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Thermodynamic behavior of the polytropic gas in cosmology

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Abstract

In this paper, we investigate on the thermodynamic behavior of Polytropic gas as a candidate for dark energy by considering the relation $P = K\rho^{1+\frac{1}{n}}$, where K and n are the Polytropic constant and Polytropic index respectively. Furthermore, P indicates the pressure and ρ is the energy density of the fluid such that $\rho = \frac{U}{V}$ where U and V represent the internal energy and volume, respectively. At first, we find an exact expression for the energy density of the Polytropic gas using thermodynamics and later on, discuss different physical parameters. Finally our study shows that the Polytropic gas may be used to describe the expansion history of the universe from the dust dominated era to the current accelerated era and it is thermodynamically stable.

Keywords: Cosmology; Dark energy; Polytropic gas; Thermodynamics.
2010 AMS subject classification: 83F05, 37D35, 82B30.‡

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1. Introduction

Cosmologists suggest that our universe expands under an accelerated expansion [1]-[7]. In the standard Friedman Lemaitre Robertson Walker (FLRW) cosmology, a new energy with negative pressure, called dark energy (DE) is responsible for this expansion [8]. The nature of the DE is still unknown and various problems have been proposed by the researchers in this field**.** About 70% of the present energy of the universe is contained in the DE. The cosmological constant with the time independent equation of state is the earliest, simplest and most traditional candidate for the dark energy which can be taken into account as a perfect fluid satisfying the relation $\rho + P = 0$. But it has some problems like fine-tuning and cosmic coincidence puzzles [9], [10]. Besides the cosmological constant, the other dark energy models are quintessence [11], phantom [12], tachyon [13], holographic dark energy [14] [15], K-essence [16] and Chaplygin gas models with various equation of state. Polytropic gas is one of the dynamical dark energy models [17].

In the present study, we want to investigate the thermodynamic behavior of the Polytropic gas. K. Karami et al. investigated the interaction between the Polytropic gas and cold dark matter and found that the Polytropic gas behaves as the phantom dark energy [18]. K. Karami and S. Ghaffari showed that the generalized second law of thermodynamics is always satisfied by a universe filled with a Polytropic gas and a cold dark matter [19]. K. Kleidis and N.K. Spyron used the first law of thermodynamics in the Polytropic gas model and they show that the Polytropic gas behaves as dark energy and this model leads to a suitable fitting with the observational data about the current expanding era [20]. H. Moradpour, A. Abri and H. Ebadi, investigated the thermo dynamical behavior and stability of the Polytropic gas [21]. M. Salti et al. discussed validity of the first and generalized second law of thermodynamics in locally rotationally symmetric Bianchi-type II space time which is dominated by a combination of Polytropic gas and baryonic matter[22]. Moreover, Muzaffer Askin et al. studied the cosmological scenarios of the Polytropic gas dark matter-energy proposal in a Friedmann-Robertson- Walker universe and they found an exact expression for the energy density of the Polytropic gas model according to the thermo dynamical point of views and a relationship between a homogeneous minimally coupled scalar field and the Polytropic gas [23].This paper is organized as follows: in section 2 we construct the basic thermodynamic formalism of the Polytropic gas model and discuss the thermodynamic behavior of this model. Finally in section 3 we provide a brief discussion.

2. Basic Formalism

In this work, we consider the following equation of state which is well known as Polytropic gas equation of state

$$P = K\rho^{1+\frac{1}{n}} \tag{1}$$

Here $K(>0)$ and $n(<0)$ are Polytropic constant and Polytropic index respectively. Moreover, P is the pressure and ρ is the energy density of the fluid such that

$$\rho = \frac{U}{V} \tag{2}$$

Where U and V are the internal energy and volume filled by the fluid respectively.

First of all, we try to find the internal energy U and energy density ρ of the polytropic gas as a function of its volume V and entropy S.

From the general thermodynamics, we have

$$\left(\frac{\partial U}{\partial V}\right)_S = -P \tag{3}$$

From the equations (1), (2) and (3), we get

$$\left(\frac{\partial U}{\partial V}\right)_S = -K\left(\frac{U}{V}\right)^{1+\frac{1}{n}} \tag{4}$$

Integrating the equation (4), we get

$$U = (-1)^{-n}\left(KV^{-\frac{1}{n}} + \xi\right)^{-n} \tag{5}$$

Where the parameter ξ is the constant of integration which may be a universal constant or a function of entropy S only

The equation (5) also can rewrite in the following form

$$U = (-1)^{-n}K^{-n}V\left(1 + \left(\frac{V}{\varepsilon}\right)^{\frac{1}{n}}\right)^{-n} \tag{6}$$

Where $$\varepsilon = \left(\frac{K}{\xi}\right)^{n} \tag{7}$$

And it has a dimension of volume.

Therefore, the energy density ρ of the Polytropic gas is

$$\rho = \frac{U}{V} = (-1)^{-n}K^{-n}\left(1 + \left(\frac{V}{\epsilon}\right)^{\frac{1}{n}}\right)^{-n} \tag{8}$$

When $n < 0$ then equation (8) gives

$$\rho \sim (-1)^{-n}K^{-n}\frac{\varepsilon}{V} \tag{9}$$

Now we will use these equations to discuss different physical parameters.

a) Pressure:

Using the equation (8) in the equation (1) we get the pressure of the Polytropic gas as a function of entropy S and volume V in the following form

$$P = (-1)^{n+1} K^{-n} \left(1 + \left(\frac{V}{\epsilon}\right)^{\frac{1}{n}}\right)^{-(n+1)} \quad (10)$$

We can rewrite the equation (10) in the following form

$$P = -\frac{\rho}{1+\left(\frac{V}{\epsilon}\right)^{\frac{1}{n}}} \quad (11)$$

When $n < 0$ and ε does not diverge then for small volume i.e. at early stage of universe, $V \ll \varepsilon$ ie $\frac{V}{\varepsilon} \ll 1$, we get

$P \simeq 0$, which represents a dust dominated universe. When $n < 0$ and ε does not diverge then for large volume i.e. at late stage of universe, $V \gg \varepsilon$ ie $\frac{V}{\varepsilon} \gg 1$, we get $P \simeq -\rho$, which indicates an accelerated expansion of the universe.

b) Caloric equation of state:

Now from the equations (8) and (10) we get the caloric equation of state parameter as

$$\omega = \frac{P}{\rho} = -\frac{1}{1+\left(\frac{V}{\epsilon}\right)^{\frac{1}{n}}} \quad (12)$$

When $n < 0$ and ε does not diverge then for small volume $V \ll \varepsilon$ ie $\frac{V}{\varepsilon} \ll 1$, we get $\omega \simeq 0$ (Dust dominated)

When $n < 0$ and ε does not diverge then for large volume $V \gg \varepsilon$ ie $\frac{V}{\varepsilon} \gg 1$, we get $\omega \simeq -1$ (Cosmological constant)

Thus the equation of state parameter (ω) of the Polytropic gas with $n < 0$ is decreased from $\omega \simeq 0$ (for small volume) to $\omega \simeq -1$ (for large volume). It indicates that the universe expands from the dust dominated era to the current accelerating era.

c) Deceleration parameter:

We get the deceleration parameter of the Polytropic gas with the help of equation (12)

$$q = \frac{1}{2} + \frac{3}{2}\frac{P}{\rho} = \frac{1}{2} - \frac{3}{2}\frac{1}{1+\left(\frac{V}{\epsilon}\right)^{\frac{1}{n}}} \qquad (13)$$

When $n < 0$ and ε does not diverge then for small volume $V \ll \varepsilon$ ie $\frac{V}{\varepsilon} \ll 1$, we get $q > 0$, which correspond to the deceleration universe.
When $n < 0$ and ε does not diverge then for large volume $V \gg \varepsilon$ ie $\frac{V}{\varepsilon} \gg 1$, we get $q < 0$, which correspond to the accelerated universe.

d) Square velocity of sound:

From the equation (11) we get the velocity of sound (V_s) as

$${V_s}^2 = \left(\frac{\partial P}{\partial \rho}\right)_S = -\frac{1}{1+\left(\frac{V}{\epsilon}\right)^{\frac{1}{n}}} \qquad (14)$$

When $n < 0$ and ε does not diverge then for small volume $V \ll \varepsilon$ ie $\frac{V}{\varepsilon} \ll 1$, we get ${V_s}^2 \simeq 0$ .Since velocity of sound is zero in vacuum. Therefore the Polytropic gas behaves like a pressure less fluid at the early stage of the universe. When $n < 0$ and ε does not diverge then for large volume $V \gg \varepsilon$ ie $\frac{V}{\varepsilon} \gg 1$, we get ${V_s}^2 \simeq -1$, which gives an imaginary speed of sound leading to a perturbation cosmology.

e) Thermodynamic stability:

The conditions of the thermodynamic stability of a fluid are

$$\left(\frac{\partial P}{\partial V}\right)_S < 0 \qquad (15)$$

And $$C_V > 0 \qquad (16)$$

Here C_V is the thermal capacity at constant volume. From the equation (10) we have

$$\left(\frac{\partial P}{\partial V}\right)_S = -\left(1 + \frac{1}{n}\right)\frac{P}{V}\frac{1}{1+\left(\frac{V}{\epsilon}\right)^{-\frac{1}{n}}} \qquad (17)$$

If $-1 < n < 0$ and $\varepsilon < 0$ then from (17), we have

$$\left(\frac{\partial P}{\partial V}\right)_S < 0$$

Thus the stability condition (15) of thermodynamics is satisfied.

