Chapter 3

Observations on the role of bulk viscosity in present scenarios of the evolution in FRW model universe

3.1 Introduction

From many of the theoretical work carried out by researchers and experimental evidence, it proposed that our universe expanded very rapidly just after big-bang within a small fraction of a second. The modern findings in cosmology tell us that the universe is expanding and accelerating (Reiss et al., 1998; Permutter et al., 1998; Sing and Devi, 2016; Reiss et al., 2004). Observations from type-Ia Supernova (Reiss et al., 1998; Amanullah et al., 2010; Astier et al., 2006, Suzuki et al., 2012), CMB radiation (Spergel et al., 2003, Tegmark et al., 2004) and LSS (Spergel at al., 2007) are the evidence that the current universe is having an accelerated expansion, rather than slowing down as predicted by the big bang theory (Silk, 1989). Scientists are trying to solve this accelerating universe by assuming various probabilities. But till today, they could not arrive at a satisfactory conclusion on such strange

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behavior of the universe. The behaviour of late-time acceleration of the universe cannot be satisfactorily described by the general theory of relativity although it is considered the most successful theory in describing the early evolution of the universe. Cosmologists have arrived at two possible approaches to answer this cosmic accelerating expansion. One of such approaches is to introduce dark energy which dominates the universe and has associated with negative pressure. The second consideration is to modify Einstein's general theory of relativity.

Recently, some of the alternative theories of gravity are studied by many researchers. Among them, the most significant theories are the Weyl theory (Weyl, 1918), Lyra geometry (Lyra, 1951), Brans-Dicke theory (Brans and Dicke, 1961), $f(R)$ theory (Nojiri and Odinstov, 2003), and $f(R,T)$ gravity (Harko et al., 2011). Weyl's theory is a generalized theory of the Riemannian manifold to unify gravitational and electromagnetic fields. But due to the drawbacks in integrability feature, this theory could not attract many researchers. Later, Lyra removed this non-integrability feature by introducing a gauge function into Reimannian structure. Many researchers investigated Lyra geometry in four or higher-dimensional spacetime. (Aygun et al., 2012) showed non-survival of the massive scalar field for an anisotropic Marder type universe in the framework of Lyra and Riemannian geometries. (Rahaman and Bera, 2001) studied Kaluza-Klein cosmological model within the context of Lyra geometry in higher dimensions. (Singh et al., 2003) have investigated five-dimensional homogeneous cosmological models by considering bulk viscosity and variable gravitational constant in Lyra geometry. Many prominent researchers (Pradhan and Pandey, 2003; Rahaman et al., 2002; Reddy, 2005; Bhowmik and Rajput, 2004) have investigated different cosmological models within the framework of Lyra geometry and modified gravity.

The cosmological study in higher dimensions has become a great significance in investigating the early evolution of the universe. For many years, scientists are trying to unify four fundamental interactions to investigate the universe in the early epoch. In view of Kaluza-Klein theories, many authors (Kaluza, 1921; Klein, 1926; Lee, 1984; Appelquist et al., 1987) have studied higher dimensional cosmological model. (Chodos and Detweller, 1980) showed the possibility of extra dimensions of space. At the very early stage, all the dimensions (4+1) exist on the same scale. Later, during evolution, the fifth dimension shrinks and becomes unobservable. (Guth, 1981; Alvarez and Gavela, 1983) in their papers presented the cosmological scenario of existing entropy on a large-scale during compactification of its extra dimension. The present spacetime in four dimension can be modeled reverting to spacetime in higher dimensions such that the universe at an early age can be thought of having more than four dimensions. (Yilmaz, 2006) solved Kaluza-Klein cosmology in five dimensions for quark matter distribution of the universe attached to cloud string and domain wall in the framework of general relativity. (Rahaman et al., 2003) investigated higher-dimensional string theory in Lyra geometry. The strong evidence stands for the concept of extra dimensions has motivated some researchers (Ibanez, 1986; Gleiser and Diaz, 1988; Banerjee and Bhui, 1990; Reddy and Rao, 2001; Khadekar and Gaikwad, 2001) to study cosmology in multi-dimensional spacetime geometry.

In the past and recent years, some researchers are showing much interest in FRW spacetime geometry because of its spatial homogeneity and isotropy. At a large-scale structure, the current universe is represented by FRW models. The FRW metric is associated with the high symmetry of these backgrounds. Due to its high degree of symmetry, FRW models give a better explanation in most of the physical situations and therefore become useful in dealing with many complicated geometries. Also, in FRW cosmology, the metric is consistent with the framework of Mach's principle (Veto, 2013). Beesham (1988) solved FRW cosmological model using the idea of time-dependent displacement vector field.

