Chapter 6

Higher Dimensional FRW model universe in scalar tensor theory of gravitation using Quadratic equation of state

6.1 Introduction

Today a challenging problem is the unification of gravity with other fundamental forces in nature. In this time, the spacetime dimensions to be more than four is the most recent effort. The detection of extra dimensions in current research interest. In recent years, multi-dimensional cosmological models have been studied by many researchers. Kaluza-Klein cosmological models in generalizes scalar tensor theory and Lyra geometry have been studied by Chakraborty and Ghosh (2001) and Rahaman and Bera (2001) respectively. Multi-dimensional cosmological models in general relativity have studied by Lorentz and Pitzold (1985), Ibanez et al. (1986), Khadekar and Gaikward (2001). five-dimensional FRW cosmological model in scalar-tensor theory of gravitation have studied by Rao et al. (2015).

The recent prediction of early inflation and the late time accelerated expansions of the universe (Reiss et. al., 1998; Perlmutter et. al., 1999) which is not explained by general theory of relativity, is interesting. This unexpected discovery of the accelerated expansion of the universe has opened the most puzzling and deepest problem in cosmology today. Hence many researchers have proposed several modifications in general relativity. The modified theory of gravity formulated by Brans-Dicke (1961), and Saez and Ballester (1986), $f(R)$ theory of gravity formulated by Nojiri and Odinstov (2003) and $f(R,T)$ theory of gravity proposed by Harko et. al. (2011) have extended General Relativity. In this scenario a scalar-theory of gravity allows the first order phase transition of the "old" inflationary model to complete, (Guth, 1981). Brans-Dicke theory (1961) is the simplest of the scalar-tensor theory in which the gravitational interaction is modified by the scalar filed ϕ as well as the tensor field g_{ij} of the Einstein's theory. In this theory, the scalar field ϕ has the dimension of the universal constant.

Subsequently, a scalar-tensor theory in which the metric is coupled with a dimensionless scalar field ϕ is developed by Saez and Ballester (1986). In this theory an anti-gravity regime appears in spite of the dimensionless character of the scalar field. In non-flat FRW cosmologies, this theory gives satisfactory description of the weak fields and suggest a possible way to solve 'missing matter' problem.

In this chapter, we have investigated five-dimensional FRW cosmological model with a quadratic Equation of state (EoS) in a scalar-tensor theory of gravitation proposed by Saez and Ballester.

6.2 Metric and field equations

We consider the five-dimensional $(t, r, \theta, \phi, \psi)$ FRW in the form

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1-kr^2)k\psi^2\right], \qquad (6.2.1)$$

where $R(t)$ is the scale factor and $k = 0, +1, -1$ is the curvature parameter for flat, open and closed universe respectively. The fifth coordinate ϕ is also assumed to spacelike.

The field equations given by Saez and Ballester for the scalar and tensor fields are

$$R_{ij} - \frac{1}{2}g_{ij}R - w\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}\phi_{,k}\phi^{,k}\right) = -8\pi T_{ij}, \tag{6.2.2}$$

and the scalar field ϕ satisfies the equation

$$2\phi^n\phi^{,i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0. \tag{6.2.3}$$

Also, we have

$$T^{ij}_{;j} = 0, \tag{6.2.4}$$

which is the consequence of the field (6.2.1) and (6.2.2). The non-vanishing components of the Einstein tensor for the metric (6.2.1) are

$$\begin{aligned} G^0_0 &= 6\frac{\dot{R}^2}{R^2} + \frac{6k}{R^2}, \\ G^1_1 = G^2_2 = G^3_3 = G^4_4 &= \frac{3\ddot{R}}{R} + \frac{3\dot{R}^2}{R^2} + \frac{3k}{R^2}. \end{aligned} \tag{6.2.5}$$

where an over-dot indicates ordinary differentiation with respect to t and $k = 0, +1, -1$ for flat, closed and open model respectively.

$$T^i{}_j = (\rho + p)u^i u_j - p g^i{}_j; \quad i, j = 0, 1, 2, 3, 4. \tag{6.2.6}$$

