CHAPTER-1

INTRODUCTION

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1.1 INTRODUCTION

The first section of this PhD thesis is the introduction, because it provides all relevant information for the topic of the thesis, allowing readers to grasp the subject quickly. In this chapter, the definition and history of the cosmology, cosmological models, General Theory of Relativity(GTR), Einstein field equations(EFE), the fate of the Universe, the current accelerated expansion of the Universe, some topic related Principles and laws, different cosmological parameters, some cosmological problems etc. have been highlighted. The objectives of the research work, methodology & tools used and a summary of the research work are also given in this chapter. We also present the literature review of articles to get a better understanding of similar works from the past and present. Humans are the most evolved and changing creatures in the world, so it is our moral obligation to learn about the unknown aspects of the cosmos, such as its origin, evolution and ultimate fate. As a member of the mathematics family, it may be done efficiently by creating mathematical models of our Universe, utilizing different theories of relativity, including Einstein’s theory of relativity as well as modified theories of gravity. The results of the model, thus, formulated may be compared with the various observational findings of present day about the origin, evolution, shape, size, physics etc. of the Universe. Motivated from this, we have considered the investigations in the thesis entitled "**A Study on Some Bianchi Type Cosmological Models in General Theory of Relativity**". This thesis includes 9 (Nine) chapters and deals with the study of some Bianchi type I, Bianchi type-III and Bianchi type-V cosmological models in 4 and 5 dimensions in general relativity as well as in Lyra’s Geometry (modified theory of general relativity). This introduction chapter is arranged as follows: In section 1.2, a brief ideas of Einstein’s general theory of relativity is presented. Section 1.3 discusses the Lyra geometry. In section 1.4, a brief idea about cosmology and cosmological models are presented. In section 1.5, the history of cosmology and accelerated expansion of Universe has been discussed. Section 1.6 deals with the brief introduction to Bianchi type space-times. Section 1.7 discusses about the string and string cosmology. Section 1.8 contains brief discussions about higher dimensional cosmology. In section 1.9, some principles and laws like cosmological Principle, Hubble's laws etc. have been discussed. In section 1.10, some cosmological parameters used are provided. In section 1.11, basic terminologies like viscosity and viscous fluid, perfect fluid & energy-momentum tensor are given. Section 1.12 discusses about some candidates of dark energy. In sections 1.13, 1.14, and 1.15 the objectives of the research work, methodology & tools used and a summary of the research work respectively have been highlighted.

1.2 GENERAL RELATIVITY

The theory of relativity is the foundation of the modern cosmology. It is basically two types namely the special relativity and general relativity. Both the relativity belonged to Albert Einstein(1879-1955). The special relativity deals with the inertial system i.e. system moving with uniform velocity and has been applied in mechanics, electromagnetism and quantum theory. In 1905, Einstein introduced the special relativity in his noted research paper "On the Electrodynamics of Moving Bodies". There are two postulates in special relativity, which are

i) "The laws of physics are the equal for all observers in the same motion relative to one another".

This first postulate states the covariance of the physical laws in every inertial system. It is a fact drawn from the Michelson-Morley experiments that failed to determine the velocity of earth relative to ether.

ii) "The speed of light in free space is the same for all inertial observers, independent of the relative velocity of the source and the observer".

This second postulate says that, the velocity of the light in vacuum is constant and it is independent of the velocity of the observer and the source. The second postulate contradicts the Galilean transformation. Consequently, Lorentz transformation has been introduced.

In modern cosmology the general relativity of Einstein is considered as most correct and complete theory of gravity which deals with the non inertial system i.e. system moving with accelerated velocity and has been applied to study the theory of gravitation and the structure of the Universe. Einstein in 1915 formulated the general theory of relativity, which expresses the relativity of all motions. This theory extends special relativity, which is concerned with the uniform motion of bodies in free space without regard for gravitational forces. It has also been used as a foundation for Universe models. Einstein's development was primarily guided by three basic principles:

i. Principle of covariance:- “The law of physics can be expressed in a form that is independent of the coordinate system”.
ii. Principle of equivalence:- “No physical experiment can tell whether a free particle's acceleration is due to a gravitational field or a frame of reference's acceleration”. As a result, the metric and gravitational systems are intimately linked.
iii. Mach's principle:- It states that the geometry of the space-time and thereby the inertial properties of test particles from the information of density and mass energy distribution in its neighborhood. In brief, according to the Mach's Principle:
    a) The inertia of the body must increases when ponder able masses are piled up in its neighborhood.
    b) A body must experience an accelerated force when neighboring masses are accelerated and in fact, the force must be in the same direction as that of acceleration.
    c) A rotating hollow body must generate inside of itself a 'Coriolis field' which deflects moving bodies in the sense of the rotation and a radial centrifugal field as well.