In cosmology, to investigate the physical scenario during the form of the early universe, the concept of string theory provides a better understanding of evolution, before particles creation in the universe. Scientists believed that just after the big-bang, the universe undergoes a spontaneous symmetry breaking during the phase transition which results in a topological stable defects called cosmic strings. Cosmic strings are the main source in rising density perturbations that are responsible for galaxy formation in the early universe (Stachel, 1980; Letelier, 1979, 1983). Also, the bulk viscosity mechanism in cosmology describes the present scenario of high entropy and accelerated expansion of the universe. At an early epoch, the coupling of neutrinos disappears, and matter distribution in the universe act as a bulk viscous fluid (Misner, 1986). The bulk viscous fluid is associated with the transition from massive superstring modes to fewer models, the occurrence of the gravitational string, and the effects of particle creation in a GUT era. Hence, the study of one-dimensional cosmic strings together with bulk viscous fluid has become an important subject in investigating cosmological models. The study of bulk viscous string cosmology in higher dimensions in Lyra manifold was started by (Mohanty et al., 2009). (Reddy et al., 2013a, b) studied Kaluza-Klein cosmology with bulk viscosity and string in five dimensions in the modified theory of gravity. (Vidyasagar et al., 2014) have discussed a Bianchi type universe filled with the same type of matter in Brans-Dicke theory of gravity. (Naidu et al., 2012, 2013) investigated a different class of Bianchi universe with bulk viscous cosmic string in the context of both in $f(R,T)$ gravity . (Kiran and Reddy, 2013) in their paper found that bulk viscous string cosmological model cannot exist in Bianchi type *III* spacetime in $f(R,T)$ gravity while in general relativity this reduces to vacuum model.

Motivated by the above kinds of literature, in the present work, we investigate the role of bulk viscosity in Friedmann-Robertson-Walker (FRW) model universe in a higher dimension in Lyra geometry. We consider one-dimensional cosmic string along with bulk viscosity as the source for energy-momentum tensor. The chapter is presented as follows. In section 3.2, we presented field equations by higher dimensional FRW metric in Lyra geometry. In Section 3.3, we solve the field equations with some acceptable physical assumptions of scale factor for flat, open, and closed models in three different cases. Kinematic and physical interpretations are given in Section 3.4. Section 3.5 has a concluding remark.

3.2 Metric and field equations

We consider Robertson-Walker metric in five-dimensional spacetime in the form

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] - S^2(t)d\psi^2, \tag{3.2.1}$$

where $R(t)$ is the scale factor of the universe, $k = 1, 0, -1$ for space of positive, vanishing and negative curvature representing closed, flat and open models of the universe respectively. The fifth co-ordinate ψ is also assumed to be space like coordinate.

The Einstein field equations based on Lyra's geometry in normal gauge is given by (Sen, 1957) as

$$R_{ij} - \frac{1}{2}Rg_{ij} + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -T_{ij}, \tag{3.2.2}$$

$$G_{ij} + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -T_{ij}, \tag{3.2.3}$$

with $\frac{8\pi G}{c^4} = 1$. The first two terms of (3.2.2) are Einstein tensor G_{ij}, ϕ_i is the displacement vector and other symbols have their usual meaning as in Riemannian geometry. The time-like displacement vector ϕ_i in (3.2.1) is given by

$$\phi_i = (0, 0, 0, 0, \beta(t)). \tag{3.2.4}$$

The non-vanishing components of the left hand side of (3.2.2) and (3.2.3) for the metric (3.2.1) are given by

$$G_0^0 = \frac{3\dot{R}^2}{R^2} + \frac{3\dot{R}\dot{S}}{RS} + \frac{3k}{R^2}, \tag{3.2.5}$$

$$G_1^1 = G_2^2 = G_3^3 = \frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{2\dot{R}\dot{S}}{RS} + \frac{\ddot{S}}{S} + \frac{k}{R^2}, \tag{3.2.6}$$

$$G_4^4 = \frac{3\ddot{R}}{R} + \frac{3\dot{R}^2}{R^2} + \frac{3k}{R^2}, \tag{3.2.7}$$

where an overhead dot indicates ordinary differentiation with respect to t.