Now we have to verify the positivity of the thermal capacity at constant volume C_V where $C_V = T\left(\frac{\partial S}{\partial T}\right)_V$ (18)

Now we determine the temperature T of the Polytropic gas as a function of its entropy S and its volume V. The temperature T of the Polytropic gas is determined from the relation

$$T = \left(\frac{\partial U}{\partial S}\right)_V \qquad (19)$$

Using (6) in (19) we get

$$T = (-1)^{n+1} V^{1+\frac{1}{n}} \left(K + \xi V^{\frac{1}{n}}\right)^{-(n+1)} \frac{d\xi}{dS} \qquad (20)$$

This gives the temperature of the Polytropic gas.
We can rewrite the equation (20) in the following form

$$T = -n \frac{\rho V^{1+\frac{1}{n}}}{1+\left(\frac{V}{\epsilon}\right)^{\frac{1}{n}}} \frac{d\xi}{dS} \qquad (21)$$

From (5) we have

$$[\xi]^{-n} = [U] \qquad (22)$$

Since $\qquad [U] = [TS] \qquad (23)$

Therefore from the equations (22) & (23) we get

$$\xi = [U]^{-\frac{1}{n}} = [T_* S]^{-\frac{1}{n}} \qquad (24)$$

Where T_* (> 0) is a universal constant with temperature dimension.
Differentiating (24) with respect to 'S' we get

$$\frac{d\xi}{dS} = -\frac{1}{n} T_*^{-\frac{1}{n}} S^{-\frac{1}{n}-1} \qquad (25)$$

Using (8) & (24) in (25) we get

$$T = (-1)^n V^{1+\frac{1}{n}} \left(T_*^{-\frac{1}{n}} S^{-\frac{1}{n}-1}\right) \left[K + T_*^{-\frac{1}{n}} S^{-\frac{1}{n}} V^{\frac{1}{n}}\right]^{-(n+1)} \qquad (26)$$

This leads to the entropy of the Polytropic gas as

$$S = \left[(-1)^{\frac{n}{n+1}} \left(\frac{T_*}{T}\right)^{\frac{1}{n+1}} - 1\right]^n \quad \frac{V}{K^n T_*} \qquad (27)$$

We know that entropy (S) of a thermo dynamical system should be positive ie $S > 0$ [24]
Here $S > 0$ if $K^n T_* > 0$
Now the thermal capacity at constant volume is

$$C_V = T \left(\frac{\partial S}{\partial T}\right)_V$$

$$= (-1)^{\frac{2n+1}{n+1}} \left(\frac{n}{n+1}\right) \frac{S}{\left[(-1)^{\frac{n}{n+1}} \left(\frac{T_*}{T}\right)^{\frac{n}{n+1}} - 1\right]} \left(\frac{T_*}{T}\right)^{\frac{1}{n+1}} \qquad (28)$$

Therefore, the condition $C_V > 0$ is satisfied if $K^n T_* > 0$. Thus both the conditions of thermo dynamic stability are satisfied. So the Polytropic gas is thermo dynamically stable.

3. Discussion

We have studied the thermo dynamical behavior of the Polytropic gas. Here, we have considered the value of $n < 0$ to study the whole work done in this article. Some important results are given below:

(i) As we have considered $n < 0$, the pressure goes more and more negative as volume increases.

(ii) The equation of state parameter (ω) of the Polytropic gas is $\omega \simeq 0$ at early stage of the universe and $\omega \simeq -1$ at late stage of the universe. This indicates that the universe expands from the dust dominated era to the present accelerated era.

(iii) The deceleration parameter (q) is investigated in the context of thermodynamics as well as Polytropic gas and our analysis shows that universe is decelerated ($q > 0$) at early stage of the universe and accelerated ($q < 0$) at late stage of the universe.

Both the conditions of the thermo dynamical stability of the Polytropic gas are studied for $K^n T_* > 0$ and our analysis shows that the Polytropic gas is thermodynamically stable.

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RECONSTRUCTION OF THE HOLOGRAPHIC POLYTROPIC GAS DARK ENERGY MODEL IN THE FLAT FRW UNIVERSE

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ABSTRACT

In this paper, we study the Polytropic gas dark energy model and holographic dark energy model in the flat FRW universe and establish a correspondence between them for the scalar fields. This correspondence allows reconstructing the potential of the Polytropic gas scalar fields and dynamics of the scalar fields according to the evolutions of the holographic dark energy, which describes the accelerated expansion of the universe

Key words: Holographic principles, Holographic Dark energy, Polytropic gas

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1. INTRODUCTION

Several cosmological experiments and observational data's prove that our universe is expanding with acceleration [1]-[7]. In the standard Friedman Lemaitre Robertson Walker (FLRW) cosmology, a new energy with negative pressure, called dark energy (DE) is responsible for this expansion [8]. The nature of the DE is still unknown and various problems have been proposed by the researchers in this field**.** About 70% of the present energy of the universe is contained in the DE. The cosmological constant with the time independent equation of state is the earliest and simplest candidate for the dark energy. The cosmological constant suffers from two well-known difficulties namely "fine-tuning" and "cosmic coincidence" problems [9-10]. Besides the cosmological constant, there are two different dynamical dark energy models with the time dependent equation of state that have been proposed by the researchers to explain the cosmic acceleration, (i) The "scalar-field models" of dark energy including quintessence[11-12], phantom[13-15], K-Essence[16-18], tachyon[19-21], dilaton[22-24] etc. (ii) The "interacting models" of dark energy including Chaplygin gas[25-27], braneworld models[28.29], holographic[30-32], agegraphic[33,34] etc. An interesting model to explain the nature of the dark energy is Polytropic gas dark energy model. The Polytropic gas DE model is a phenomenological model of dark energy where the pressure is a function of energy density [35, 36].

On the other hand, we consider the holographic model as a dark energy model to explain the cosmic acceleration of the universe. The concept of holographic model was proposed by Gerard't Hooft [37] while studying black holes thermodynamics which was elaborated and developed by Leonard Susskind [38]. This model is based on the principle of holography and the principle states that the degree of freedom of a system scales with its area rather than with its volume. In other words, Susskind says that the nature can be explained using two-dimensional lattice at the spatial boundaries of the world in the place of a three-dimensional lattice [38]. The model considers the relation connecting UV(ultraviolet) and IR(infrared)cut-off, so that if ρ_Λ is the zero point energy density of a quantum system which is caused by a short-distance cut-off, then the total energy inside the region L should not exceed the mass of the black hole of the same size[39], where the equation $L^3\rho_\Lambda \leq LM_P^2$ emerges. The largest L is chosen by saturating the inequality in order to obtain the holographic dark energy density

$$\rho_\Lambda = 3c^2M_P^2L^{-2} \tag{1}$$

Where c is a dimensionless numerical constant, $M_P$is the reduced Planck mass such that $M_P^{-2} = 8\pi G$ and L is considered as the size of the current universe.

2. POLYTROPIC GAS DARK ENERGY MODEL

The equation of state (EOS) of the Polytropic gas [40] is given by

$$P_\Lambda = K\rho_\Lambda^{1+\frac{1}{n}} \tag{2}$$

Where$P_\Lambda, \rho_{\Lambda,}$ K and n are the pressure, energy density, polytropic constant and polytropic index respectively.

The conservation equation for the dark energy in the FRW universe is given by

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + P_\Lambda) = 0 \tag{3}$$

Where H the Hubble parameter and overhead dot is denotes the differentiation with respect to the cosmological time

Using the EOS (1) into the conservation equation (2) and integrating we get

$$\rho_\Lambda = \left[Ba^{3/n} - K\right]^{-n} \tag{4}$$

Where B is a positive integration constant and $a(t)$ is a time scale factor of the universe .

The corresponding pressure takes the following form

$$P_\Lambda = K\left[Ba^{3/n} - K\right]^{-n-1} \tag{5}$$

Using equations (3) & (4), the EOS parameter for the Polytropic gas dark energy model is obtained as

$$\omega_\Lambda = \frac{P_\Lambda}{\rho_\Lambda} = -1 + \frac{Ba^{3/n}}{Ba^{3/n} - K} \tag{6}$$

When $K > Ba^{3/n}$, from (5), we see that $\omega_\Lambda < -1$ which corresponds to a universe dominated by phantom field. The phantom field lead to accelerated expansion of the universe

The pressure and energy density for the scalar field φ are given by

$$P_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \quad (7)$$

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \quad (8)$$

From the equations (3), (4), (7) and (8), we get the scalar potential and the kinetic energy terms for the Polytropic gas as

$$V(\varphi) = \frac{\frac{B}{2}a^{3/n} - K}{\left(Ba^{3/n} - K\right)^{n+1}} \quad (9)$$

$$\dot{\varphi}^2 = \frac{Ba^{3/n}}{\left(Ba^{3/n} - K\right)^{n+1}} \quad (10)$$

When $K > Ba^{3/n}$, then from (10), we see that $\dot{\varphi}^2 < 0$ and it indicates that the scalar field φ is a phantom field.