In this theory, following Riemannian metric described the space-time

$$ds^2 = g_{ij}dx^i dx^j \quad ; \quad i\,,j = 1,2,3 \; and \; 4 \tag{1.1}$$

Here g_{ij} is the fundamental metric tensor. The gravitational field equation or simply the Einstein’s field equations in general relativity introduced by Albert Einstein in 1915, are given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -\frac{8\pi G}{C^4} T_{ij} \tag{1.2}$$

Here R_{ij} is the Ricci tensor, R is the curvature (Ricci) scalar, T_{ij} is the energy momentum tensor of the source producing the gravitational field, G is the gravitational constant and C is the speed of light. The LHS of Einstein's field equations represents the curvature of space-time as determined by metric tensor (geometry) and the expression on RHS represents the matter/energy content of space-time. Matter in the space-time tells space-time to curve and curvature in the space-time tells matter how to move. The negative sign is inserted for later convenience. In Einstein's field equations, the gravitational constant G serves as a coupling constant between matter and geometry.

Initially, Einstein found a non-static solution of his field equations but he was not satisfied with it as he strongly believed in the static Universe. In order to study static cosmological model, Einstein modified his field equation in 1917, by introducing another term Λ as

$$R_{ij} - \frac{1}{2} R g_{ij} + \Lambda g_{ij} = -\frac{8\pi G}{C^4} T_{ij} \tag{1.3}$$

Here Λ is known as the cosmological constant. Einstein first proposed this constant, which he later abandoned, calling it "worst blunder of his life".

However, in recent years, the cosmological constant scenario has drawn the attention of many researchers in general relativity. The cosmological constant(Λ) is proportional to the vacuum energy density (ρ), given by

$$\Lambda = \frac{8\pi G}{3C^2} \rho \tag{1.4}$$

The unit of the cosmological constant Λ is $\frac{1}{Second^2}$.

In the present Universe the value of Λ is exactly not known but it may be zero, positive or negative.

The field equations given by (1.3) yield

$$T^{i}_{i;j} = 0 \tag{1.5}$$

This is analogous to the energy momentum conservation equations and it also gives us the equation of motion of matter.

General relativity has proved highly successful as a physical theory and is regarded as the most important tool for cosmologists to describe the nature of the Universe in various ways. Based on these principles Einstein wrote his field equations for the gravity, which become the core of the general relativity. German cosmologist K. Schwarzschild(1873-1916) solved the first non trivial exact solution of the Einstein's field equations in 1915 and published it in the month of January 1916. Prior to 1920, it was believed that our galaxy, the Milky Way, made up the whole Universe. American astronomer Harlow Shapley(1885-1972) also supported this concept.

Current observations reveal that the expansion of Universe is accelerating, such that the velocity with which a galaxy fades away from us is continuously increasing with time. This accelerated expansion of the Universe was proved by two well known projects namely "The Supernova Cosmology Project" headed by S. Perlmutter [Perlmutter et al. (1998, 1999)] and "The High red-shift Supernova Search Team" headed by A.G. Riess & B.P. Schmidth[Riess et al.(1998, 2004)] measuring the distances of remote supernovae (exploding stars) of Type Ia (SNe Ia) and using their red-shift. Some astrophysical and cosmological observation's data that supported it are The Wilkinson Microwave Anisotropy Probe (WMAP), which is a spacecraft operated by NASA between 2001 and 2010, mapped the sky and measured the differences in temperature in the CMB radiation across the sky, Sloan Digital Sky Survey (SDSS), using a 2.5 meter wide-angle optical telescope at Apache Point Observatory, New Mexico maps the sky to examine the history and large scale structures of the Universe, Chandra X-Ray Observatory, which is a flagship class telescope launched on July 23, 1999 and one of the NASA's Great observatories through which the scientists are able to understand the structure and evolution of the Universe by collecting, observing and analyzing X-ray radiation. The Baryon Acoustic Oscillations (BAO) is considered as 'standard ruler' for the length scale measurement in cosmology and its measurements help the cosmologists to know more about the nature of dark energy that indicates the accelerated expansion of the Universe by constraining cosmological parameters.

A good numbers of research papers have been published based on these data. Various authors like Spergel et al.(2003, 2007), Bennett et al.(2003, 2003a, 2011, 2013), Tegmark et al.(2004), Hinshaw et al.(2007, 2009, 2013) analyzed data from WMAP. York et al.(2000), Gunn et al.(2006), Eisenstein et al.(2011), Blanton et al.(2017), analyzed data from SDSS. Anderson, et al.(2012) for BOSS analyzed data from BAO. Allen(1998), Allen et al.(2001, 2007) analyzed data from Chandra X-Ray Observatory.

1.3 LYRA GEOMETRY

So far, several Riemannian geometry modifications have been suggested in an attempt to unify gravitation, electromagnetic fields and a variety of other effects in the Universe. Weyl(1918) attempted to bring gravity and electromagnetism together in single space-time geometry. Weyl's theory, however, was dismissed due to its reliance on the non-integrability of length transfer. Later, Lyra(1951) made a modification to Riemannian geometry by the introduction of a gauge function β, which is a displacement vector in Riemannian space-time that eliminates the non-integrability condition of a vector under parallel transport. Lyra geometry is the name given to this modified geometry. Subsequently, Sen(1957) and Sen & Dunn(1971) proposed a new scalar tensor theory of gravitation based on Lyra's geometry and formulated an analogue of Einstein's field equations that can be written as (in geometrized unit where, $8\pi G = 1, C = 1$)

$$R_{ij} - \frac{1}{2} R g_{ij} + \frac{3}{2}\phi_i \phi_j - \frac{3}{4} g_{ij}\phi_k \phi^k = -T_{ij} \tag{1.6}$$

Here $\phi_i = \big(0,0,0,\beta(t)\big)$ is the displacement field vector.

