The Polytropic gas equation of state (Das & Singh, 2020a; Karami et al., 2009) is

$$P_\Lambda = K{\rho_\Lambda}^{1+\frac{1}{n}} \tag{7.5}$$

Here K is a Polytropic constant and n (>0) is a Polytropic index. (Das & Singh, 2020a; Setare & Darabe, 2013)

Using the metric equation (7.4) & (7.1), the field equations of Einstein are

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} = \rho_\Lambda \tag{7.6}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} = -P_\Lambda \tag{7.7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -P_\Lambda \tag{7.8}$$

The energy conservation equation (7.3) takes the following forms

$$\dot{\rho}_\Lambda + \left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)(\rho_\Lambda + P_\Lambda) = 0 \tag{7.9}$$

The mean Hubble parameter H and scalar expansion θ are given by

$$H = \frac{1}{3}\theta = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}) \tag{7.10}$$

The spatial volume of the Universe is expressed here by V.

The average parameter of anisotropy Δ and shear scalar σ^2 are obtained as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{7.11}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} {H_i}^2 - 3H^2\right) \tag{7.12}$$

Here H_i, ($i = 1,2,3$)represents the directional parameters of the Hubble in the axes of x, y, z respectively

7.3 Field equation's solutions

We presume that the scalar expansion (θ) is proportional to the shear scalar (σ^2) for solving the field equations of Einstein (Das & Basak, 2018d). For it we consider the relation

$$B = A^m \tag{7.13}$$

With a positive constant m

The field equations (7.6), (7.7) and (7.8) are

$$(2m+1)\frac{\dot{A}^2}{A^2} = \rho_\Lambda \tag{7.14}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} = -P_\Lambda \tag{7.15}$$

$$(m+1)\frac{\ddot{A}}{A} + m^2\frac{\dot{A}^2}{A} = -P_\Lambda \tag{7.16}$$

Solving the equations (7.15) & (7.16) we get

$$\left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2}\right) + (m+2)\frac{\dot{A}^2}{A^2} = 0 \tag{7.17}$$

Integrating it we get

$$A(t) = a_0[t_0 + (m+2)t]^{\frac{1}{m+2}} \tag{7.18}$$

Where a_0 and t_0 are integration constants.

Equations (7.13) and (7.18) give

$$B(t) = \left[a_0\{t_0 + (m+2)t\}^{\frac{1}{m+2}}\right]^m$$

$$= a_0^m[t_0 + (m+2)t]^{\frac{m}{m+2}} \tag{7.19}$$

By a suitable choice of constants ($a_0 = 1, t_0 = 0$), the metric equation (7.1) can be written as

$$ds^2 = dt^2 - [(m+2)t]^{\frac{1}{m+2}}(dx^2 + dy^2) - [(m+2)t]^{\frac{m}{m+2}}dz^2 \tag{7.20}$$

7.4 Physical and cosmological properties of the model

We get the energy density from the equation (7.14) as

$$\rho_\Lambda = \frac{(2m+1)}{(m+2)^2t^2} \tag{7.21}$$

We get the pressure from the equation (7.21) & (7.5) as

$$P_\Lambda = K\left[\frac{(2m+1)}{(m+2)^2t^2}\right]^{1+\frac{1}{n}} \tag{7.22}$$

Also

$$H = \frac{1}{3t} \tag{7.23}$$

$$\Theta = \frac{1}{t} \tag{7.24}$$

The spatial volume (V) of the Universe is obtained as

$$V = (m+2)t \tag{7.25}$$

The shear scalar σ^2 is given by

$$\sigma^2 = \frac{(m-1)^2}{(m+2)^2t^2} = \frac{(m-1)^2\theta^2}{(m+2)^2} \tag{7.26}$$

The mean anisotropy parameter (Δ) is obtained as

$$\Delta = \frac{2(m-1)^2}{(m+2)^2} = \frac{2\sigma^2}{\theta^2} \tag{7.27}$$

The deceleration parameter(q) is given by

$$q = -\frac{\dot{H}}{H^2} - 1 = 2 \tag{7.28}$$

The equation of state parameter(ω) is given by (Sadeghi, J. et al., 2013).

$$\omega_\Lambda = \frac{P_\Lambda}{\rho_\Lambda} = K\left[\frac{(2m+1)}{(m+2)^2t^2}\right]^{\frac{1}{n}} \tag{7.29}$$

Also the energy density parameter(Ω) is given by

$$\Omega = \frac{\rho_\Lambda}{3H^2} = \frac{3(2m+1)}{(m+2)^2} \tag{7.30}$$

For $m = 1$, we have $A = B, \Delta = 0, \sigma^2 = 0, \Omega = 1$, this implies an isotropic flat Universe and the Universe would expand indefinitely, but after an infinite period of time, the rate of expansion would slow, we have Δ=constant, $\frac{\sigma}{\theta}$ =constant, $\Omega < 1$ when $m \neq 1$, this leads to an anisotropic open Universe, and the Universe would expand indefinitely.

7.5 Conclusion

There has been an analysis of the cosmological model of Bianchi type I with the Polytropic gas. The physical and cosmological parameters that play a key role are obtained in the model's analysis. It is noted that the Universe's spatial volume is zero at t=0 and grows indefinitely as $t \rightarrow \infty$. At t= 0, the mean Hubble parameter, the pressure, the energy density, expansion scalar and shear scalar are infinite at $t = 0$ and approach 0 as $t \rightarrow \infty$. The Universe is isotropic and flat when m=1 and it is anisotropic and open when m≠1. The model is thus anisotropic in the Universe's evolution, except for m=1.