The energy-momentum tensor T_{ij} for cloud of massive strings with bulk viscosity is given by (Landau and Lifshitz, 1959),

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta(u_i u_j - g_{ij}), \quad i,j = 0,1,2,3,4. \tag{3.2.8}$$

Here, ρ is the rest energy density of the cloud of strings with particles attached to them, λ is the string tension density of the strings and ξ is the co-efficient of bulk coefficient. If the particle density of the configuration is denoted by ρ_p, then we have

$$\rho = \rho_p + \lambda. \tag{3.2.9}$$

The velocity u^i describes the five-velocity, which has components (1,0,0,0,0) for a cloud of particles and x^i represents the direction of string that satisfies the condition

$$u^i u_i = x^i x_j = -1, \quad u^i x_i = 0. \tag{3.2.10}$$

So that we have

$$\begin{aligned}
&T_0^0 = \rho,\\
&T_1^1 = T_2^2 = T_3^3 = \xi\theta,\\
&T_4^4 = \xi\theta + \lambda,\\
&T = T_0^0 + T_1^1 + T_2^2 + T_3^3 + T_4^4 = \rho + 2\xi\theta + \lambda.
\end{aligned} \tag{3.2.11}$$

Using co-moving co-ordinates, the field equations based on Lyra geometry (3.2.2) and (3.2.3) together with (3.2.4) – (3.2.10) for the metric (3.2.1) can be obtained as

$$\frac{3\dot{R}^2}{R^2} + \frac{3\dot{R}\dot{S}}{RS} + \frac{3k}{R^2} - \frac{3}{4}\beta^2 = \rho, \tag{3.2.12}$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{2\dot{R}\dot{S}}{RS} + \frac{\ddot{S}}{S} + \frac{k}{R^2} + \frac{3}{4}\beta^2 = \xi\theta, \tag{3.2.13}$$

$$\frac{3\ddot{R}}{R} + \frac{3\dot{R}^2}{R^2} + \frac{3k}{R^2} + \frac{3}{4}\beta^2 = \xi\theta + \lambda. \tag{3.2.14}$$

The energy conservation equation $T^{j}_{i;j}$ leads to

$$\dot{\rho} + \left(3\frac{\dot{R}}{R} + \frac{\dot{S}}{S}\right)\rho - \lambda\frac{\dot{S}}{S} - \xi(n+3)^2\frac{\dot{R}^2}{R^2} = 0, \tag{3.2.15}$$

and

$$\left(R^j_i - \frac{1}{2}Rg^j_i\right)_{;j} + \frac{3}{2}\left(\phi_i\phi^j\right)_{;j} - \frac{3}{4}\left(g^j_i\phi_k\phi^k\right)_{;j} = 0, \tag{3.2.16}$$

which leads to the following equation

$$\frac{3}{2}\beta\dot{\beta} + \frac{3}{2}\beta^2\left(\frac{3\dot{R}}{R} + \frac{\dot{S}}{S}\right) = 0. \tag{3.2.17}$$

3.3 Solution of the field equations

The field equations (3.2.11), (3.2.12) and (3.2.13) are a system of three independent equations having six unknowns R, S, β, ρ, λ and ξ. To get the determinate solution, let the deceleration parameter to be a constant (Berman, 1983), i.e.

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -\frac{\dot{H} + H^2}{H^2} = b \text{ (constant)}. \tag{3.3.1}$$

The above equation may be rewritten as

$$\frac{\ddot{R}}{R} + b\frac{\dot{R}^2}{R^2} = 0. \tag{3.3.2}$$

On integrating (3.2.15), we get the exact solutions as

$$\begin{aligned} R(t) &= (Ct + D)^{\frac{1}{1+b}} \qquad b \neq -1, \\ \text{or } R(t) &= R_0 e^{H_0 t} \qquad b = -1, \end{aligned} \tag{3.3.3}$$

where C, D, R_0 and H_0 are constants of integration.

We consider a power law equation because of the existence of anisotropy for the flat and homogeneous universe and $\theta \propto \sigma_{ij}$ (shear tensor). Hence we use the following polynomial relation between the metric co-efficient.

$$S = R^n \tag{3.3.4}$$

where n is an arbitrary constant.

Therefore, from (3.3.2) and (3.3.3), we get

$$\begin{aligned} S(t) &= (Ct + D)^{\frac{n}{1+b}} \qquad b \neq -1, \\ or\ S(t) &= R_0 e^{nH_0 t} \qquad b = -1. \end{aligned} \tag{3.3.5}$$

Case I: $b \neq -1$ The metric in (3.2.1) for FRW model takes the form

$$ds^2 = dt^2 - (Ct + D)^{\frac{2}{1+b}} \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] - (Ct + D)^{\frac{2n}{1+b}} d\psi^2. \tag{3.3.6}$$