3. HOLOGRAPHIC DARK ENERGY MODEL IN THE FLAT FRW UNIVERSE

The energy density for a flat universe is given by [41]

$$\rho_\Lambda = 3c^2 M_P^2 R_h^{-2} \quad (11)$$

Where R_h is the proper size of the future event horizon and it is defined as

$$R_h = a(t)\int_t^\infty \frac{dt'}{a(t')} = \int_a^\infty \frac{da'}{H' a'^2} \quad (12)$$

But $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}$ and $\rho_{cr} = 3M_p^2 H^2$ is the critical energy density of our universe, so from the equation (11) we have $HR_h = \frac{c}{\sqrt{\Omega_\Lambda}}$ (13)

Now differentiating the equation (12) with respect to't' we get

$$\dot{R_h} = HR_h - 1 = \frac{c}{\sqrt{\Omega_\Lambda}} - 1 \quad (14)$$

Using the equation (11), the changing rate of the holographic dark energy with time is

$$\frac{d\rho_\Lambda}{dt} = -6c^2 M_P^2 R_h^{-3}\dot{R_h} = -2H\left(1 - \frac{\sqrt{\Omega_\Lambda}}{c}\right)\rho_\Lambda \quad (15)$$

For the conservation of the energy momentum tensor, the equation of the energy density of the dark energy is

$$\frac{d}{da}(a^3\rho_\Lambda) = -3a^2 P_\Lambda$$

And we have

$$P_\Lambda = -\frac{1}{3}\frac{d\rho_\Lambda}{dlna} - \rho_\Lambda \quad (16)$$

The equation of state (EoS) parameter for the holographic dark energy is

$$\omega_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} = -\frac{dln\rho_\Lambda}{dlna} - 1 = -\frac{1}{3}\left(1 + \frac{2\sqrt{\Omega_\Lambda}}{c}\right) \quad (17)$$

Where $dlna = Hdt$ is applied.

From the result (17), we see that $\omega_\Lambda \cong -1$ when any other type of energy dominated and $\omega_\Lambda = -\frac{1}{3}\left(1+\frac{2}{c}\right)$ when dark energy dominated($\Omega_\Lambda \to 1$). If c=1 then it behaves almost like the cosmological constant and so the universe expand with acceleration.

4. CORRESPONDENCE BETWEEN THE POLYTROPIC GAS AND HOLOGRAPHIC DARK ENERGY MODELS

First of all, we create a correspondence between the Polytropic gas dark energy model and holographic dark energy model for the scalar fields. We create this correspondence, by comparing the holographic dark energy density (11) with the Polytropic gas model density (4) and equating the EoS parameter of the holographic dark energy (17) with the EoS parameter of the Polytropic gas model (6) as follows

Now equating equations (4), (11) and then using (13), we get

$$3M_p^2H^2\Omega_\Lambda = \left[Ba^{3/n} - K\right]^{-n} \tag{18}$$

And

$$K = Ba^{3/n} - \left(3M_p^2H^2\Omega_\Lambda\right)^{-\frac{1}{n}} \tag{19}$$

Again equating equations (6), (17) and using (19), we get

$$K = \left(3M_p^2H^2\Omega_\Lambda\right)^{-\frac{1}{n}}\left(-\frac{1}{3}-\frac{2}{3}\frac{\sqrt{\Omega_\Lambda}}{c}\right) \tag{20}$$

Putting (20) in (19) we get

$$B = \left(3M_p^2H^2\Omega_\Lambda a^3\right)^{-\frac{1}{n}}\left(-\frac{2}{3}-\frac{2}{3}\frac{\sqrt{\Omega_\Lambda}}{c}\right) \tag{21}$$

Now, we reconstruct the scalar potential (9) and the kinetic energy term (10) with the help of equations (20) & (21) as follows

$$\dot{\varphi}^2 = M_p^2H^2\left(2\Omega_\Lambda - 2\frac{(\Omega_\Lambda)^{3/2}}{c}\right) \tag{22}$$

$$V(\varphi) = M_p^2H^2\left(2\Omega_\Lambda + 2\frac{(\Omega_\Lambda)^{3/2}}{c}\right) \tag{23}$$

The equation (23) represents the scalar potential of the Polytropic gas dark energy according to the evolutions of the holographic dark energy.

By the definition $\dot{\varphi} = \varphi' H$, we can rewrite the equation (22) in terms of the derivative with respective to $x = lna$, as

$$\varphi' = M_p\left(2\Omega_\Lambda - 2\frac{(\Omega_\Lambda)^{3/2}}{c}\right)^{\frac{1}{2}} \tag{24}$$

Finally, the evolutionary form of the scalar field is

$$\varphi(a) - \varphi(0) = \int_0^{lna} M_p\left(2\Omega_\Lambda - 2\frac{(\Omega_\Lambda)^{3/2}}{c}\right)^{\frac{1}{2}} \mathrm{dx} \tag{25}$$

Here we take $lna_0 = 0$ at the present time. Thus the corresponding scalar field of the Polytropic gas dark energy model has reconstructed according to the evolutions of holographic dark energy

5. CONCLUSION

In this work, we consider the Polytropic gas dark energy model and holographic dark energy model in the flat FRW universe. For $K > Ba^{3/n}$, the Polytropic gas model behaves as a phantom like dark energy model and it can explain that the universe expands with acceleration. The holographic dark energy model also almost behaves as a cosmological constant when $c = 1$ and dark energy dominated. We have proposed a correspondence between the Polytropic gas dark energy model and holographic dark energy model for the scalar fields. We reconstruct the potential of the Polytropic gas scalar fields and dynamics of the scalar fields according to the evolutions of the holographic dark energy, which describes the accelerated expansion of the universe.

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Polytropic gas has plausibility in preventing big rip singularity

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***Abstract*: In this paper, we have studied the polytropic gas defined by the equation of state $P_\Lambda = K\rho_\Lambda^{1+\frac{1}{n}}$, where$P_\Lambda, \rho_\Lambda, K$ and n are the pressure, energy density, polytropic constant and polytropic index respectively as dark energy. We have verified that if cosmic dark energy behaves like a fluid with equation of state parameter $P_\Lambda = \omega_\Lambda\rho_\Lambda$, $\omega_\Lambda < -1$ as well as polytropic gas then Big rip singularity does not arise and the scale factor is found to be regular for all time.**

***Keywords*: Dark energy, Accelerated universe, Big rip, Polytropic gas.**

Introduction:

Many Cosmological experiments and observations such as Type 1a Supernovae [1]-[3], Cosmic Microwave Background Radiation [4], Large Scale Structure [5], [6], Wilkinson Microwave Anisotropy Probe [7], Sloan Digital Sky Survey [8], etc. indicates that our universe expands under an accelerated expansion. In standard Friedman Lemaitre Robertson Walker (FLRW) cosmology, a new energy with negative pressure and positive energy density, called dark energy (DE) is responsible for this expansion [9]-[10]. The nature of the DE is still unknown and various problems have been proposed by the researchers in this field. About 70% of the present energy of the universe is contained in the DE. The cosmological constant with the time independent equation of state is the earliest and simplest candidate for the dark energy [11]. The dark energy is usually described by an equation of state parameter $\omega = \frac{P}{\rho}$, where P is the pressure and ρ is the energy density [12]. For the different values of ω, different types of dark energy models are obtained. If $\omega < -1$ then it is phantom dark energy model and it is quintessence for $-1 < \omega < -\frac{1}{3}$. When the parameter ω crosses the phantom divide line $\omega = -1$ then it is termed as quintom. Moreover from the observations of the Wilkinon Microwave Anisotropy Probe (WMAP) it is clear that the value of the equation of state parameter $\omega \equiv -1.10$ and it means that our universe is dominated by phantom energy $(\omega < -1)$ [13]-[15]. The phantom dominated universe ends up with a finite time future singularity called Big rip [16]. The existence of this future singularity is often considered as a negative feature of phantom dark energy models and therefore the researchers are trying to construct various models with $\omega < -1$ to avoid this future singularity [17]-[20]. In this paper, we consider a model where the dark energy behaves like a fluid with the equation of state parameter $P_\Lambda = \omega_\Lambda\rho_\Lambda$, $\omega_\Lambda < -1$ as well as polytropic gas to avoid the Big rip singularity. The polytropic gas is one of the dynamical dark energy models to explain the cosmic acceleration of the universe and it is a phenomenological model of dark energy where the pressure is a function of energy density [21]-[23].

Field equation and it solution

The Equation of state (EOS) of the polytropic gas is given by

$$P_\Lambda = K\rho_\Lambda^{1+\frac{1}{n}} \qquad (1)$$

Where$P_\Lambda, \rho_\Lambda, K$ and n are the pressure, energy density, polytropic constant and polytropic index respectively.

We consider the line element for the spatially homogeneous flat FRW universe

$$ds^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2] \qquad (2)$$

Where x, y, z are the space coordinate, t is the time component and $a(t)$ is the scale factor.

For this line element, the field equations are obtained as

$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3}$ (3)

And $2\dot{H}+3H^2 = -P$ (4)

The conservation equation for the dark energy in the FRW universe is given by

$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + P_\Lambda) = 0$ (5)

Where H the Hubble parameter and the overhead dot denotes the differentiation with respect to the cosmological time

Using the EOS (1) into the conservation equation (2) and integrating we get

$\rho_\Lambda = \left[Ba^{3/n} - K\right]^{-n}$ (6)

Where B is a positive integration constant and $a(t)$ is a time scale factor of the universe . The corresponding pressure takes the following form

$P_\Lambda = K\left[Ba^{3/n} - K\right]^{-n-1}$ (7)

In this present model it is assumed that the dark energy behaves like polytropic gas obeying equation (1) as well as fluid with equation of state

$P_\Lambda = \omega_\Lambda \rho_\Lambda$ With $\omega_\Lambda < -1$ (8)

Using equations (6) & (7), the EOS parameter for the polytropic gas dark energy model is obtained as

$\omega_\Lambda = \frac{P_\Lambda}{\rho_\Lambda} = -1 + \frac{Ba^{3/n}}{Ba^{3/n} - K}, \qquad K > Ba^{3/n}$ (9)

At the present time $t = t_0$, the equation (9) gives

$K = \frac{B\omega_0\, a_0^{\frac{n}{3}}}{\omega_0+1}$ (10)

Where $a_0 = a(t_0),\ \ \omega_0 = \omega(t_0)$

From (6) and (10) we get

$\rho = \left[Ba^{\frac{3}{n}} - \frac{B\omega_0\, a_0^{\frac{n}{3}}}{\omega_0+1}\right]^{-n}$ (11)

The energy density and pressure of the scalar field $\phi(t)$ and potential $V(\phi)$ in the homogeneous universe are given by

$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ (12)

$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$ (13)

Where $\frac{1}{2}\dot{\phi}^2$ is the kinetic energy and $V(\phi)$ is the potential energy of the scalar field ϕ

From (12) & (13) we get

$\dot{\phi}^2 = \rho_\phi + P_\phi$ (14)

Using (6), (7) and (10) in (14) we get

$\dot{\phi}^2 = \frac{Ba^{\frac{3}{n}}(\omega_0+1)^{n+1}}{\left[Ba^{\frac{3}{n}}\left\{(\omega_0+1) - \omega_0\left(\frac{a_0}{a}\right)^{\frac{3}{n}}\right\}\right]^{n+1}}$ (15)

From the equation (15) we see that $\dot{\phi}^2 > 0$ when $\omega_0 > -1$ and $\dot{\phi}^2 < 0$ when $\omega_0 < -1$.Thus $\omega_0 > -1$ and $\omega_0 < -1$ represents a case of quintessence and phantom fluid dominated universe respectively. Hence dual behavior of dark energy fluid obeying equations (1) and (8) is possible for scalars.