In this section, some of the relevant work carried out by various authors related to Lyra's geometry in different contexts has been presented.

Lyra(1951) and Scheibe(1952) completed the study about Lyra's Geometry. In contrast to Weyl's geometry, in Lyra's geometry, the attachment is metric sustaining as Riemannian geometry and length transfers as integrable. This alternating theory is of interest since it produces effects similar to Einstein's theory.

Sen(1960) showed that, unlike Riemannian geometry, the auto parallels associated with the affine connection in Lyra geometry did not coincident with the geodesics arises from the metric. In the Lyra's geometry, they also constructed a new scalar tensor theory where both the scalar and tensor field had natural geometrical significance.

Halford(1970) proposed a cosmological theory based on Lyra's geometry, demonstrating that the constant displacement vector field in Lyra's geometry serves as the cosmological constant in normal general relativistic studies.

Sen & Vanstone(1972), in their paper "On Weyl and Lyra Manifolds", demonstrated that the Lyra's geometry and Weyl's geometry are specific cases of configurations with more general correlations. Also, they showed the relationship between Lyra's geometry and Weyl's geometry and the relationship of them with Riemannian geometry was obtained by providing a global formulation of Lyra's geometry.

Bhamra(1974) obtained a spherically symmetric cosmological model of class-one in Lyra's geometry and showed that the static Universe is physically unrealistic whereas the non-static Universe is similar to Lemaitre's model in Riemannian geometry in which the mass-energy conservation law did not hold.

Reddy & Innaiah(1985) formulated an anisotropic and spatially homogeneous Bianchi type-I model in Lyra's manifold with perfect fluid as a source of gravitational field by considering energy density equal to pressure.

Also, Reddy & Innaiah(1986) constructed a plane-symmetric cosmological model in Lyra manifold with perfect fluid as a source of gravitational field by taking energy density equal to pressure.

T. Singh & G.P. Singh(1991, 1991a, 1991b, 1991c, 1992, 1993) and G.P. Singh & Desikan(1997) investigated Bianchi types I, II, III, V, VI0, VIII, IX, Kantowski-Sachs, and a new class of cosmological model universes with and without time-dependent displacement field in the context of Lyra geometry. They also conducted a comparison study of the cosmological theory based on Lyra's geometry and the Friedmann-Robertson-Walker (FRW) model universes with a constant DP in Einstein's theory of relativity.

Using on Lyra geometry, Rahaman et al.(2005) obtained two model universes namely axially symmetric Bianchi type-I and Kantowski-Sach cosmological models with negative constant DP.

Kumar & Singh(2008) studied a spatially homogeneous and anisotropic Bianchi type-I in the presence of perfect fluid and obtained a cosmological model Universe based on Lyra geometry. They obtained the accurate solutions of the field equations using the special law of Hubble's parameter, which gives a constant DP and is consistent with recent observational data from supernovae of type Ia. (SNe Ia).

Studying an inhomogeneous Bianchi type-I metric in presence of an electromagnetic field, Megied et al.(2009) obtained a cosmological model in the context of Lyra geometry.

Investigating plane-symmetric metric under the influence of perfect fluid, Yadav(2010) obtained an inhomogeneous cosmological model Universe with electromagnetic field based on Lyra geometry and the exact solutions of the field equations for this model are agreeing with the recent observational data from supernovae type Ia(SNe Ia).

In the framework of Lyra geometry, Gad(2011) developed a new class of axially symmetric cosmological model universes in presence of the mesonic stiff fluid with time-dependent displacement field which are expanding, shearing and non-rotating.

Adhav(2011) examined an LRS Bianchi type-I line element under the influence of anisotropic fluid to obtain an dark energy model based on Lyra geometry. Exact solutions of the field equations for constant and time-dependent displacement fields were determined using exponential volumetric expansion and the isotropic properties of the space and fluid were investigated.

In the framework of Lyra geometry, Mahanta & Biswal(2012) obtained cosmological model universes for both domain walls and string cloud with quark matter by solving the Einstein’s field equations using anisotropy property of the Universe, time-dependent displacement field and special law for Hubble’s parameter that gives the constant deceleration parameter.

R.K. Dubey et al.(2017) investigated an anisotropic and spatially homogeneous Bianchi type-II Universe field with anisotropic dark energy within the framework of Lyra geometry with displacement field factor. This model shows that the Universe is anisotropic for all times. .

Various authors like Halford(1972), Rahaman et al.(2002), Pradhan & Vishwakarma (2004), Pradhan et al.(2005), Pradhan & Chauhan(2006), Rao & Vijaya Santhi(2008), R.P. Singh & L. Yadav(2009), Bali Raj & Naresh K. Chandnani(2009), Vineet K. Yadav & Lallan Yadav (2011), S. Parikh et al.(2018) studied various cosmological models in Lyra geometry in different contexts.