The displacement vector β is obtained as

$$\beta = c_1 (Ct + D)^{-\frac{3+n}{1+b}}. \tag{3.3.7}$$

The expansion scalar θ is obtained as

$$\theta = \frac{C(n+3)}{(1+b)(D+Ct)}. \tag{3.3.8}$$

From (3.2.12), we get

$$\rho = \frac{3c^2(1+n)}{(1+b)^2(D+Ct)^2} + \frac{1}{4}(D+Ct)^{-\frac{2(3+n)}{1+b}} \left[12k(D+Ct)^{\frac{2(2+n)}{1+b}} - 3c_1^2\right]. \tag{3.3.9}$$

From (3.2.13), we get

$$\begin{aligned} \xi\theta &= \frac{4C^2 - 8bC^2 + 8nC^2 - 4nC^2(1+b-n) + 4(1+b)^2 k(D+Ct)^{\frac{2b}{1+b}} + 3(1+b)^2(D+Ct)^{2-\frac{2(3+n)}{1+b}} c_1^2}{4(1+b)^2(D+Ct)^2} \\ \xi &= \frac{4C^2 - 8bC^2 + 8nC^2 - 4nC^2(1+b-n) + 4(1+b)^2 k(D+Ct)^{\frac{2b}{1+b}} + 3(1+b)^2(D+Ct)^{2-\frac{2(3+n)}{1+b}} c_1^2}{4C(n+3)(1+b)(D+Ct)}. \end{aligned} \tag{3.3.10}$$

From (3.2.14), we get

$$\lambda = \frac{4C^2 - 10bC^2 + 4nC^2 - 2nC^2(1+b-n) + 8(1+b)^2k(D+Ct)^{\frac{2b}{1+b}} + 3(1+b)^2(D+Ct)^{2-\frac{2(3+n)}{1+b}}c_1^2}{2(1+b)^2(D+Ct)^2}. \tag{3.3.11}$$

From (3.3.8) and (3.3.10) together with (3.2.9), we get

$$\rho_p = \frac{4C^2 - 20bC^2 + 20nC^2 - 4nC^2(1+b-n) + 28(1+b)^2k(D+Ct)^{\frac{2b}{1+b}} + 3(1+b)^2(D+Ct)^{2-\frac{2(3+n)}{1+b}}c_1^2}{4(1+b)^2(D+Ct)^2}. \tag{3.3.12}$$

For $k = 0$, flat model:

In this particular case, the model becomes

$$ds^2 = dt^2 - (Ct+D)^{\frac{2}{1+b}}[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)] - (Ct+D)^{\frac{2n}{1+b}}d\psi^2 \tag{3.3.13}$$

The energy density for this model is given by

$$\rho = \frac{3c^2(1+n)}{(1+b)^2(D+Ct)^2} - c_1^2\frac{3}{4}(D+Ct)^{-\frac{2(3+n)}{1+b}}. \tag{3.3.14}$$

The bulk viscosity is

$$\xi = \frac{4C^2 - 8bC^2 + 8nC^2 - 4nC^2(1+b-n) + 3(1+b)^2(D+Ct)^{2-\frac{2(3+n)}{1+b}}c_1^2}{4C(n+3)(1+b)(D+Ct)}. \tag{3.3.15}$$

The tension density is given by

$$\lambda = \frac{4C^2 - 10bC^2 + 4nC^2 - 2nC^2(1+b-n) + 3(1+b)^2(D+Ct)^{2-\frac{2(3+n)}{1+b}}c_1^2}{2(1+b)^2(D+Ct)^2}. \tag{3.3.16}$$

The particle density is

$$\rho_p = \frac{4C^2 - 20bC^2 + 20nC^2 - 4nC^2(1+b-n) + 3(1+b)^2(D+Ct)^{2-\frac{2(3+n)}{1+b}}c_1^2}{4(1+b)^2(D+Ct)^2}. \tag{3.3.17}$$

For $k = 1$, closed model:

The metric for FRW model takes the form

$$ds^2 = dt^2 - (Ct + D)^{\frac{2}{1+b}} \left[\frac{dr^2}{1 - r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right] - (Ct + D)^{\frac{2n}{1+b}} d\psi^2. \quad (3.3.18)$$

The energy density for this model is given by

$$\rho = \frac{3c^2(1+n)}{(1+b)^2(D+Ct)^2} + \frac{1}{4}(D+Ct)^{-\frac{2(3+n)}{1+b}} \left[12(D+Ct)^{\frac{2(2+n)}{1+b}} - 3c_1^2 \right]. \quad (3.3.19)$$