Here

$$u^i u_i = 1 \quad \text{and} \quad u^i u_j = 0. \tag{6.2.7}$$

So,

$$T_0^0 = \rho,$$
$$T_1^1 = T_2^2 = T_3^3 = T_4^4 = -p \quad \text{and}$$
$$T = T_0^0 + T_1^1 + T_2^2 + T_3^3 + T_4^4 = \rho - 4p. \tag{6.2.8}$$

Using the co-moving coordinates, the field equations (6.2.2) - (6.2.4) with the help of (6.2.5) - (6.2.8) for the metric (6.2.1) can be written as

$$\frac{6\dot{R}^2}{R^2} + \frac{6k}{R^2} - \frac{w}{2}\frac{\dot{\phi}^2}{\phi^2} = -8\pi\rho, \tag{6.2.9}$$
$$\frac{\ddot{R}}{R} + \frac{3\dot{R}^2}{R^2} + \frac{3k}{R^2} + \frac{w}{2}\frac{\dot{\phi}^2}{\phi^2} = 8\pi p, \tag{6.2.10}$$
$$\frac{\ddot{\phi}}{\phi} + \frac{4\dot{R}}{R}\frac{\dot{\phi}}{\phi} + \frac{n}{2}\frac{\dot{\phi}^2}{\phi^2} = 0, \tag{6.2.11}$$
$$\dot{\rho} + \frac{4\dot{R}}{R}(\rho + p) = 0. \tag{6.2.12}$$

6.3 Solutions and the models

6.3.1 Scale factor: de Sitter universe, $R = e^{\alpha t}$, α is constant

Taking de Sitter universe

$$R = e^{\alpha t}, \tag{6.3.1}$$

where α is constant as scale factor. The quadratic equation of state is written as,

$$p = \alpha\rho^2 - \rho, \tag{6.3.2}$$

where ρ, p and α is enenrgy density, pressure and constant. Using (6.3.1) and (6.3.2) in (6.2.12), we get

$$\rho = \frac{1}{4\alpha^2 t + C}, \tag{6.3.3}$$

where C is constant of integration. Putting (6.3.3) in (6.3.2), we get

$$p = \frac{K_1 - 4\alpha^2 t}{(4\alpha^2 t + C)^2}, \tag{6.3.4}$$

where K_1 is the constant. From (6.2.9), we get

$$\phi = \frac{\Gamma(C + 4\alpha^2 t)}{2\sqrt{2}\alpha^2} + \frac{\pi \log[2\pi + 3e^{-2\alpha t}(C + 4\alpha^2 t) + 3\alpha^2(C + 4\alpha^2 t) + \sqrt{3}Q\Gamma(C + 4\alpha^2 t)]}{\sqrt{6}Q\alpha^2}, \tag{6.3.5}$$

where

$$Q = \sqrt{e^{-2\alpha t}k + \alpha^2}, \quad \Gamma = \sqrt{\frac{4\pi + 3e^{-2\alpha t}k(C + 4\alpha^2 t) + 3\alpha^2(C + 4\alpha^2 t)}{(C + 4\alpha^2 t)}}.$$

Thus, the directional Hubble parameters are given by

$$H_r = H_\theta = H_\phi = H_\psi = \alpha. \tag{6.3.6}$$

The mean Hubble parameter is given by

$$H = \alpha. \tag{6.3.7}$$

The anisotropy parameter is

$$\Delta = \frac{(\alpha - e^{\alpha t})}{\alpha^2}. \tag{6.3.8}$$

The shear scalar

$$\sigma^2 = \frac{3}{2}\left(\alpha - e^{\alpha t}\right)^2. \tag{6.3.9}$$

The spatial volume

$$V = e^{4\alpha t}. \tag{6.3.10}$$

The scalar of expansion is given by

$$\theta = 4\alpha. \tag{6.3.11}$$

Also, the deceleration paramater has the form

$$q = -1. \tag{6.3.12}$$

The energy density parameter is

$$\Omega = \frac{4\alpha^2 t + C}{4\alpha^2}. \tag{6.3.13}$$

6.4 Physical Interpretations of the solutions

We have constructed higher dimensional FRW model universe in scalar tensor theory of gravitation using quadratic equation of state. Taking $\alpha = 1,\ C = 1$, the variation of the parameters of the model are shown by the Figures. The physical and geometrical behavior of model can be discussed as follows:

From the expression (6.3.10) of spatial volume V, we observed that it is an increasing function of cosmic time t increasing exponentially and evolves with small finite volume at $t = 0$ and it becomes infinite as $t \to \infty$. Fig. 6.1 depicts this behaviour of volume V.

