Chapter 7

Study on the nature of variable Gravitational and Cosmological constant in higher dimensional FRW model universe

7.1 Introduction

During last few decades, a great change seems to have taken place in cosmology. The keen interest for higher dimensional spacetime for the study of universe among the cosmologists and scientists begin with the latest development of super-string theory and gravitational theory. A number of authors (Sahadev (1984), Emelyanov et al. (1986), Chatterjee et al. (1990) and (1993) have investigated the physics of universe in higher-dimensional spacetime. Overdium and Wesson (1987) have introduced an excellent review of higher-dimensional unified theories, in which the cosmological and astrophysical implifications of extra-dimension have been discussed. In the Einstein's field equations there are two parameters, the cosmological constant and gravitational constant G. The Newtonian constant of gravitation, G, plays the role of a coupling constant between the geometry and the matter of the

Einstein field equations. Ozar and Taha (1986) and (1987) have proposed a model in which the cosmological constant is time dependent and the cosmic density ρ_c equals the Einstein-de Sitter critical density ρ_c . In a separate development, independent of critical density ρ_c assumption, Chen and Wu (1990) suggested that is proportional to R^{-2}. They have shown that such a behaviour is deducible from the simple general principle in the line with quantum gravity. Generally a variable cosmological constant implies creation of radiation and matter and non-conservation of entropy (Burman, 1991; Rahman, 1990). The condition $\rho = \rho_c$, and the requirement of the increasing entropy completely determines in terms of Robertson-Walker scalar factor 'R'. We consider the two fold energy content, formed by dust matter and energy density ρ_m and by vacuum term with equation of state $p = -\rho_\Lambda$. Then the total energy and pressure are given by $\rho = \rho_m + \rho_\Lambda$ and $p = -\rho_\Lambda$. One of the most important and outstanding problem in the cosmology is the cosmological constant problem. The recent observations indicate that $\Lambda \sim 10^{-55} cm$ while particle physics prediction for is greater than this value by factor of order 10^{120}. This discrepancy is known as cosmological constant problem. Some of the recent discussions on the cosmological constant 'problem' and the consequences on cosmology with matter varying cosmological constant are investigated by Ratra and Peebles (1988), Dolgov et al. (1983, 1990, 1997) There are many cosmological solutions dealing with higher-dimensional models containing a variety of matter fields. However, few work in literature is available where variable G and have been considered in higher dimensions. In the present paper we considered higher dimensional, R-W model, specially flat, decaying cosmology, in the realm of the model where the gravitational constant G varies with cosmological scale. We show that for late times, such a cosmology is in accordance with the observed value of the cosmological parameter.

7.2 Field equations

Here we consider the five-dimensional FRW metric in the form

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{(1-kr^2)} + r^2(d\theta^2 + \sin^2\theta d\theta^2) + (1-kr^2)d\psi^2\right] \tag{7.2.1}$$

where $R(t)$ is the scale factor $k = 0, -1 \quad \text{or} \quad +1$ is the curvature paramter for flat, open and closed universe respectively. The universe is assumed to be filled with distribution of matter represented by energy-momentum tensor of a perfect fluid

$$T_{ij} = (\rho + p)u_i v_j - p g_{ij}, \tag{7.2.2}$$

where ρ is the energy density of the cosmic matter and p is its pressure and u_i is the five-velocity vector such that u_i is the five-velocity vector such that $u_i u^i = 1$. The Einstein field equations with time-dependent cosmological and gravitational constants is given by

$$R_{ij} - \frac{1}{2}g_{ij}R = 8\pi G(t)T_{ij} + \Lambda(t)g_{ij} \tag{7.2.3}$$

where R_{ij} is the Ricci tensor, $G(t)$ and $\Lambda(t)$ being the variable gravitational and cosmological constants. The divergence of (7.2.3), taking into account the Bianchi identity, gives

$$\left(8\pi G T_{ij} + \Lambda g_{ij}\right)^{;j} = 0 \tag{7.2.4}$$

Equation (7.2.3) and (7.2.4) may be considered as the fundamental equations of gravity with G and Λ coupling parameters. Using co-moving coordinates

$$u_j = (1, 0, 0, 0, 0) \tag{7.2.5}$$

in (7.2.2) and with the line element (7.2.1), Einstein's field (7.2.3) yeilds

$$8\pi G(t)\rho = \frac{6\dot{R}^2}{R^2} + \frac{6k}{R^2} - \Lambda(t), \tag{7.2.6}$$

$$8\pi G(t)p = -\frac{3\dot{R}}{R} - \frac{3\dot{R}^2}{R^2} - \frac{3k}{R^2} + \Lambda(t) \tag{7.2.7}$$

where dot denotes derivative w.r.t t.