The bulk viscosity is

$$\xi = \frac{4C^2 - 8bC^2 + 8nC^2 - 4nC^2(1+b-n) + 4(1+b)^2(D+Ct)^{\frac{2b}{1+b}} + 3(1+b)^2(D+Ct)^{2-\frac{2(3+n)}{1+b}} c_1^2}{4C(n+3)(1+b)(D+Ct)}. \quad (3.3.20)$$

The tension density is given by

$$\lambda = \frac{4C^2 - 10bC^2 + 4nC^2 - 2nC^2(1+b-n) + 8(1+b)^2(D+Ct)^{\frac{2b}{1+b}} + 3(1+b)^2(D+Ct)^{2-\frac{2(3+n)}{1+b}} c_1^2}{2(1+b)^2(D+Ct)^2}. \quad (3.3.21)$$

The particle density is

$$\rho_p = \frac{4C^2 - 20bC^2 + 20nC^2 - 4nC^2(1+b-n) + 28(1+b)^2(D+Ct)^{\frac{2b}{1+b}} + 3(1+b)^2(D+Ct)^{2-\frac{2(3+n)}{1+b}} c_1^2}{4(1+b)^2(D+Ct)^2}. \quad (3.3.22)$$

For $k = -1$, open model:

The metric for FRW model takes the form of

$$ds^2 = dt^2 - (Ct + D)^{\frac{2}{1+b}} \left[\frac{dr^2}{1 + r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right] - (Ct + D)^{\frac{2n}{1+b}} d\psi^2. \quad (3.3.23)$$

The energy density for this model is given by

$$\rho = \frac{3c^2(1+n)}{(1+b)^2(D+Ct)^2} + \frac{1}{4}(D+Ct)^{-\frac{2(3+n)}{1+b}} \left[-12(D+Ct)^{\frac{2(2+n)}{1+b}} - 3c_1^2 \right]. \quad (3.3.24)$$

The bulk viscosity is

$$\xi = \frac{4C^2 - 8bC^2 + 8nC^2 - 4nC^2(1+b-n) - 4(1+b)^2(D+Ct)^{\frac{2b}{1+b}} + 3(1+b)^2(D+Ct)^{2-\frac{2(3+n)}{1+b}}c_1^2}{4C(n+3)(1+b)(D+Ct)}. \tag{3.3.25}$$

The tension density is given by

$$\lambda = \frac{4C^2 - 10bC^2 + 4nC^2 - 2nC^2(1+b-n) - 8(1+b)^2(D+Ct)^{\frac{2b}{1+b}} + 3(1+b)^2(D+Ct)^{2-\frac{2(3+n)}{1+b}}c_1^2}{2(1+b)^2(D+Ct)^2}. \tag{3.3.26}$$

The particle density is

$$\rho_p = \frac{4C^2 - 20bC^2 + 20nC^2 - 4nC^2(1+b-n) - 28(1+b)^2(D+Ct)^{\frac{2b}{1+b}} + 3(1+b)^2(D+Ct)^{2-\frac{2(3+n)}{1+b}}c_1^2}{4(1+b)^2(D+Ct)^2}. \tag{3.3.27}$$

Case II: $b = -1$

$$ds^2 = dt^2 - R_0^2 e^{2H_0 t}\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] - R_0^{2n} e^{2nH_0 t} d\psi^2. \tag{3.3.28}$$

The displacement vector β is obtained as

$$\beta = c_0 e^{-H_0(3+n)t}. \tag{3.3.29}$$

The expansion scalar θ is

$$\theta = (n+3)H_0. \tag{3.3.30}$$

From (3.2.12), we get

$$\rho = 3H_0^2(n+1) + \frac{3k}{R_0^2 e^{2H_0 t}} - \frac{3}{4}e^{-2H_0(n+3)t}c_0^2. \tag{3.3.31}$$

From (3.2.13), we get

$$\xi\theta = \frac{3}{4}c_0^2 e^{-2(3+n)H_0 t} + \frac{k}{R_0^2 e^{2H_0 t}} + (3+2n+n^2)H_0^2$$
$$\xi = \frac{\frac{3}{4}c_0^2 e^{-2(3+n)H_0 t} + \frac{k}{R_0^2 e^{2H_0 t}} + (3+2n+n^2)H_0^2}{(n+3)H_0}. \tag{3.3.32}$$

From (3.2.14), we get

$$\lambda = \frac{4k}{R_0^2 e^{2H_0 t}} + (9+2n+n^2)H_0^2. \tag{3.3.33}$$

The particle density is

$$\rho_p = -\frac{3}{4}e^{-2H_0(n+3)t}c_0^2 + 3H_0^2(5n-6) + \frac{7k}{R_0^2 e^{2H_0 t}}. \tag{3.3.34}$$