The expression for energy density ρ given by (6.3.3) shows that the energy

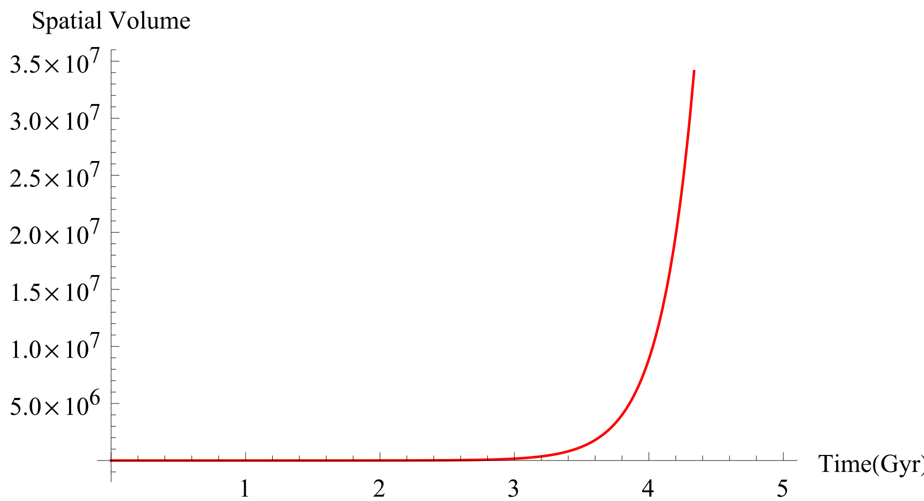

Figure 6.1: Variation of Volume V vs. time t.

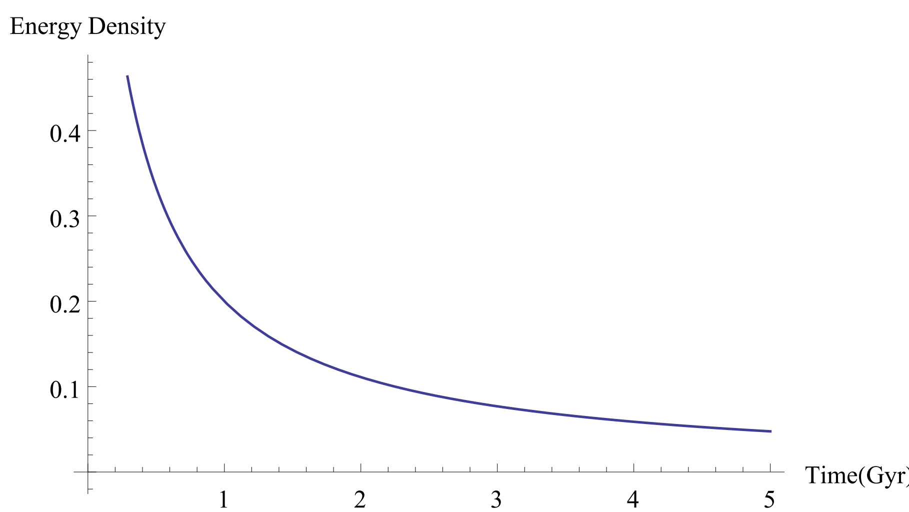

Figure 6.2: Variation of energy density ρ vs. time t.