From the equations (3) and (11) we get

$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\Omega_0 {H_0}^2}{2}\left[1 - \frac{\omega_0\left(\frac{a_0}{a}\right)^{\frac{3}{n}}}{1+\omega_0}\right]^{-n} = \frac{\Omega_0 {H_0}^2}{2}\left[1 - \frac{|\omega_0|\left(\frac{a_0}{a}\right)^{\frac{3}{n}}}{|\omega_0|-1}\right]^{-n}$ (16)

Where$|\omega_0| = -\omega_0$, H_0 is the present value of the Hubble constant and $\Omega_0 = \frac{\rho_0}{\rho_{cr,0}}$, with $\rho_{cr,0} = \frac{3{H_0}^2}{8\pi G}$ (G being the Newtonian gravitational constant)

Taking square root to the both sides of equation (16) we get

$$H = \frac{\dot{a}}{a} = \frac{\sqrt{\Omega_0}}{\sqrt{2}} H_0 \left[1 - \frac{|\omega_0|\left(\frac{a_0}{a}\right)^{\frac{3}{n}}}{|\omega_0| - 1}\right]^{-\frac{n}{2}} \tag{17}$$

Expanding right hand side of equation (17) and neglecting higher powers of $\frac{|\omega_0|\left(\frac{a_0}{a}\right)^{\frac{3}{n}}}{|\omega_0|-1}$ we get

$$H = \frac{\dot{a}}{a} = \frac{\sqrt{\Omega_0}}{\sqrt{2}} H_0 \left[1 + \frac{n|\omega_0|\left(\frac{a_0}{a}\right)^{\frac{3}{n}}}{2(|\omega_0| - 1)}\right] \tag{18}$$

Integrating (18) we get

$$a(t) = \frac{a_0}{\{2(|\omega_0|-1)\}^{\frac{n}{3}}} \left[\{n + 2(|\omega_0| - 1)\} e^{\frac{\sqrt{\Omega_0}}{\sqrt{2}} H_0 (t - t_0)}\right]^{\frac{3}{n}} \tag{19}$$

From (19) we see that $a(t) \to \infty$ as $t \to \infty$

In this case the Hubble distance is given by

$$H^{-1} = \frac{\sqrt{2}}{\sqrt{\Omega_0} H_0} \left[1 - \frac{n|\omega_0|\left(\frac{a_0}{a}\right)^{\frac{3}{n}}}{2(|\omega_0| - 1)}\right] \tag{20}$$

Equation (20) shows the growth of Hubble's distance $H^{-1} \to \frac{\sqrt{2}}{\sqrt{\Omega_0} H_0} \neq 0$ as $t \to \infty$

It means that the galaxies will not disappear when $t \to \infty$, avoiding big rip singularity.

Conclusion

In the FRW universe, we observed that when the cosmic dark energy behaves simultaneously like a fluid with equation of state $P_\Lambda = \omega_\Lambda \rho_\Lambda$, $\omega_\Lambda < -1$ as well as polytropic gas with equation of state $P_\Lambda = K \rho_\Lambda^{1+\frac{1}{n}}$, then the concluding remarks are as follows

For the positive kinetic energy i.e. $\dot{\phi}^2 > 0$ we have $\omega_0 > -1$ which represents a quintessence fluid dominated universe and for the negative kinetic energy i.e. $\dot{\phi}^2 < 0$ we have $\omega_0 < -1$ which represents a phantom fluid dominated universe.

This results match with results obtained by Hoyle-Narlikar in C-field with negative kinetic energy for steady state theory of universe.

) For $t \to \infty$, we have $a(t) \to \infty$ which indicates that the present model is free from finite time future singularity.

) For $t \to \infty$ we have $H^{-1} \neq 0$ which indicates that the galaxies will not disappear as $t \to \infty$, avoiding big rip singularity. Therefore one can conclude that when cosmic dark energy behaves like a fluid with equation of state parameter $P_\Lambda = \omega_\Lambda \rho_\Lambda$, $\omega_\Lambda < -1$ as well as polytropic gas then the Big rip singularity does not arise and the scale factor is found to be regular for all time.

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Correspondence between the quintessence and dilaton scalar fields with the polytropic gas dark energy model

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Abstract

In this paper, we study the correspondence between the quintessence and dilaton scalar fields with the polytropic gas dark energy model. This correspondence allows us to reconstruct the dynamics and potential of these scalar fields in the context of the polytropic gas dark energy model, which describes the accelerated expansion of the universe.

Keywords: *Dark energy, Polytropic gas, quintessence, dilaton*

1. Introduction

Cosmologist's belief that our universe expands under an accelerated expansion [1]-[7].In the standard Friedman Lemaitre Robertson Walker (FLRW) cosmology, a new energy with negative pressure, called dark energy (DE) is responsible for this expansion [8]. The nature of the DE is still unknown and various problems have been proposed by the researchers in this field. About 70% of the present energy of the universe is contained in the DE. The cosmological constant with the time independent equation of state is the earliest and simplest candidate for the dark energy. Besides the cosmological constant, there are many dynamical dark energy models with the time dependent equation of state that have been proposed to explain the cosmic acceleration. Polytropic gas is one of the dynamical dark energy models [9]. In this work, we focus on the polytropic gas model as a DE model. The polytropic gas DE model is a phenomenological model of dark energy where the pressure is a function of energy density [10]. The quintessence [12, 13] and dilaton [14]-[16] scalar fields are considered as a source of dark energy. In this paper, we establish a correspondence between the quintessence and dilaton scalar fields with the polytropic gas dark energy model. This correspondence allows us to reconstruct the dynamics and potential of these fields in the context of the polytropic gas dark energy model, which describes the accelerated expansion of the universe.

2. Polytropic quintessence model

Equation of state (EOS) of the polytropic gas is given by [11]

$$P_\Lambda = K\rho_\Lambda^{1+\frac{1}{n}} \qquad (1)$$

Where P_Λ, ρ_Λ, K and n are the pressure, energy density, polytropic constant and polytropic index respectively.

The conservation equation for the dark energy in the FRW universe is given by

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + P_\Lambda) = 0 \qquad (2)$$

Where H is the Hubble parameter and overhead dot denotes the differentiation with respect to the cosmological time

Using the EOS (1) into the conservation equation (2) and integrating we get

$$\rho_\Lambda = \left[Ba^{3/n} - K\right]^{-n} \qquad (3)$$

Where B is a positive integration constant and $a(t)$ is a time scale factor of the universe .

The corresponding pressure takes the following form

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$$P_\Lambda = K\left[Ba^{3/n} - K\right]^{-n-1} \quad (4)$$

Using equations (3) & (4), the EOS parameter for the polytropic gas dark energy model is obtained as

$$\omega_\Lambda = \frac{P_\Lambda}{\rho_\Lambda} = -1 + \frac{Ba^{3/n}}{Ba^{3/n}-K} \quad (5)$$

When $K > Ba^{3/n}$, from (5), we see that $\omega_\Lambda < -1$ which corresponds to a universe dominated by phantom field. The phantom field lead to accelerated expansion of the universe

The action for the quintessence scalar field φ is given by [12], [13]

$$S = \int\left[\frac{1}{2}g^{ij}\,\partial_i\varphi\,\partial_j\varphi - V(\varphi)\right]\sqrt{-g}\,d^4x \quad (6)$$

Where φ *is* the quintessence field with the potential $V(\varphi)$ and g is the determinant of g_{ij}

The pressure and energy density for the quintessence scalar field φ are given by

$$P_\varphi = \frac{1}{2}\dot\varphi^2 - V(\varphi) \quad (7)$$

$$\rho_\varphi = \frac{1}{2}\dot\varphi^2 + V(\varphi) \quad (8)$$

From the equations (3), (4), (7) and (8), we get the scalar potential and the kinetic energy terms for the polytropic gas as

$$V(\varphi) = \frac{\frac{B}{2}a^{3/n}-K}{\left(Ba^{3/n}-K\right)^{n+1}} \quad (9)$$

$$\dot\varphi^2 = \frac{Ba^{3/n}}{\left(Ba^{3/n}-K\right)^{n+1}} \quad (10)$$

The equation of state for the scalar field φ is given by

$$\omega_\varphi = \frac{P_\varphi}{\rho_\varphi} = \frac{\frac{1}{2}\dot\varphi^2 - V(\varphi)}{\frac{1}{2}\dot\varphi^2 + V(\varphi)} = \frac{\dot\varphi^2 - 2V(\varphi)}{\dot\varphi^2 + 2V(\varphi)} \quad (11)$$