For $k = 0$, flat model

$$ds^2 = dt^2 - R_0^2 e^{2H_0 t}\left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] - R_0^{2n} e^{2nH_0 t} d\psi^2. \tag{3.3.35}$$

The expansion scalar θ is

$$\theta = (n+3)H_0. \tag{3.3.36}$$

The energy density ρ is

$$\rho = 3H_0^2(n+1) - \frac{3}{4}e^{-2H_0(n+3)t}c_0^2. \tag{3.3.37}$$

The bulk viscosity is

$$\xi = \frac{\frac{3}{4}c_0^2 e^{-2(3+n)H_0 t} + (3+2n+n^2)H_0^2}{(n+3)H_0}. \tag{3.3.38}$$

The string tension density λ is

$$\lambda = (9+2n+n^2)H_0^2. \tag{3.3.39}$$

The particle density ρ_p is

$$\rho_p = -\frac{3}{4}e^{-2H_0(n+3)t}c_0^2 + 3H_0^2(5n-6). \tag{3.3.40}$$

For $k = 1$, closed model

$$ds^2 = dt^2 - R_0^2 e^{2H_0 t}\left[\frac{dr^2}{1-r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] - R_0^{2n} e^{2nH_0 t} d\psi^2. \tag{3.3.41}$$

The energy density ρ is

$$\rho = 3H_0^2(n+1) + \frac{3}{R_0^2 e^{2H_0 t}} - \frac{3}{4}e^{-2H_0(n+3)t}c_0^2. \tag{3.3.42}$$

The bulk viscosity is

$$\xi = \frac{\frac{3}{4}c_0^2 e^{-2(3+n)H_0 t} + \frac{1}{R_0^2 e^{2H_0 t}} + (3+2n+n^2)H_0^2}{(n+3)H_0}. \tag{3.3.43}$$

The string tension density λ is

$$\lambda = \frac{4}{R_0^2 e^{2H_0 t}} + (9+2n+n^2)H_0^2. \tag{3.3.44}$$

The particle density ρ_p is

$$\rho_p = -\frac{3}{4}e^{-2H_0(n+3)t}c_0^2 + 3H_0^2(5n-6) + \frac{7}{R_0^2 e^{2H_0 t}}. \tag{3.3.45}$$

For $k = -1$, open model

$$ds^2 = dt^2 - R_0^2 e^{2H_0 t}\left[\frac{dr^2}{1+r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] - R_0^{2n} e^{2nH_0 t} d\psi^2. \tag{3.3.46}$$

The energy density ρ is

$$\rho = 3H_0^2(n+1) - \frac{3}{R_0^2 e^{2H_0 t}} - \frac{3}{4}e^{-2H_0(n+3)t}c_0^2. \tag{3.3.47}$$

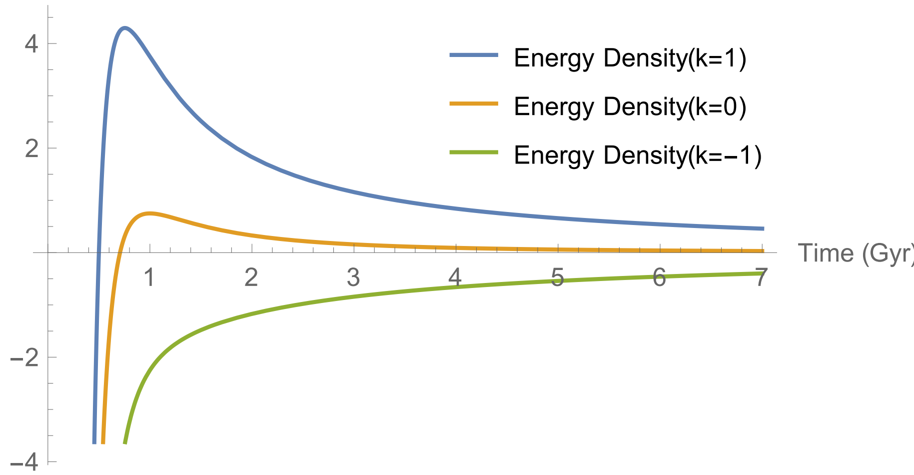

Figure 3.1: Variation of ρ with t in Gyr for $b \neq -1$

The bulk viscosity is

$$\xi = \frac{\frac{3}{4}c_0^2 e^{-2(3+n)H_0 t} - \frac{1}{R_0^2 e^{2H_0 t}} + (3 + 2n + n^2)H_0^2}{(n+3)H_0}. \tag{3.3.48}$$

The tension density λ is

$$\lambda = -\frac{4}{R_0^2 e^{2H_0 t}} + (9 + 2n + n^2)H_0^2. \tag{3.3.49}$$

The particle density is

$$\rho_p = -\frac{3}{4}e^{-2H_0(n+3)t}c_0^2 + 3H_0^2(5n - 6) - \frac{7}{R_0^2 e^{2H_0 t}}. \tag{3.3.50}$$

Observations on the role of bulk viscocity in present scenarios of the evolution in FRW model.