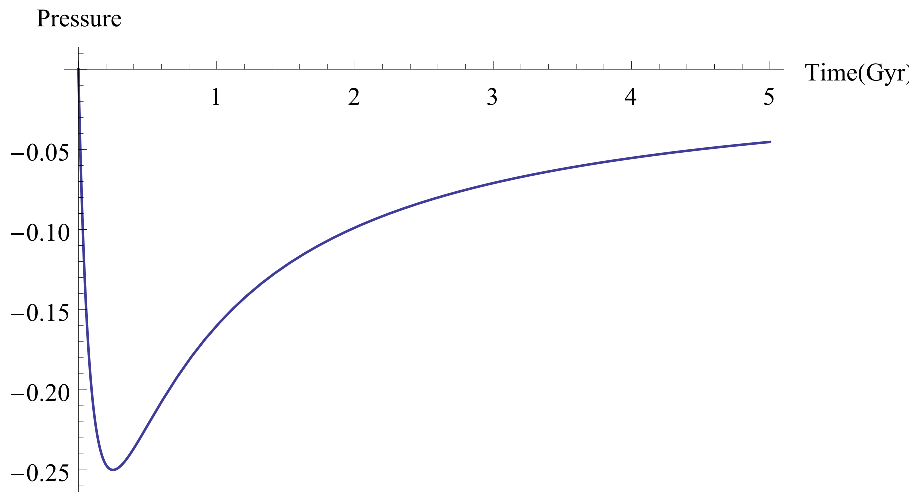

Figure 6.3: Variation of pressure p vs. time t.

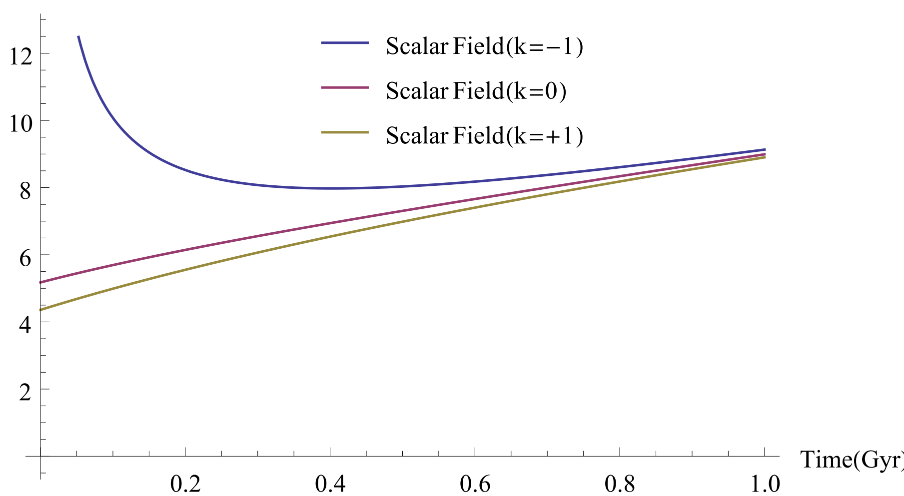

Figure 6.4: Variation of Scalar field vs. time t.

density ρ is a decreasing function of cosmic time t and satisfy the energy condition $\rho \geq 0$ for all $C \geq 0$. The variation of energy density against cosmic time is presented in Fig. 6.2, which also shows that it is a decreasing function of time t.

From the expression for pressure p given by (6.3.4) and its graphical recantation it is seen that the pressure is negative throughout the evolution for $\alpha \leq C$.

The expression in (6.3.5) shows that the Scalar field ϕ and the Fig. 6.4 presented the variation of scalar field verses time for flat ($k = 0$), open ($k = +1$) and closed ($k = -1$) universe respectively. For flat ($k = 0$) and open ($k = +1$) universe, the scalar field ϕ increases with increases of time. For closed ($k = -1$) universe the Scalar field ϕ decreases with increases of time. The variation of Scalar field vs. time t for flat ($k = 0$), open ($k = +1$) and closed ($k = -1$) universe are shown in the Fig. 6.4.

The expression in (6.3.12) shows that the deceleration parameter q is a negative constant equal to -1 (de Sitter universe). It means that this model is found to be exponential expanding with time. And since $H > 0, q < 0$ for $\alpha > 0$, our model universe obtained here shows the expanding and accelerating universe.

From (6.3.11) and (6.3.7) both the expansion scalar θ and the Hubble's parameter H are finite positive constant for $\alpha > 0$.

The expression of Ω in (6.3.13)and its graphical recantation in Fig. 6.5 it is seen that Ω is increasing function of time. at the initial point has a small finite value and then increases with increases of time and finally attains infinite value at $t \to \infty$.

Eqn. (6.3.9) and its graphical presentations shown in Fig. 6.6 give us an idea about shear scalar σ which is non zero, explaining a shearing model universe at infinite time.