So that $-1 \le \omega_\varphi \le 1$

If the kinetic term $\dot\varphi^2$ dominates then $\omega_\varphi \approx 1$ and if the potential term $V(\varphi)$ dominates then $\omega_\varphi \approx -1$

The equation of motion for the field $\varphi = \varphi(t)$ on a FLRW background is given by the Klein Gordon equation

$$\ddot\varphi + 3H\dot\varphi + V'(\varphi) = 0 \quad (12)$$

Where $H = \frac{\dot a}{a}$ is the Hubble parameter, overhead dot denotes differentiation with respect to cosmic time t and $V'(\varphi) = \frac{dV}{d\varphi}$

The Friedmann equation and acceleration equation are given by

$$H^2 = \frac{1}{3}\left[\frac{1}{2}\dot\varphi^2 + V(\varphi)\right] \quad (13)$$

$$\frac{\ddot a}{a} = -\frac{1}{3}\left[\dot\varphi^2 - V(\varphi)\right] \quad (14)$$

If $\dot\varphi^2 < V(\varphi)$ then $\ddot a > 0$ this corresponds to the accelerated expansion of the universe

If $\frac{1}{2}\dot\varphi^2 \ll V(\varphi)$ (Slow roll approximation) then $H^2 \approx \frac{1}{3}V(\varphi)$ and $\omega_\varphi \approx -1$ (like cosmological constant) which represents a potential dominated scalar field. In the slow roll approximation as the term $\dot\varphi^2$ is considered negligible and the potential $V(\varphi)$ can be considered to fulfill

$$3H\dot\varphi \approx -V'(\varphi) \quad (15)$$

Therefore this slow rolling potential dominated scalar field can accelerated the expansion of the universe and act as a dark energy candidate.

3. Polytropic dilaton model

The pressure and energy density of the dilaton scalar field are given by [14]-[16]

$$P_\varphi = -\chi + ce^{\lambda\varphi}\chi^2 \quad (16)$$

$$\rho_\varphi = -\chi + 3ce^{\lambda\varphi}\chi^2 \quad (17)$$

Where c and λ are positive constants and $\chi = \frac{\dot\varphi^2}{2}$

So the equation of state parameter for the dilaton scalar field is obtain as

$$\omega_d = \frac{P_\varphi}{\rho_\varphi} = \frac{-1+ce^{\lambda\varphi}\chi}{-1+3ce^{\lambda\varphi}\chi} \quad (18)$$

To establish the correspondence between the dilaton equation of state parameter(ω_d) and polytropic equation of state parameter(ω_Λ) , equating the equations (5) and (18) we get,

$$ce^{\lambda\varphi}\chi = \frac{\omega_\Lambda - 1}{3\omega_\Lambda - 1} = \frac{2K - Ba^{3/n}}{4K - Ba^{3/n}} \quad (19)$$

Putting $\chi = \frac{\dot{\varphi}^2}{2}$ in the above equation (19) we get,

$$ce^{\lambda\varphi}\dot{\varphi}^2 = \frac{4K - 2Ba^{3/n}}{4K - Ba^{3/n}} \quad (20)$$

The above equation can be written as

$$e^{\frac{\lambda\varphi}{2}}\dot{\varphi} = \left[\frac{1}{c}\frac{4K - 2Ba^{3/n}}{4K - Ba^{3/n}}\right]^{\frac{1}{2}} \quad (21)$$

And its integration yields

$$\varphi(a) = \frac{2}{\lambda} ln\left[e^{\frac{\lambda\varphi(a_0)}{2}} + \frac{\lambda}{2\sqrt{c}}\int_{a_0}^{a}\frac{1}{aH}\left[\frac{4K - 2Ba^{3/n}}{4K - Ba^{3/n}}\right]^{\frac{1}{2}} da\right] \quad (22)$$

Thus we reconstructed the potential and dynamics of the dilaton scalar field in the context of the polytropic gas.

3. Conclusion

In this paper, we have studied the polytropic gas dark energy model with the quintessence and dilaton scalar fields. The polytropic quintessence model indicates a potential dominated scalar field universe in the slow roll approximation that corresponds to the accelerated expansion of the universe. We established a correspondence between the polytropic gas and the scalar fields. We also reconstructed the potential and dynamics of the scalar fields in the context of polytropic gas.

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Modified Polytropic $f(T)$ gravity model

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Abstract: In this paper, we reconstruct the $f(T)$ gravity model with the polytropic gas dark energy. We also find the equation of state of the $f(T)$ gravity model in the context of polytropic gas to explain the acceleration of the universe.

Keywords: *Dark energy, Polytropic gas,* $f(T)$ gravity.

I. INTRODUCTION

Many Cosmological experiments and observations such as Type 1a Supernovae [1]-[3], Cosmic Microwave Background Radiation [4], Large Scale Structure [5], [6], Wilkinson Microwave Anisotropy Probe [7], Sloan Digital Sky Survey [8], etc. indicates that our universe expands under an accelerated expansion. In standard Friedman Lemaitre Robertson Walker (FLRW) cosmology, a new energy with negative pressure, called dark energy (DE) is responsible for this expansion [9], [10]. The nature of the DE is still unknown and various problems have been proposed by the researchers in this field. About 70% of the present energy of the universe is contained in the DE. The cosmological constant with the time independent equation of state is the earliest and simplest candidate for the dark energy [11]. Besides the cosmological constant, there are many dynamical dark energy models with the time dependent equation of state that have been proposed to explain the cosmic acceleration. Polytropic gas is one of the dynamical dark energy models to explain the cosmic acceleration of the universe [12]-[14]. The polytropic gas DE model is a phenomenological model of dark energy where the pressure is a function of energy density [15]. An interesting alternative to General Relativity is the so-called $f(T)$ gravity, which has received considerable attention as a possible explanation of the late time acceleration of the universe [16]-[18]. It is based on the old idea of the "teleparallel" equivalence of the General Relativity [19]. : In this paper, we reconstruct the $f(T)$ gravity model with the polytropic gas dark energy. We also find the equation of state of the $f(T)$ gravity model in the context of polytropic gas to explain the acceleration of the universe.

II. THE BASIC EQUATION OF THE $f(T)$ GRAVITY MODEL

The action of the $f(T)$ gravity is given by [17], [18]

$$S = \frac{1}{2k^2}\int d^4x\sqrt{-g}[f(T) + L_m] \quad (1)$$

Where$k^2 = 8\pi G$, T is the torsion scalar, $f(T)$ is the general differentiable function of the torsion$T$ and L_m is the Lagrangian density of the matter inside the universe

The modified Friedmann equation in the case of $f(T)$ gravity for the spatially flat FRW universe are given by

$$H^2 = \frac{1}{3}(\rho_m + \rho_T) \quad (2)$$

$$2\dot{H} + 3H^2 = -(p_m + p_T) \quad (3)$$

Where

$$\rho_T = \frac{1}{2}(2Tf_T - f - T) \quad (4)$$

$$p_T = -\frac{1}{2}\left[-8\dot{H}f_{TT} + (2T - 4\dot{H})f_T - f + 4\dot{H} - T\right] \quad (5)$$

$$T = -6H^2 \quad (6)$$

Here $H = \frac{\dot{a}}{a}$ is the Hubble parameter, ρ_m and p_m are the energy density and pressure of the matter inside the universe, ρ_T and p_T are the torsion contributions to the energy density and pressure. Also $f_T$and f_{TT} denotes one and two times derivatives with respect to the torsion scalar T .

The energy conservation laws are

$$\dot{\rho}_T + 3H(\rho_T + p_T) = 0 \quad (7)$$

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0 \quad (8)$$

Using (4) & (5), the equation of state (EOS) due to torsion contribution is defined by

$$\omega_T = -1 + \frac{4\dot{H}(2Tf_{TT} + f_T - 1)}{2Tf_T - f - T} \quad (9)$$

The scale factor $a(t)$ is represented by [21]

$$a(t) = a_o(t_s - t)^{-h}, t \le t_s, h > 0 \quad (10)$$

From (5) & (9) one can write

$$H = \frac{h}{t_s - t} \quad (11)$$

$\dot{H} = \frac{H^2}{h} = \frac{h}{(t_s - t)^2}$ (12)

$T = -6\left(\frac{h}{t_s - t}\right)^2$ (13)

From the equations (10) & (11), the scale factor $a(t)$ can be rewritten as

$a(t) = a_o\left(\frac{H}{h}\right)^h$ (14)

From the equation (12), we see that $\dot{H} > 0$ which represent a supper accelerated FRW universe with a big rip singularity at $t = t_s$. Also from the equation (13), we see that

When $t = t_s$, $T = -\infty$ (15)

III. POLYTROPIC $f(T)$ GRAVITY MODEL

The equation of state (EOS) of the polytropic gas is given by [20]

$p_\Lambda = k\rho_\Lambda^{1+\frac{1}{n}}$ (16)

Wherep_Λ,ρ_Λ,k,and n are the pressure, energy density, polytropic constant and polytropic index respectively.