In uniform cosmology $G = G(t)$ and $\Lambda(t)$ so that the conservation (7.2.4) is given by

$$\dot{\rho} + 4(\rho + p)H = -\left(\frac{\dot{G}}{G} + \frac{\dot{\Lambda}}{8\pi G}\right) \tag{7.2.8}$$

The usual energy-momentum conservation relation $T^{\mu\nu}_{;\nu}$ leads to

$$\dot{\rho} + 4(\rho + p)H = 0. \tag{7.2.9}$$

Therefore (7.2.8) yields

$$\frac{\dot{G}}{G}\rho + \frac{\dot{\Lambda}}{8\pi G} = 0. \tag{7.2.10}$$

The field equations (7.2.6) - (7.2.7) can also be written as

$$\frac{3\ddot{R}}{R} = -4\pi G(t)\left[2p + \rho - \frac{\Lambda(t)}{8\pi G(t)}\right], \tag{7.2.11}$$

$$\frac{6\dot{R}^2}{R^2} = 8\pi G(t)\left[\rho + \frac{\Lambda(t)}{8\pi G(t)}\right]. \tag{7.2.12}$$

Equation (7.2.11) and (7.2.12) can be written in terms of the Hubble paramater $H = \frac{\dot{R}}{R}$ to give the below the equation respectively

$$\dot{H} + H^2 = -\frac{4\pi}{3}G(t)(2p + \rho) + \frac{\Lambda(t)}{6}, \tag{7.2.13}$$

$$H^2 = \frac{4\pi}{3}G(t)\rho + \frac{\Lambda(t)}{6} - \frac{k}{R^2}. \tag{7.2.14}$$

Taking the scale factor

$$R = e^{\beta t}, \tag{7.2.15}$$

where β is a constant. The energy density is given by

$$\rho = \frac{1}{e^{4\gamma\beta t}}. \tag{7.2.16}$$

The pressure is given by

$$p = \frac{(\gamma - 1)}{e^{4\gamma\beta t}}. \tag{7.2.17}$$

The gravitational constant $G(t)$ with time t is

$$G(t) = \frac{3k}{8\pi\gamma} e^{(4\gamma-1)\beta t}. \tag{7.2.18}$$

The cosmological constant $\Lambda(t)$ with time t is

$$\Lambda(t) = 6\left[\beta^2 - \frac{k}{\gamma}\frac{1}{e^{\beta t}} + \frac{k}{e^{2\beta t}}\right]. \tag{7.2.19}$$

The deceleration parameter is

$$q = -1. \tag{7.2.20}$$

The Hubble's parameter is

$$H = \beta. \tag{7.2.21}$$

7.2.1 Case : I Taking $\gamma = 1$ and $k = 1$

The energy density is given by

$$\rho = \frac{1}{e^{4\beta t}}. \tag{7.2.22}$$

The pressure is given by

$$p = 0. \tag{7.2.23}$$

The gravitational constant $G(t)$ with time t is

$$G(t) = \frac{3}{8\pi\gamma} e^{3\beta t}. \tag{7.2.24}$$

The cosmological constant $\Lambda(t)$ with time t is

$$\Lambda(t) = 6\left[\beta^2 - \frac{1}{e^{\beta t}} + \frac{1}{e^{2\beta t}}\right]. \tag{7.2.25}$$

7.2.2 Case : II Taking $\gamma = 1$ and $k = -1$

The energy density is given by

$$\rho = \frac{1}{e^{4\beta t}}. \tag{7.2.26}$$

The pressure is given by

$$p = 0. \tag{7.2.27}$$

The gravitational constant $G(t)$ with time t is

$$G(t) = \frac{-3}{8\pi\gamma} e^{3\beta t}. \tag{7.2.28}$$

The cosmological constant $\Lambda(t)$ with time t is

$$\Lambda(t) = 6\left[\beta^2 + \frac{1}{e^{\beta t}} - \frac{1}{e^{2\beta t}}\right]. \tag{7.2.29}$$