1.4 COSMOLOGY AND COSMOLOGICAL MODELS

Cosmology is a systematic and scientific investigation of the Universe. It is a branch of astronomy that studies the origin, evolution and ultimate fate of the Universe from the Big Bang to today and beyond. The word "Cosmology" and "Cosmetology" are both derived from the Greek word "Kosmos", which means "harmony or order". It deals with issues that are central to the human condition. Cosmologists look for answers to questions like: “what is the Universe made up of?, is it finite or infinite in spatial extent?, did it have a beginning sometime in the past?, will it come to an end sometime in the future?” etc.. Cosmologists, like other scientists, rely on experiments and observations to progress in their quest to learn more about our cosmic past. The nebulae, stars, star clusters, pulsars, galaxies, quasars, cosmic rays, background radiation etc. are the main elements of the Universe. Some areas relevant to the cosmology are Astronomy, Astrophysics, Particle Physics, Plasma Physics and Quantum Mechanics. Since we are an element of the Universe, therefore cosmology attracts all of us. Cosmology has a very lengthy history and there are many statements and theories proposed during this long era. Various opinions have been developed by the cosmologists about the origin and evolution of the Universe.

Cosmological models are mathematical as well as astrophysical justification about the origin, evolution, geometry, contents, behavior and ultimate fate of the Universe. The cosmological models are based on direct experiments and observations. Most of them are based on the Einstein’s general relativity, field equations and cosmological

Principles. In the general relativity, gravity has a geometric property of the space-time and the field equations indicate the relations between the curvature of the space-time and the energy-momentum. So, the solutions of the field equations describe the evolution of the Universe. The simplest and well known cosmological model is the ΛCDM model. It is based on the general relativity and the cosmological Principle. In this model, the Universe contains cosmological constant Λ which deals with dark energy and dark matter(cold). It is considered as the 'Standard cosmological model' of the Universe. It provides the existence and structure of the cosmic microwave background (CMB), large scale structures, abundance of Hydrogen, Helium & Lithium and the accelerating expansion of the Universe. Most of the cosmological models are based on this model.

1.5 HISTORY OF COSMOLOGY AND ACCELERATED EXPANSION OF UNIVERSE

Today, Einstein's general relativity is the most important tool for the cosmologists to describe the nature of the Universe in different ways. Based on these principles Einstein wrote his field equations for the gravity, which become the core of the general theory of relativity.

Greek Philosophers Aristotle(384 BC-322 BC) promulgated that the Universe had no beginning, but is and always was. In 1654, Bishop Ussher(1581-1656), after studying biblical chronology had announced that the Universe was created at 9 A.M on 23rd October, 4004 BC as per the Proleptic Julian calendar. But in the 19th century, Cosmologists confirmed that Bishop Ussher's creation date was wrong. Nobody can give a particular date and time about the formation of the Universe till now. The Cosmologists believed that the Universe was created only after a sudden explosion. English Mathematician, Physicist and Astronomer Sir Isaac Newton(1642-1727) did not agree with the James Ussher's creation date of the Universe. He imagined that the Universe was finite in space. He also realized that gravity is the most important force to understand the large-scale structure of the Universe. He assumed that at the beginning, the Universe was sufficiently cold so that only gravitational attraction played a role; at that time there was also no pressure.

Aristotle projected that the spherical earth is enclosed by concentric celestial spheres containing the stars and therefore the Universe is static, has finite extent and exists throughout eternity. Supported the Aristotelian model, Ptolemy projected that the Universe revolves around the stationary earth and therefore the planets move on circular orbits whose centre's once more move in a very larger circular orbit with a centre close to the earth. The Ptolemic geocentric theory was the accepted theory till sixteenth century. Numerous philosophers and astronomers particularly Italian astronomer and man of science Galileo Galilei(1564-1642) opposed this theory controversy that if the Universe were geocentric, then the sun and therefore the different heavenly bodies would have to be compelled to orbit the planet on massive ways with tremendous speed that wasn't attainable. Probably, the Greek Mathematician and Astronomer Aristarchus of Samos(310BC-230BC) was the primary to propose a heliocentric model of the Universe inserting the sun at the centre and therefore the earth orbiting it on a circular path whereas it's rotating on its axis. He additionally projected that the stars are fixed and therefore the centre of the sphere containing all the stars is at the sun. Within the middle of 16^{th} century, astronomer Nicolaus Copernicus(1473–1543) revived Aristarchu's theory of heliocentric Universe and argued that astronomical information might be explained higher if the earth revolved on its axis and the earth with the other planets rotated around the sun and therefore the sun were placed at the centre of the Universe. The idea of the earth's rotation on its axis is very older. Johannes Kepler established the famous laws known as Kepler's laws of planetary motions between 1609 and 1619. In 1687, Sir Issac Newton highlighted in the Principia Mathematica, how the heavens move. Using Kepler's laws of planetary motions, Newton proposed his famous law of gravitation and this law was considered as a suitable explanation of the gravitational force between masses for more than two hundred years.