Using the EOS (16) into the conservation equation (7) and integrating we get

$\rho_\Lambda = \left[-k + Ba^{3/n}\right]^{-n}$ (17)

Where B is a positive integration constant and $a(t)$ is a time scale factor of the universe [13]

From (14) and (17) one can obtain

$\rho_\Lambda = \left[-K + \alpha(-6H^2)^{\frac{3h}{2n}}\right]^{-n}$ (18)

Where $\alpha = Ba_0^{\frac{3}{n}}(-6H^2)^{\frac{-3h}{2n}}$ (19)

Equating (4) and (18) we get

$(-12H^2 f_T - f + 6H^2) - \left[-K + \alpha(-6H^2)^{\frac{3h}{2n}}\right]^{-n} = 0$ (20)

Using the solution of (20) into (9), the EOS of parameter of torsion contribution is given by [20]

$\omega_T = -1 - \frac{1}{\frac{K}{B}\left[a_0\left(\frac{H}{h}\right)^h\right]^{\frac{-3}{n}} - 1}$, h>0 (21)

Equation (12) can be rewritten as

$\frac{\dot{H}}{H} = \frac{H}{h}$ (22)

Using (22) in (21) we get

$\omega_T = -1 - \frac{1}{\frac{K}{B}\left[a_0\left(\frac{\dot{H}}{H}\right)^h\right]^{\frac{-3}{n}} - 1}$, h>0 (23)

When $\frac{K}{B}\left[a_0\left(\frac{\dot{H}}{H}\right)^h\right]^{\frac{-3}{n}} > 1$ then from (23) we see that $\omega_T < -1$ which represents a phantom like accelerated universe.

IV CONCLUSION

We see from above discussion that the equation of state of the $f(T)$ gravity model in the context of polytropic gas represent a phantom like accelerated universe.

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LRS BIANCHI TYPE-I POLYTROPIC GAS DARK ENERGY MODELS IN COSMOLOGY

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Abstract: In this paper, we have studied the Locally Rotationally Symmetric (LRS) Bianchi Type-I cosmological model in presence of Polytropic gas as dark energy in the form $P = K\rho^{1+\frac{1}{n}}$, Where $P, \rho, K$and n are the pressure, energy density, polytropic constant and polytropic index respectively and to solve the Einstein's field equations for the LRS Bianchi Type-I space time, we have applied the relation that the scalar expansion (θ) is proportional to the shear scalar(σ).Some physical and cosmological properties of the model have been obtained and discussed.
***Index Terms*- LRS Bianchi Type-I space time, Polytropic gas, Dark energy.**

I. INTRODUCTION

Cosmology is the scientific study of the large scale properties of the universe as a whole. Cosmology is study of the motion of heavenly bodies of the universe. Most of the Cosmological experiments and observations such as Type 1a Supernovae [1]-[3], Cosmic Microwave Background Radiation [4], Large Scale Structure [5], [6], Wilkinson Microwave Anisotropy Probe [7], Sloan Digital Sky Survey [8], etc. indicates that our universe expands under an accelerated expansion. In standard Friedman Lemaitre Robertson Walker (FLRW) cosmology, a new energy with negative pressure, called dark energy (DE) is responsible for this expansion [9] & [10]. But the nature of the DE is still unknown and various problems have been proposed by the researchers in this field**.** About 70% of the present energy of the universe is contained in the DE. The cosmological constant with the time independent equation of state is the earliest and simplest candidate for the dark energy [11]. Besides the cosmological constant, there are many dynamical dark energy models with the time dependent equation of state that have been proposed to explain the cosmic acceleration. Polytropic gas is one of the dynamical dark energy models to explain the cosmic acceleration of the universe [12]-[14]. The polytropic gas DE model is a phenomenological model of dark energy where the pressure is a function of energy density [15]. From the different observational data's, it is clear that our universe is homogeneous and isotropic on a large scale; however no physical evidence denies the chances of an anisotropic universe. In fact, many theoretical arguments are present promoting the existence of an anisotropic phase of the universe that approaches the isotropic phase [16]-[19]. The anisotropy plays a vital rule in the early phase of evolution of the universe and therefore studying homogeneous and anisotropic cosmological models are considered most important. The Bianchi type models are spatially homogeneous and generally anisotropy. The simplicity of the field equations made by the Bianchi type space time is very useful in constructing cosmological models which are spatially homogeneous and anisotropic. K.S.Adhav et al. have investigated Bianchi type models in different contexts [20]-[26]. Also several authors such as S.D. Katore and D.V. Kapse [27], C.P.Singh and S.Kumar [28], A.Pradhan and A.K. Vishwakarma [29], M.A. Rahman and M. Ansari [30] have investigated various cosmological models in this field.

In this paper, we have investigated Bianchi type-1 cosmological model in presence of Polytropic gas in the form $P = K\rho^{1+\frac{1}{n}}$,Where $P, \rho, K$and n are the pressure, energy density, polytropic constant and polytropic index respectively and the field equation have been solved by using the physical condition that that the scalar expansion (θ) is proportional to the shear scalar(σ). Some physical and cosmological properties of the model have been obtained and discussed.

II. METRIC AND FIELD EQUATION

We consider the LRS Bianchi type-I metric in the form

$$ds^2 = dt^2 - A(dx^2 + dy^2) - Bdz^2 \tag{1}$$

Where A and B are the metric functions of cosmic time$'t'$ only.

The Einstein's field equations are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \tag{2}$$

Where R_{ij} is the Ricci tensor, R is the Ricci scalar and T_{ij} is the energy momentum tensor for bulk viscous cosmology
The energy conservation equation is given by

$$T^{ij}_{;j} = 0 \quad (3)$$

Where $T^{ij}_{;j} = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^j}\left(T^{ij}\sqrt{-g}\right) + T^{jk}\Gamma^i_{jk}$

The energy momentum tensor for the bulk viscous fluid is given by

$$T_{ij} = (\rho + P)u_i u_j + P g_{ij} \quad (4)$$

Where ρ is the energy density, P is the pressure and u^i is the four velocity vector satisfying $g_{ij}u^i u^j = 1$

In this model we have considered that the universe is filled with a fluid named polytropic gas.
The equation of state of the polytropic gas is given by

$$P = K\rho^{1+\frac{1}{n}} \quad (5)$$

Where K and n (>0) are constants known as polytropic constant and polytropic index respectively.
Using equation (4) for the metric (1), the Einstein's field equations becomes

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} = \rho \quad (6)$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} = -P \quad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -P \quad (8)$$

Where overhead dot represents the differentiation with respect to cosmic time t
Using equation (1), the energy conservation equation (3) takes to the following forms

$$\dot{\rho} + \left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)(P + \rho) = 0 \quad (9)$$

The mean Hubble parameter H and scalar expansion θ are given by

$$H = \frac{1}{3}\theta = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \quad (10)$$

Where V is the spatial volume of the universe
The average anisotropy parameter Δ and shear scalar σ^2 are given by

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \quad (11)$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^{\,2} - 3H^2\right) \quad (12)$$

Where $H_i, i = 1,2,3$ represents the directional parameters in the directions of x, y, z respectively and $\Delta = 0$ corresponds to the isotropic expansion of the universe.

III. SOLUTIONS OF THE FIELD EQUATIONS

To solve the Einstein's Field equations (6), (7) & (8) we assume that the scalar expansion ( ) is proportional to the shear scalar (σ^2) . Thus by assuming it we can take a relation as follows

$$B = A^m \quad (13)$$

Where $m(> 0)$ is a constant
With the help of the equation (13), the field equations (6), (7) and (8) can be reduced to the following forms

$$(2m + 1)\frac{\dot{A}^2}{A^2} = \rho \quad (14)$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} = -P \quad (15)$$

$$(m + 1)\frac{\ddot{A}}{A} + m^2\frac{\dot{A}^2}{A} = -P \quad (16)$$

Solving the equations (15) & (16) we get

$$\left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2}\right) + (m + 2)\frac{\dot{A}^2}{A^2} = 0 \quad (17)$$

Integrating it we get

$$A(t) = a_0[t_0 + (m + 2)t]^{\frac{1}{m+2}} \quad (18)$$

Where a_0 and t_0 are constants of integration.
From the equations (13) and (18) we get

$$B(t) = \left[a_0\{t_0 + (m + 2)t\}^{\frac{1}{m+2}}\right]^m$$

$$= a_0^m[t_0 + (m + 2)t]^{\frac{m}{m+2}} \quad (19)$$

By a suitable choice of constants ($a_0 = 1, t_0 = 0$), the metric equation (1) can be written as

$$ds^2 = dt^2 - [(m+2)t]^{\frac{1}{m+2}}(dx^2 + dy^2) - [(m+2)t]^{\frac{m}{m+2}} dz^2 \quad (20)$$

IV. PHYSICAL AND COSMOLOGICAL PROPERTIES OF THE MODEL

From the equation (14) we get the energy density as

$$\rho = \frac{(2m+1)}{(m+2)^2 t^2} \quad (21)$$

Using the equation (21) in the equation (5) we get the pressure as

$$P = K\left[\frac{(2m+1)}{(m+2)^2 t^2}\right]^{1+\frac{1}{n}} \quad (22)$$

The mean Hubble parameter (H) and scalar expansion $(\theta$)are given by

$$H = \frac{1}{3t} \quad (23)$$

$$\theta = \frac{1}{t} \quad (24)$$

The spatial volume (V) of the universe is given by

$$V = (m+2)t \quad (25)$$

The shear scalar σ^2 is given by

$$\sigma^2 = \frac{(m-1)^2}{(m+2)^2 t^2} = \frac{(m-1)^2\theta^2}{(m+2)^2} \quad (26)$$

The average anisotropy parameter (Δ) is given by

$$\Delta = \frac{2(m-1)^2}{(m+2)^2} = \frac{2\sigma^2}{\theta^2} \quad (27)$$

The deceleration parameter(q) is given by

$$q = -\frac{\dot{H}}{H^2} - 1 = 2 \quad (28)$$

The equation of state parameter(ω) is given by

$$\omega = \frac{P}{\rho} = K\left[\frac{(2m+1)}{(m+2)^2 t^2}\right]^{\frac{1}{n}} \quad (29)$$

Also the energy density parameter(Ω) is given by

$$\Omega = \frac{\rho}{3H^2} = \frac{3(2m+1)}{(m+2)^2} \quad (30)$$

For $m = 1$, we have $A = B, \Delta = 0, \sigma^2 = 0, \Omega = 1$, this corresponds to an isotropic flat universe and the universe would expand forever but the rate of expansion would slow after infinite amount of time, Also for $m \neq 1$, we have Δ=constant, $\frac{\sigma}{\theta}$ =constant,$\Omega < 1$,this corresponds to an anisotropic open universe and the universe would expand forever.