7.2.3 Case : III Taking $\gamma = \frac{1}{3}$ and $k = 1$

The energy density is given by

$$\rho = \frac{1}{e^{\frac{4}{3}\beta t}}. \tag{7.2.30}$$

The pressure is given by

$$p = -\frac{2}{3e^{\frac{4}{3}\beta t}}. \tag{7.2.31}$$

The gravitational constant $G(t)$ with time t is

$$G(t) = \frac{9}{8\pi} e^{\frac{1}{3}\beta t}. \tag{7.2.32}$$

The cosmological constant $\Lambda(t)$ with time t is

$$\Lambda(t) = 6\left[\beta^2 - \frac{3}{2e^{\beta t}} + \frac{1}{e^{2\beta t}}\right]. \tag{7.2.33}$$

7.2.4 Case : IV Taking $\gamma = \frac{1}{3}$ and k=-1

The energy density is given by

$$\rho = \frac{1}{e^{\frac{4}{3}\beta t}}. \tag{7.2.34}$$

The pressure is given by

$$p = -\frac{2}{3e^{\frac{4}{3}\beta t}}. \tag{7.2.35}$$

The gravitational constant $G(t)$ with time t is

$$G(t) = -\frac{9}{8\pi}e^{\frac{1}{3}\beta t}. \tag{7.2.36}$$

The cosmological constant $\Lambda(t)$ with time t is

$$\Lambda(t) = 6\left[\beta^2 + \frac{3}{2e^{\beta t}} - \frac{1}{e^{2\beta t}}\right]. \tag{7.2.37}$$

7.2.5 Case : V Taking $\gamma = 2$ and $k = 1$

The energy density is given by

$$\rho = \frac{1}{e^{8\beta t}}. \tag{7.2.38}$$

The pressure is given by

$$p = \frac{1}{e^{8\beta t}}. \tag{7.2.39}$$

The gravitational constant $G(t)$ with time t is

$$G(t) = \frac{3}{16\pi}e^{7\beta t}. \tag{7.2.40}$$

The cosmological constant $\Lambda(t)$ with time t is

$$\Lambda(t) = 6\left[\beta^2 - \frac{1}{2e^{\beta t}} + \frac{1}{e^{2\beta t}}\right]. \tag{7.2.41}$$

7.2.6 Case : VI Taking $\gamma = 2$ and $k = -1$

The energy density is given by

$$\rho = \frac{1}{e^{8\beta t}}. \tag{7.2.42}$$

The pressure is given by

$$p = \frac{1}{e^{8\beta t}}. \tag{7.2.43}$$

The gravitational constant $G(t)$ with time t is

$$G(t) = -\frac{3}{16\pi}e^{7\beta t}. \tag{7.2.44}$$

The cosmological constant $\Lambda(t)$ with time t is

$$\Lambda(t) = 6\left[\beta^2 + \frac{1}{2e^{\beta t}} - \frac{1}{e^{2\beta t}}\right]. \tag{7.2.45}$$

7.3 Physical interpretation

From the above analytical expression, their physical behaviours can be smoothly understood even in the absence of graphical representation. In the cases of $k = 0$, the gravitational constant with time t is zero. So, in all the cases pressure and energy density are same condition that is when the time increases, pressure and density decreases. Hubble's parameter is constant for all cases with time change and deceleration parameter is negative which represents an expanding universe with an accelerated rate and is consistent with the present observation.

7.4 Conclusion

In this chapter, we have investigated a higher dimensional flat FRW model with variable G and Λ. The cosmological parameters and state finder parameters have been obtained for dust, radiation and stiff matter. The different models are obtained for different stages of the universe. We have discussed the physical parameters of the models.

The constant G and Λ are allowed to depend on the cosmic time t. We hope that our results may throw some light in understanding of the real universe. This study will throw some light on the structure formation of the universe, which has astrophysical significance. The expanding universe has singular at $t = 0$. In this way the unified description of early evolution of the universe is possible with variables G and Λ in the framework of higher dimensional space time.