Prior to 1920, it was believed that our galaxy, the Milky Way, made up the whole Universe. During 1922-1923, American Astronomer, Edwin Powell Hubble(1889-1953) measured the distances of spiral nebulae by Hooker Telescope and found that Andromeda and Triangulum galaxies and many other galaxies are well outside the boundary of our galaxy and concluded that there are a great number of galaxies in the

Universe with vast tracks of empty space between them. In 1912, after measuring the first Doppler shift of spiral nebula, American Astronomer Vesto Melvin Slipher(1875–1969) discovered that almost all such nebulae were moving away from the earth [Slipher(1913)]. Later on, in 1924, using Slipher's data, American Astronomer Milton La Salle Humason(1891–1972) obtained velocities and Edwin Hubble measured distances of spiral nebulae and found that the farthest nebulae are receding faster than the nearest nebulae from the earth. In 1917, using the Einstein general theory of relativity, Dutch Astronomer William de-Sitter formulated a model of expanding Universe. Alexander Friedmann(1888-1925), a Russian Cosmologist and Mathematician in 1922 obtained an exact solution of Einstein's field equations which predicts that the red-shift of a galaxy should be directly proportional to its distance from us.

In 1927, Belgian Cosmologist and Astronomer Georges Edouard Lemaitre(1894-1966) derived Friedmann's equations severally and ended that the recession of the nebulae was because of the growth of the Universe. However the works of Friedmann and Edouard Lemaitre wasn't responded at that point because everyone believed that the Universe was static, even Einstein was additionally positive regarding it. In 1935, their works became known once similar models were derived severally by American Mathematician and Scientist H.P. Robertson(1903–1961) and British Scientist A.G. Walker(1909-2001) in response to Hubble's discovery of the uniform growth of the Universe. The image of increasing Universe dates back to solely 1929 once Edwin Hubble supported the observations of Cepheid variable stars in distant galaxies, detected that the galaxies are receding quicker than the nearer ones with velocities proportional to their distances from us. As the growth causes matter and energy to cool down and unfolded with time, it reveals that our Universe should have began from some extent mass abundant hotter and denser than it is these days. The scientific model of the evolution of the Universe, that explains however the current day Universe developed from an especially hot and dense starting, is thought as the Big-Bang theory. The Big-Bang theory was projected by Georges Edouard Lemaitre in 1927 and was developed in 1948 by Soviet-American theoretical scientist and Astronomer George Gamow(1904-1968), American Astronomer Ralph Asher Alpher(1921–2007) and United States Scientist Robert Herman(1914-1997). According to this theory, the

Universe began as a super-hot, high-density, high-pressure state known as the big bang. Following the Big Bang, the Universe experienced a brief period of extremely rapid expansion referred to as inflation and because the Universe cooled down, it's been increasing since then. During the primary three hundred thousand years, it had been thus hot that matter might exist solely in an exceedingly dense state of plasma. Galaxies began to make by the time the Universe was a couple of billion year old. Stars have shaped in those galaxies, producing significant components that were recycled into later generations of stars. This theory was supported by Hubble’s discovery of galactic red shift in 1929 and also the discovery of CMBR by American scientist Arno Allan Penzias and American astronomer Robert Woodrow Wilson in 1964. The Big-Bang theory is currently the foremost accepted theory for the origin and evolution of the Universe. But still the scientists aren't certain whether or not the Universe started after an explosion from some extent singularity with high density and high pressure or it simply started increasing from a state having some volume, density and pressure.

In 1920s and 1930s, most of the Cosmologists favored steady state Universe. In 1981, another vital development happened in cosmology, when A.H. Guth & E.J. Weinberg [Guth & Weinberg(1983)] projected inflationary model of the early Universe. Guth inferred that if the growth of the very early Universe is accelerated in such some way that the scale factor of the Universe grows by an element throughout a brief period just when the Big-Bang, then, the ‘flatness’ and ‘ horizon’ issues of the Big-Bang theory might be avoided. Till the late 1990s, the Astronomers thought that the expansion of the Universe was decelerated because of the attraction of the masses in it. But in 1998, two independent groups of Astronomers, one guided by A. G. Riess and B. P. Schmidt [Riess et al.(1998)] and also the other by S. Perlmutter [Perlmutter et al.(1998)] proved that the growth of the Universe wasn't decelerating, rather it had been accelerating and therefore the expansion history of our Universe over the past 5-6 billion years. Riess et al.(1998) within the High Red-shift supernova search team analyzed 16 distant and 34 close supernovae and Perlmutter et al.(1998) within the expanding Cosmology Project Team analyzed 18 close supernovae and 42 high red shift supernovae. Their experimental information powerfully indicates that the current Universe is increasing with acceleration and therefore the present rate of expansion is more than the rate of expansion of the Universe five billion years past.

Later on, it was established by a good number of cosmological observations such as Large Scale Structure(LSS), Wilkinson Microwave Anisotropy Probe(WMAP), Cosmic Microwave Background(CMB), Baryon Acoustic Oscillations(BAO) and their cross-relations. But the Cosmologists are unable to discover the actual cause of this acceleration. Most of the Cosmologists suggest that an energy with negative pressure known as dark energy is the actual cause of this acceleration. But the nature of the dark energy is still unknown. There are many candidates for the dark energy proposed by the researchers. The cosmological constant Λ is the simplest candidate for the dark. Moreover, Quintessence, Phantom, Tachyon, K-essence, Dilaton, Polytropic gas, Chaplygin gas, Holographic dark energy, Holographic Ricci dark energy, Wet dark fluid, Viscous fluid, Quintom etc. are also considered as candidates for the dark energy.