V. CONCLUSION

A study of the Bianchi type I cosmological model with the polytropic gas has been done. The physical and cosmological parameters which play a key role in the discussion of the model are obtained. It is noted that the spatial volume of the universe is zero at $t = 0$ and increases infinitely as $t \to \infty$. The energy density, the pressure, the average Hubble parameter, expansion scalar and shear scalar are infinite at $t = 0$ and approaches 0 as $t \to \infty$. The universe is isotropic and flat when $m = 1$ and it is anisotropic and open when $m \neq 1$. Therefore the model is anisotropic throughout the evolution of the universe except at $m = 1$.In this model the average anisotropy parameter (Δ) and energy density parameter(Ω) are independent of the cosmic time.

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Universe dominated by dark energy in presence of polytropic gas

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Abstract: Polytropic gas has been thought as an alternative dark energy model by many cosmologists. Here we have discussed about the cases of dominance of phantom dark energy or quintessence in universe due to the presence of polytropic gas.

Keywords: ***Dark energy, Polytropic gas.***

1. INTRODUCTION :- Many Cosmological experiments and observations such as Type 1a Supernovae [1]-[3], Cosmic Microwave Background Radiation [4], Large Scale Structure [5], [6], Wilkinson Microwave Anisotropy Probe [7], Sloan Digital Sky Survey [8], etc. indicates that our universe expands under an accelerated expansion. In standard Friedman Lemaitre Robertson Walker (FLRW) cosmology, a new energy with negative pressure, called dark energy (DE) is responsible for this expansion [9],[10]. The nature of the DE is still unknown and various problems have been proposed by the researchers in this field. About 70% of the present energy of the universe is contained in the DE. The cosmological constant with the time independent equation of state is the earliest and simplest candidate for the dark energy [11]. Besides the cosmological constant, there are many dynamical dark energy models with the time dependent equation of state that have been proposed to explain the cosmic acceleration. Polytropic gas is one of the dynamical dark energy models to explain the cosmic acceleration of the universe [12]-[14]. The polytropic gas DE model is a phenomenological model of dark energy where the pressure is a function of energy density [15].

2. FORMULATION OF THE PROBLEM:- The polytropic gas has been proposed as an alternative dark energy model to explain the acceleration of the universe and its equation of state (EOS) is given by [13]

$$p_\Lambda = k\rho_\Lambda^{1+\frac{1}{n}} \qquad (1)$$

Where p_Λ, ρ_Λ, k, and n are the pressure, energy density, polytropic constant and polytropic index respectively.

The conservation equation for the dark energy in the FRW universe is given by

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = 0 \qquad (2)$$

Where H is the Hubble parameter and a dot are denotes the differentiation with respect to the cosmological time.

Using the EOS (1) into the conservation equation (2) and integrating we get

$$\rho_\Lambda = \left[-k + Ba^{3/n}\right]^{-n} \qquad (3)$$

Where B is a positive integration constant and $a(t)$ is a time scale factor of the universe [13]

When $k < Ba^{3/n}$, we see that $\rho_\Lambda > 0$ for any arbitrary value of n; when $k > Ba^{3/n}$, we see that $\rho_\Lambda > 0$ for even value of n.Also when $k = Ba^{3/n}$,we see that $\rho_\Lambda \to \infty$ and the polytropic gas has a finite time singularity at $a_s = \left(\frac{k}{B}\right)^{n/3}$.

Using equations (1) & (3), the EOS parameter of the polytropic gas dark energy model is obtained as

$$\omega_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} = -1 + \frac{Ba^{3/n}}{Ba^{3/n} - k} \quad (4)$$

When $k > Ba^{3/n}$, we see that $\omega_\Lambda < -1$ which corresponds to a universe dominated by phantom dark energy; when $k < Ba^{3/n}$, we see that $\omega_\Lambda > -1$ which corresponds to a quintessence like accelerated universe; also when $k = Ba^{3/n}$, we see that $\omega_\Lambda \to \infty$ which corresponds to a singularity at $a_s = \left(\frac{k}{B}\right)^{n/3}$.

Now we assume the scalar field and potential dark energy model. The energy density and pressure of the scalar field $\phi(t)$ and potential $V(\phi)$ are given by

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad (5)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad (6)$$

Where $\frac{1}{2}\dot{\phi}^2$ is the kinetic energy and $V(\phi)$ is the potential energy of the scalar field ϕ

Using equations (1) &(3) into the equations (5)&(6) we can find the scalar potential and kinetic energy terms for the polytropic gas as

$$V(\phi) = \frac{\frac{B}{2}a^{3/n} - K}{\left(Ba^{3/n} - K\right)^{n+1}} \quad (7)$$

$$\dot{\phi}^2 = \frac{Ba^{3/n}}{\left(Ba^{3/n} - K\right)^{n+1}} \quad (8)$$

When $k > Ba^{3/n}$, we see that $\dot{\phi}^2 < 0$(negative kinetic energy), therefore the scalar field is a phantom field. The phantom field lead to super accelerated expansion of the universe. When $k < Ba^{3/n}$, we see that, $\dot{\phi}^2 > 0$(positive kinetic energy), therefore the scalar is a quintessence field.

Using equations (5) & (6), the EOS parameter for the scalar fields is

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)} = -1 + \frac{2\dot{\phi}^2}{\dot{\phi}^2 + 2V(\phi)} \quad (9)$$

When $\dot{\phi}^2 = 0$, then equation (9) gives $\omega_\phi = -1$, when $V(\phi) = 0$, then equation (9) gives $\omega_\phi = 1$. Here $\omega_\phi = -1$and $\omega_\phi = 1$ representing the vacuum fluid and stiff fluid dominated universe respectively. When $V(\phi) > \frac{1}{2}\dot{\phi}^2$, then equation (9) gives $\omega_\phi > -1$ which corresponds a quintessence dominated universe.

Considering negative kinetic energy in the equations (5) & (6), we get as follows

$$\rho_\phi = -\frac{1}{2}\dot{\phi}^2 + V(\phi) \quad (10)$$

$$p_\phi = -\frac{1}{2}\dot{\phi}^2 - V(\phi) \quad (11)$$

Using equations (10) & (11), the EOS of parameter for the scalar field is

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = \frac{-\frac{1}{2}\dot{\phi}^2 - V(\phi)}{-\frac{1}{2}\dot{\phi}^2 + V(\phi)} = \frac{\dot{\phi}^2 + 2V(\phi)}{\dot{\phi}^2 - 2V(\phi)} = -1 + \frac{2\dot{\phi}^2}{\dot{\phi}^2 - 2V(\phi)} = -1 + \frac{\dot{\phi}^2}{\frac{1}{2}\dot{\phi}^2 - V(\phi)} \quad (12)$$

When $\dot{\phi}^2 = 0$, then equation (12) gives $\omega_\phi = -1$; when $V(\phi) = 0$, then equation (12) gives $\omega_\phi = 1$,When $V(\phi) > \frac{1}{2}\dot{\phi}^2$, then equation (12) gives $\omega_\phi < -1$ which corresponds a phantom field. The phantom field lead to super accelerated expansion of the universe.

3. CONCLUSIONS :- Due the presence of polytropic gas in the form $p_\Lambda = k\rho_\Lambda^{1+\frac{1}{n}}$, where p_Λ, ρ_Λ , k and n are pressure, energy density, polytropic constant and polytropic index, a universe may be dominated by phantom dark energy or quintessence dark energy according as $\omega_\Lambda \left(= \frac{p_\Lambda}{\rho_\Lambda}\right) \lesseqgtr -1$. Also for $V(\phi) > \frac{1}{2}\dot{\phi}^2$ the universe may be dominated by phantom dark energy or quintessence dark energy according as negative kinetic energy$(-\frac{1}{2}\dot{\phi}^2)$ or positive kinetic energy$(\frac{1}{2}\dot{\phi}^2)$.

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Correspondence between the polytropic gas dark energy model and Tachyon scalar field

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Abstract: In this work we establish a correspondence between the polytropic gas dark energy model and Tachyon scalar field. Also we find the dynamics and potential of the Tachyon field in the context of polytropic gas dark energy model.

Keywords: *Dark energy, Polytropic gas, Tachyon field.*

1. Introduction

Cosmologist's belief that our universe expands under an accelerated expansion [1]-[7].In standard Friedman Lemaitre Robertson Walker (FLRW) cosmology, a new energy with negative pressure, called dark energy (DE) is responsible for this expansion [8]. The nature of the DE is still unknown and various problems have been proposed by the researchers in this field**.** About 70% of the present energy of the universe is contained in the DE. The cosmological constant with the time independent equation of state is the earliest and simplest candidate for the dark energy. Besides the cosmological constant, there are many dynamical dark energy models with the time dependent equation of state that have been proposed to explain the cosmic acceleration. Polytropic gas is one of the dynamical dark energy models [9]. In this work, we focus on the polytropic gas model as a DE model. The polytropic gas DE model is a phenomenological model of dark energy where the pressure is a function of energy density [10]. The Tachyon scalar field can be considered as a source of dark energy. In this paper we establish a correspondence between the polytropic gas dark energy model and Tachyon scalar field. Also we find the dynamics and potential of the Tachyon field in the context of polytropic gas dark energy model.