Since $\frac{\sigma^2}{\theta^2}$ = constant for $t \to \infty$, so the model does not approach isotropy for large value of time t (Asgar and Ansari, 2014). The anisotropy parameter $\Delta \neq 0$ for $t \to \infty$. Thus the model universe is anisotropic throughout its evolution.

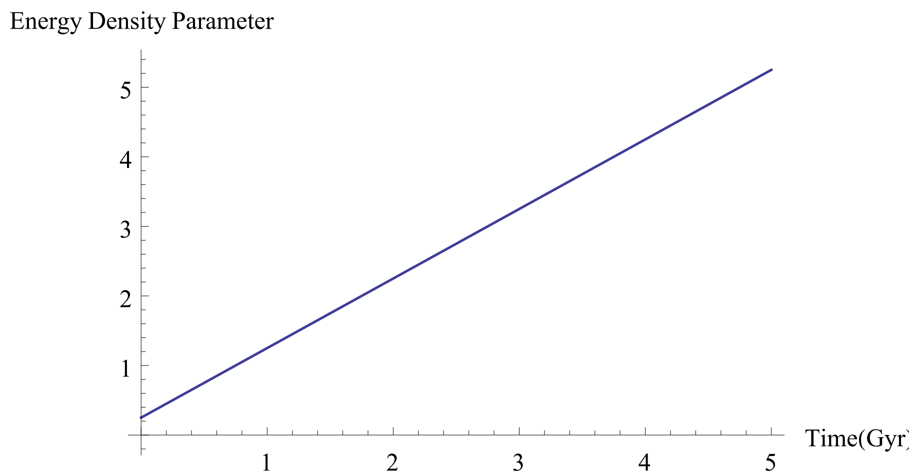

Figure 6.5: Variation of Ω vs. time t.

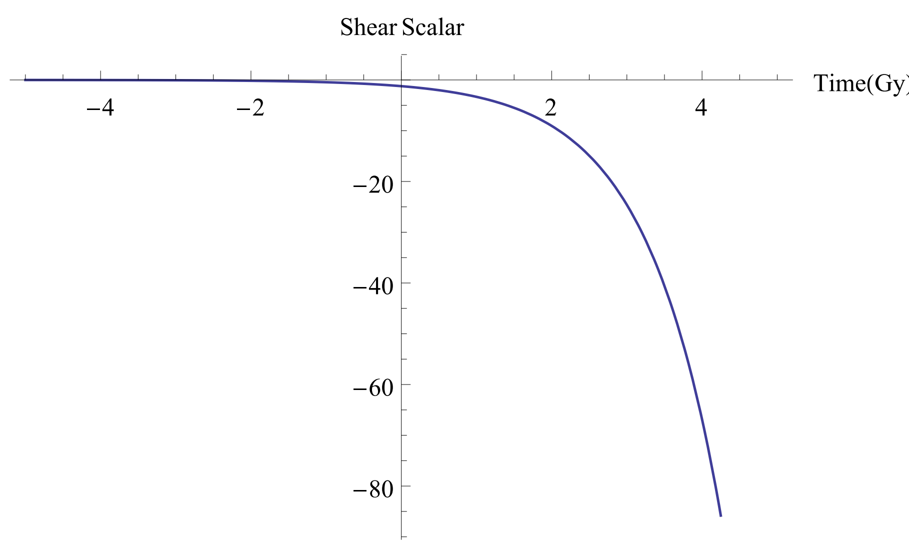

Figure 6.6: Variation of σ vs. time t.

6.5 Conclusion

In this chapter, we attempt to explain the behaviour some of unknown phenomenon of the universe. five-dimensional FRW model universe in scalar tensor theory of gravitation using quadratic equation of state is studied with the use of certain physical assumptions, which are agreeing with the present observational findings. The field equations for five-dimensional FRW model universe in scalar tensor theory of gravitation have been obtained and exact solutions are obtained. The model represents to have anisotropic phase throughout the evolution of the universe which is in agreement with the present observational data made by COBE (Cosmic Background Explorer) and WMAP (The Wilkinson Microwave Anisotropy Probe). Also, the model represents an expanding universe that starts with small finite volume at cosmic time $t = 0$ and expands with acceleration. Our model satisfies the energy conditions $\rho \geq 0$. Also, the shear scalar become non zero as $t \to \infty$. So, our model represents a shearing cosmological model universe for large values of cosmic time t.