Some researchers recommend that the acceleration of the Universe is because of the repulsive gravitational interaction of antimatter[Benoit–Levy & Chardin(2012); Hajdukovic(2012)] or deviation of attraction laws from general theory of relativity. Consistent with information offered from the observations by Planck team[Ade et al. (2014, 2014a)], the Universe is concerning 13.799 billion years old. The age of the Universe was firm by mapping fluctuations in temperature within the cosmic microwave background radiation(CMBR).

1.6 BIANCHI TYPE SPACE-TIMES

The cosmological principle is supposed to be deduced from the Copernican principle. Further the analogy of the assumption may be stated as: the world line of our galaxy is not special because if our Universe is isotropic about our world line then it is also isotropic about the world line of other galaxies. In the year 2016 [P. Bull et al.(2016)], showed that Einstein's GTR is a perfect theory to describe our expanding Universe by using Friedmann-Robertson-Walker(FRW) space-time which describes isotropic, homogeneous on large scale. But the isotropic CMB radiation is not certainly able to explain about isotropic space-time. Considering these limitations, it does not suit us to curb ourselves, only in the study of isotropic homogeneous cosmological models; rather it leads to study more conveniently anisotropic and inhomogeneous cosmological models. Since the solution of the Einstein's field equation for study of inhomogeneous cosmological model universes appears to be more complicated and sometimes becomes very hard nut to crack for the researchers to solve, hence, many cosmologists opt to undergo the spatially homogeneous and anisotropic Bianchi type models instead of inhomogeneous models. Generally, Bianchi type models represent a mid way between the FRW model and inhomogeneous & anisotropic universes, hence its important role is found in modern cosmology. In order to understand the properties and structure of the space of all cosmological solutions of the Einstein's field equations, a spatially homogeneous cosmological model plays a vital role [McCallum(1979)].

It has been discovered through the study of various texts by various authors that Bianchi cosmological models are significant because they are both anisotropic & homogeneous, allowing for the investigation of the process of isotropization of the Universe over the time. The Bianchi type models have the advantage of providing a true dynamics model of general relativity, despite the fact that the metric components in the invariant basis are only functions of time. Also, anisotropic models have been found to be more general than isotropic models from a theoretical standpoint. The Bianchi Type cosmological models are also important in the construction of anisotropic and spatially homogeneous cosmological models because of the simple structure of the field equations and comparable case of solutions in Bianchi space-times. Thus it can be said that Bianchi type cosmological models play a crucial role in the description of the model universes. Anisotropic Universe has received a lot of attention over the last three decades. The most basic are the well-known nine types of Bianchi models[Taub (1951)], which are spatially homogeneous by definition. Di Pietro & Demaret (1999) classified the 9 (nine) different types of Bianchi models into 2 (two) groups 1 and 2. They placed BT-I, II, VI_0, VII_0, VIII and IX models in group 1 whereas the BT-III, IV, V, VI_h and VII_h are in group 2. The BT- I and BT-VII_0 represents the generalized flat FRW models while the BT- V and BT-VII_h represents the generalized open FRW models. It should also be noted that BT- II, VI, VIII and IX are the most general non-flat models.

A line-element or metric is the elementary distance between two events in a coordinate system. For Bianchi models, the line element can be expressed as

$$ds^2 = -dt^2 + g_{ij}dx^i dx^j \qquad (1.7)$$

Here $g_{ij}dx^i dx^j$ is the three/four dimensional line element. It plays a crucial role in determining the geometric properties of the space-time.

The line elements of various Bianchi type models are given bellow:

BT-I metric: $ds^2 = -dt^2 + a^2dx^2 + b^2dy^2 + c^2dz^2$

LRS BT-I metric: $ds^2 = -dt^2 + a^2dx^2 + b^2(dy^2 + dz^2)$

BT-II metric: $ds^2 = -dt^2 + a^2(dx^2 - zdy^2) + b^2dy^2 + c^2dz^2$

LRS BT-II metric: $ds^2 = -dt^2 + a^2(dx^2 - zdy^2) + b^2(dy^2 + dz^2)$

BT-III metric: $ds^2 = -dt^2 + a^2dx^2 + e^{-2\alpha x}b^2dy^2 + c^2dz^2$

BT-IV metric: $ds^2 = -dt^2 + e^{-2z}[adx^2 + (az^2 + b)dy^2 + 2azdxdy] + cdz^2$

BT-V metric: $ds^2 = -dt^2 + a^2dx^2 + e^{-2\alpha x}b^2dy^2 + e^{-2\alpha x}c^2dz^2$

BT VI$_0$ metric: $ds^2 = -dt^2 + a^2dx^2 + e^{-2\alpha x}b^2dy^2 + e^{2\alpha x}c^2dz^2$