2. Polytropic gas Tachyon field model

Equation of state (EOS) of the polytropic gas is given by [11]

$$p_\Lambda = k\rho_\Lambda^{1+\frac{1}{n}} \qquad (1)$$

Where p_Λ , ρ_Λ , $k,$ and n are the pressure, energy density, polytropic constant and polytropic index respectively. The polytroic index is considered to be even.

The conservation equation for the dark energy in the FRW universe is given by

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = 0 \qquad (2)$$

Where H is the Hubble parameter and a dot are denotes the differentiation with respect to the cosmological time.

Using the EOS (1) into the conservation equation (2) and integrating we get

$$\rho_\Lambda = \left[Ba^{3/n} - k\right]^{-n} \qquad (3)$$

Where B is a positive integration constant and $a(t)$ is a time scale factor of the universe .

When $k > Ba^{3/n}$, we see that $\rho_\Lambda > 0$ for even values of n .Also when $k = Ba^{3/n}$, we see that $\rho_\Lambda \to \infty$ and the polytropic gas has a finite time singularity at $a_s = \left(\frac{k}{B}\right)^{n/3}$.

Using equations (1) & (3), the EOS parameter of the polytropic gas dark energy model is obtained as

$$\omega_\phi = \frac{p_\Lambda}{\rho_\Lambda} = -1 + \frac{Ba^{3/n}}{Ba^{3/n} - k} \qquad (4)$$

When $k > Ba^{3/n}$, we see that $\omega_\Lambda < -1$ which corresponds to a universe dominated by phantom field; when $k < Ba^{3/n}$, we see that $\omega_\Lambda > -1$ which corresponds to a quintessence dominated universe; also when $k = Ba^{3/n}$, we see that $\omega_\Lambda \to \infty$ which corresponds to a singularity at $a_s = \left(\frac{k}{B}\right)^{n/3}$

The energy density and pressure of the Tachyon scalar field are

$$\rho_\phi = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} \qquad (5)$$

$$p_\phi = -V(\phi)\sqrt{1-\dot{\phi}^2} \qquad (6)$$

The EOS parameter of the Tachyon field is given by

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = \dot{\phi}^2 - 1 \qquad (7)$$

When $\dot{\phi}^2 < 0$ then from (7) we see that $\omega_\phi < -1$ which represents a phantom field. The phantom field lead to accelerated expansion of the universe and hence the Tachyon field can interpret the accelerated expansion of the Universe.

Using equation (4) in (15) and (3) in (13) we get

$$\omega_\phi = -1 + \frac{Ba^{3/n}}{Ba^{3/n} - k} = \dot{\phi}^2 - 1 \qquad (8)$$

$$\rho_\Lambda = \left[Ba^{3/n} - k\right]^{-n} = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} \qquad (9)$$

From the equations (8) and (9) we can find the dynamics and potential of the Tachyon field in the context of polytropic gas dark energy model as follows

$$\dot{\phi}^2 = \frac{Ba^{3/n}}{Ba^{3/n} - k} \qquad (10)$$

$$V(\phi) = \left[Ba^{3/n} - k\right]^{-n}\sqrt{1-\dot{\phi}^2} \qquad (11)$$

If $k > Ba^{3/n}$, then from the equation (10) we see that $\dot{\phi}^2 < 0$ which represents the phantom behavior of the Tachyon field and from the equation (11) we see that $V(\phi) > 0$

3. Conclusion

The correspondence between the polytropic gas dark energy model and Tachyon scalar field can be established and we see that the Tachyon field behaves as phantom field. Therefore the Tachyon field can interpret the accelerated expansion of the Universe. Also we can find the dynamics and potential of the Tachyon field in the context of polytropic gas dark energy model.

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Research Article

K-ESSENCE POLYTROPIC GAS DARK ENERGY MODEL

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A B S T R A C T

In this work, we study the correspondence between the K-Essence scalar field and the polytropic gas dark energy model. This correspondence allows for reconstructing the K-Essence scalar field with reference to the polytropic gas dark energy model, which describe accelerated expansion of the universe.

INTRODUCTION

Many Cosmological experiments and observations such as Type 1a Supernovae[1],[2], Cosmic Microwave Background Radiation[3], Large Scale Structure[4],[5], Wilkinson Microwave Anisotropy Probe[6], Sloan Digital Sky Survey[7]etc. indicates that our universe expands under an accelerated expansion. In standard Friedman Lemaitre Robertson Walker (FLRW) cosmology, a new energy with negative pressure, called dark energy (DE) is responsible for this expansion [8]. The nature of the DE is still unknown and various problems have been proposed by the researchers in this field**.** About 70% of the present energy of the universe is contained in the DE. The cosmological constant with the time independent equation of state is the earliest and simplest candidate for the dark energy. Besides the cosmological constant, there are many dynamical dark energy models with the time dependent equation of state that have been proposed to explain the cosmic acceleration. Polytropic gas is one of the dynamical dark energy models [9]. The polytropic gas DE model is a phenomenological model of dark energy where the pressure is a function of energy density [10]. In this work, we study the correspondence between the K-Essence scalar field and the polytropic gas dark energy model. This correspondence allows to reconstructing the K-Essence scalar field with reference to the polytropic gas dark energy model, which describe accelerated expansion of the universe.

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Reconstruction of K-Essence Polytropic Gas Dark Energy Model

Equation of state (EOS) of the polytropic gas is given by [11]

$$p_\Lambda = k\rho_\Lambda^{1+\frac{1}{n}} \qquad (1)$$

Wherep_Λ,ρ_Λ ,k,and n are the pressure, energy density, polytropic constant and polytropic index respectively.

The conservation equation for the dark energy in the FRW universe is given by

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = 0 \qquad (2)$$

Where H is the Hubble parameter and a dot are. denotes the differentiation with respect to the cosmological time.

Using the EOS (1) into the conservation equation (2) and integrating we get

$$\rho_\Lambda = \left[Ba^{3/n} - k\right]^{-n} \qquad (3)$$

Where B is a positive integration constant and $a(t)$ is a time scale factor of the universe .

When $k > Ba^{3/n}$, we see that $\rho_\Lambda > 0$ for even values of n . Also when $k = Ba^{3/n}$, we see that $\rho_\Lambda \to \infty$ and the polytropic gas has a finite time singularity at$a_s = \left(\frac{k}{B}\right)^{n/3}$.

Using equations (1) & (3), the EOS parameter of the polytropic gas dark energy model is

$$\omega_\phi = \frac{p_\Lambda}{\rho_\Lambda} = -1 + \frac{Ba^{3/n}}{Ba^{3/n-k}} \qquad (4)$$

When $k > Ba^{3/n}$, then from (4) we see that $\omega_\Lambda < -1$ which corresponds to a universe dominated by phantom dark energy; when $k < Ba^{3/n}$, then from (4) we see that $\omega_\Lambda > -1$ which corresponds to a quintessence dominated universe; also when $K = Ba^{3/n}$, we see that $\omega_\Lambda \to \infty$ which corresponds to a singularity at $a_s = \left(\frac{k}{B}\right)^{n/3}$.

The K-Essence scalar field is given by

$$S = \int d^4x\sqrt{-g}\; p(\phi,\chi) \tag{5}$$

Where the Lagrangian density $p(\phi,\chi)$ correspondence to the pressure density and energy density via the following equations

$$p(\phi,\chi) = f(\phi)(-\chi + \chi^2) \tag{6}$$

$$\rho(\phi,\chi) = f(\phi)(-\chi + 3\chi^2) \tag{7}$$

The EOS parameter of K-Essence is given by

$$\omega_k = \frac{p(\phi,\chi)}{\rho(\phi,\chi)} = \frac{\chi-1}{3\chi-1} \tag{8}$$

When $\frac{1}{3} < \chi < \frac{1}{2}$, then $\omega_k < -1$ which represents phantom energy and hence the K-Essence scalar field can interpret the accelerated expansion.
Using (4) in (8) we get

$$\omega_\phi = -1 + \frac{Ba^{3/n}}{Ba^{3/n} - k} = \frac{\chi-1}{3\chi-1} \tag{9}$$

The parameter χ can be obtained as

$$\chi = \frac{2 + \frac{Ba^{3/n}}{k - Ba^{3/n}}}{4 + 3\frac{Ba^{3/n}}{k - Ba^{3/n}}} \tag{10}$$

Using $2\chi = \dot{\phi}^2$ in (10) we get

$$\dot{\phi}^2 = \frac{4 + 2\frac{Ba^{3/n}}{k - Ba^{3/n}}}{4 + 3\frac{Ba^{3/n}}{k - Ba^{3/n}}} \tag{11}$$

When $k > Ba^{3/n}$, then from (11) we see that $\dot{\phi}^2 > 0$, (positive kinetic energy), therefore the scalar field is a quintessence field. When $k < Ba^{3/n}$, then from (11) we see that $\dot{\phi}^2 < 0$(negative kinetic energy), therefore the scalar field is a phantom field. The phantom field lead to super accelerated expansion of the universe.

CONCLUSIONS

Due the presence of polytropic gas in the form $p_\Lambda = k\rho_\Lambda^{1+\frac{1}{n}}$, wherep_Λ,ρ_Λ, k and n are pressure, energy density, polytropic constant and polytropic index , an universe may bedominated by the K-Essence scalar field which can interpret the accelerated expansion when$\frac{1}{3} < \chi < \frac{1}{2}$and may be dominated by phantom field or quintessence field according as $k < Ba^{3/n}$or $k > Ba^{3/n}$.

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