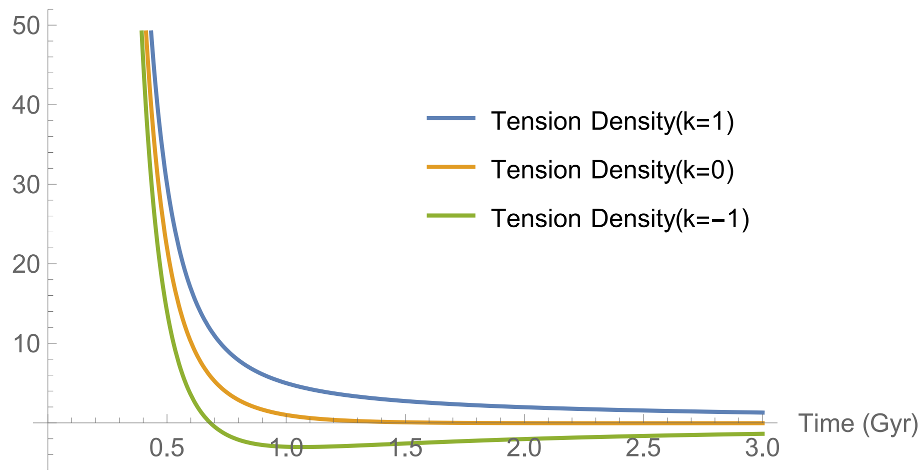

Figure 3.2: Variation of λ with t in Gyr for $b \neq -1$

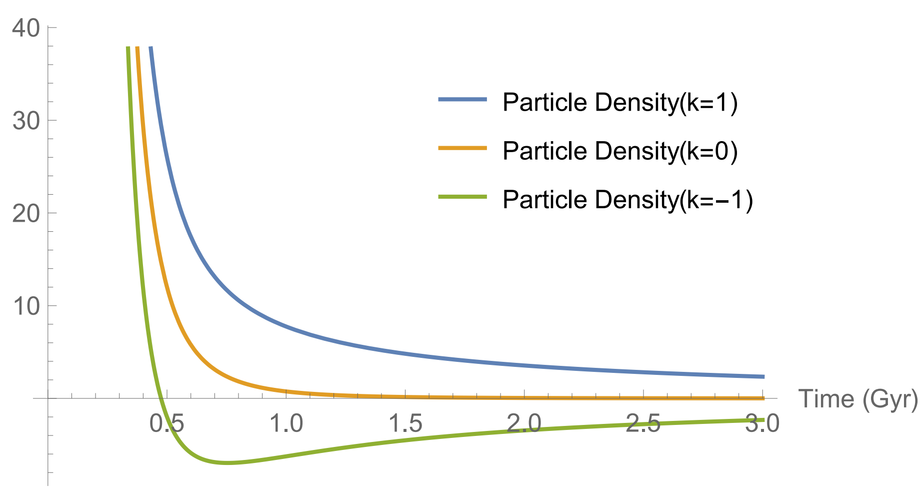

Figure 3.3: Variation of ρ_p, with t in Gyr for $b \neq -1$

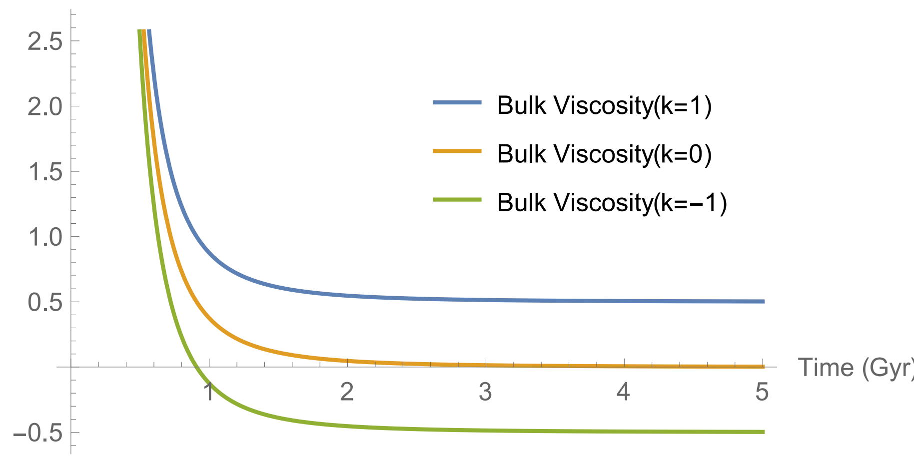

Figure 3.4: Variation of ξ with t in Gyr for $b \neq -1$

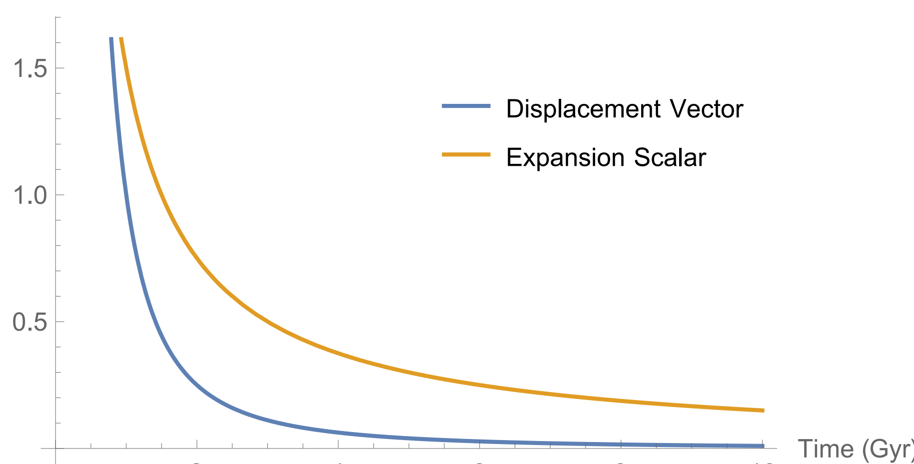

Figure 3.5: Variation of β, θ with t in Gyr for $b \neq -1$

3.4 Discussion

Fig. 3.1 depicts the behaviour of energy density versus time. We observed that ρ changes sign from negative to positive value after some finite time and reaches large value and decreases with the time approaching a small positive value at late times for closed model ($k = 1$). For a flat model ($k = 0$), the energy density is negative at the initial epoch and maintains a small positive value at an early stage, and is almost coincident with zero at late times. For the open model ($k = -1$), energy density is a negative increasing function of time converging to a small negative value at late times. The string tension density and particle density versus time for all flat, closed, and open models have been plotted in Fig 3.2 and Fig 3.3. Both particle and string tension density are always positive in the closed model ($k = 1$) whereas, they decrease more sharply with the cosmic time and approaches zero in the flat model ($k = 0$). However, in the open model ($k = -1$), ρ_p and λ quickly passes through zero and approaches negative values. It is observed that the string tension density disappears more rapidly than particle density leaving particles only indicating the matter-dominated universe at late times as anticipated. The variation of bulk viscosity with time is shown in Fig. 3.4. In the flat model ($k = 0$), the bulk viscosity decreases with time leading to an inflationary model and vanishes for infinitely large time t. In the