BT VI$_h$ metric: $ds^2 = -dt^2 + a^2dx^2 + e^{2x}b^2dy^2 + e^{2hx}c^2dz^2$

BT VII$_0$ metric: $ds^2 = -dt^2 + (acos^2z + bsin^2z)dx^2 + (asin^2z + bcos^2z)dy^2 + (a + b)sin2zdxdy + cdz^2$

BT VII$_h$ metric: $ds^2 = -dt^2 + b^2dx^2 + a^2dy^2\left(a^2sinh^2y + b^2cosh^2y\right)dz^2 -2b^2coshydxdz$

BT-IX metric: $ds^2 = -dt^2 + a^2dx^2 + (a^2cos^2y + b^2sin^2y)dz^2 - 2a^2cosydxdz$

Here a, b and c are the directional scale factors and are functions of time t only.

In this section, a few relevant works done by different authors connected to the Bianchi type Metric (Models) in GR has been discussed.

Singh & Kumar(2006) investigated a spatially homogeneous and anisotropic LRS Bianchi type-II perfect fluid model with constant DP. A separate law of variation for Hubble's parameter is introduced in anisotropic space-time, yielding a constant DP.

Bali Raj & Pradhan Anirudh(2007) studied Bianchi Type-III cosmological models with bulk viscosity (time dependent) assuming the coefficient of bulk viscosity ξ is inversely proportional to the expansion θ and the model's expansion θ is proportional to the shear scalar σ. The model shows that the expansion decreases as the time increases.

G. Mohanty & G.C. Samanta(2009) investigated the bulk viscous fluid distribution in the LRS Bianchi type-I space time with massive string. In general theory of relativity, they built two five-dimensional string cosmological models with bulk viscous fluid and obtained the exact solution of the field equations using the equations of $\rho = -\lambda$ and $\rho = \lambda$. They observed that the bulk viscous fluid survive for $\rho = \lambda$ and it doesn't survive for $\rho = -\lambda$.

G.C. Samanta et al.(2011) developed five five-dimensional string cosmological models, one of which is physically unrealistic. In addition, they found that all models are inflationary and have an initial singularity.

A. Pradhan et al.(2012) discussed about spatially homogeneous and anisotropic BT-I models representing massive strings in GR.

A. Singh et al.(2013) investigated some Bianchi type-III bulk viscous massive string cosmological models in the presence of an electromagnetic field. They discovered that at both the early and late stages of the evolution of the Universe, there arose two types of scenarios: (i) the Universe is dominated by massive strings and (ii) the Universe is dominated by strings depending on the nature of the constants.

M.K. Singh et al.(2013) obtained exact solutions for a viscous fluid with time-dependent gravitational and cosmological constants in totally anisotropic Bianchi-type II space-time. The power-law expansion model has a Big-Bang singularity at $t = 0$, whereas the exponential expansion model has a singularity in the infinite past.

K. Priyokumar Singh & K. Manihar Singh(2014) studied on Bianchi type-III string cloud universes containing strange quark matter which is expanding, anisotropic.

H.R. Ghate et al.(2015) investigated "Bianchi type-IX cosmological models with variable equation of state (EoS) parameter in general relativity when Universe is filled with dark energy".

R.K. Jumale et al.(2016) studied a spatially homogeneous and anisotropic five dimensional Bianchi type cosmological models with perfect fluid. The variation law for generalized Hubble's parameters proposed by Berman(1983) in FRW models that yield a constant value of DP, was used to solve the Einstein's field equations obtained from five-dimensional space-time. Exponential and power law expansion are observed as two physically feasible models of the Universe($n = 0$, $n \neq 0$).

S.D. Katore & D.V. Kapse(2018) investigated the Bianchi type-I Universe with the Polytropic EoS in Barber's second self-creation theory of gravitation. The field equations were solved using (i) Berman's special law of variation for Hubble's parameter and (ii) the power law relation between the average scale factors 'a' and thus the scalar field 'a'. Aspects of the models' physical and kinematic properties are also discussed.

In the framework of general relativity, T.S. Trivedi & S. Shrimali(2018) studied a decelerating anisotropic Bianchi type-VI_0 cosmological model. By assuming $A = B^n$ and $\rho + \bar{\rho} = 0$, they were able to find a determinant solution. They discovered that as cosmic time progresses θ, σ and H decrease while V increases, implying an expanding Universe.

R.K. Muharlyamov & T.N. Pankratyeva(2018) explored a spatially homogeneous and anisotropic cosmological model of Bianchi type-I space-time. The model includes dark matter in the form of a pressureless perfect fluid and dark energy with anisotropic pressure.

In general relativity, Reddy(2018) investigated an inflationary LRS Bianchi type-II model in the vicinity of a scalar field and a massive string. Their model depicts a modern cosmology scenario in which the Universe expands at a faster rate due to inflation.

Recently, S. Sharma & L. Poonia(2021) have studied the cosmic expansion of the universe in existence of massless scalar field having flat potential with bulk viscosity, considering Bianchi type-IX space time. The presence of bulk viscosity provides the inflationary phase of the current model.