closed model ($k = 1$), ξ decreases, remain positive throughout the evolution whereas, for the open model ($k = -1$), it reaches negative values. The function of the bulk viscosity is to retard the expansion of the universe and since bulk viscosity ξ decreases with time, retardedness also decreases which supports the expansion at a faster rate in the late stages of the evolution of the universe. The displacement vector β and the expansion scalar θ have been found out for all three flat, open, and closed models and are plotted in Fig. 3.5. We noticed that β and θ decreases with the increase in the age of the universe. At the initial epoch of time, the gauge function β^2 is found to be infinite and ultimately $\beta^2 \to 0$ when $t \to \infty$.

3.5 Conclusion

In this chapter, we investigated the role of bulk viscous fluid attached to the string cloud by considering a time-dependent deceleration parameter in present scenarios of the evolution in FRW model universe in the context of Lyra Geometry. Here, we restricted our study to a constant deceleration parameter as predicted from observation. The solutions of the model have been obtained for flat, closed, and open bulk viscous string FRW universe in five dimensions. The physical parameters have been plotted for $b \neq -1$. However, in the case of $b = -1$, all the parameters vanish rapidly within a short period of time. This fact indicates that the solution represents an early era of the evolution of the universe. The incorporation of bulk viscosity in our investigation is to replace the condition of material distribution other than perfect fluid. The bulk viscosity plays a significant role in the present scenario of the evolution of the universe. So, our model will contribute to a better understanding of spatially homogeneous and isotropic accelerating universe (Bamba, 2012) in five-dimensions.