Various authors like Pant & Oli (2002), Wang(2006), R. Bali et al.(2008), K.S. Adhav et al.(2010), Atul Tyagi & Keerti Sharma(2010), A. Pradhan(2011)**,** H. Amirhashchi et al.(2011), S.P. Kandalkar et al.(2012), C. Chawla et al.(2012), A. Pradhan et al.(2013), C.P. Singh(2013), R. Venkateswarlu & J. Satish(2014)**,** V. Humad et al.(2014), Soni &

Shrimali(2015), R.K. Dubey et al.(2017), S.D. Katore & D.V. Kapse(2017), Y. Aditya & D.R.K. Reddy(2018), L. Poonia et al.(2021), S. Sharma et al.(2021), Basumatary & Dewri(2021), Mahanta & Das(2021), N. Sarma(2021) studied various Bianchi type cosmological models in general relativity and modified theories of relativity in different contexts.

1.7 STRING AND STRING COSMOLOGY

Despite the vicissitudes of literary tastes and temperament, the Big-Bang theory is the most prevailing theory of the formation of our Universe; however, we are still unable to provide an exact clear statement about the origin and evolution of Universe that is why the origin of Universe is regarded as one of the biggest mysteries among cosmologists. Therefore the study of the exact physical state of the Universe in its early stages of evolution becomes a matter of concern. Cosmologists explore the concept of string theory to describe the events at the beginning phases of evolution of our Universe, with cosmic string being one of the most important objects of study. In order to study about the period before the creation of the particle in the Universe, string theory is used by cosmologists. During the initial stages of the evolution, it might have passage through a number of phase transitions just after the Big-Bang as it cooled from its hot initial state (i.e. the Universe passage through its critical temperatures). During the phase transitions at the early stage of our Universe, the symmetry of the Universe might have been broken spontaneously and as a result, various topological defects like cosmic string, textures, domain walls, monopole etc. [Kibble(1976); Mermin(1979)] were formed. In the year 1998, Pando and his co-authors [Pando et al.(1998)] suggested that these defects were responsible for the creation of the structure of the Universe. Vilenkin (1985) and Vilenkin & Shellard (1994) showed that amongst the all topological defects, only string can explain to very interesting cosmological consequences like galaxy formation and double quasar problem. Different works of literatures reveal that the string theory is also treated as one of the most important contenders for bringing all forces together. The presence of a large-scale network of strings in the early stages of our Universe also agrees with current configurations of our Universe. Strings are also treated as one of the main causes of density perturbations which are necessary for the configuration of the large-scale structure in our Universe. Due to the stress energy possessed by the strings they could have produced a gravitational field. Therefore the gravitational effects arising from the strings are also treated as an interesting topic of study. Letelier(1979, 1983) and Stachel(1980) were the first who introduced the strings into the general theory of relativity. Considering that the massive strings were formed from the massless geometric strings with particle attached along its extension, Letelier(1979) formulated the equation of energy momentum tensor for a cloud of massive strings as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \qquad (1.8)$$

Here ρ denotes the energy density of a string cloud with particles attached to it and λ is the tension density of string. Also u^i is the particle's four velocity vector, x^i is the unit space like vector that represents the direction of string. The relation between ρ and λ is

$$\rho = \rho_p + \lambda \qquad (1.9)$$

In which ρ_p is the particle density of the configuration. Many authors like Zeldovich et al.(1974), Kibble(1976), Letelier(1979), Stachel(1980) showed that existence of the string in the early Universe may be explained by using GUT (grand unified theories). Brief discussion of the number of the important chronological achievements in string theory was presented by Schwarz(2001). Recently strings and string cosmological models have got increasing importance in the cosmological society.

1.8 HIGHER DIMENSIONAL COSMOLOGY

In the general relativistic physics, our present Universe seems to be four-dimensional, of which three are used to denote usual spatial dimensions and the fourth dimension represents time. But many researchers established their theories about the Universe in higher-dimensional space-time mainly due to the significant achievement in solving long-standing problems relating to the stability of the results in general relativity and quantum mechanics. Before Einstein, two mathematicians namely, Herman Weyl(1918) and Theodor Kaluza(1921a) attempted to unify gravity with the electromagnetic force. In the standard four-dimensional space-times, the first unified theory was suggested by Herman Weyl on the basis of generalizing the Riemannian geometry. But in the five-dimensional space-times, a unified theory of gravitation and electromagnetic force was established first time by the mathematician Kaluza. Also in the year 1926 Oskar Klein, Swedish physicist, suggested the unification law of the gravitational force and the electromagnetic force by using the fifth dimension. This theory is known as Kaluza-Klein theory. Later on, it was established that their approaches were to some extent erroneous, but this theory provides a basis to the researchers for further investigation over the last few decades. Later on Einstein(1927) showed that in general relativity, the Kaluza's idea gives a rational foundation for Maxwell's electromagnetic equations and combines them with gravitational equations to a formal whole.

In this thesis, works are concerned to the four and five dimensional space time. So, the works done by numerous authors on five dimensional space times has also been highlighted.

Rahaman et al.(2003) discussed a five-dimensional spherically symmetric line element in presence of a homogeneous perfect fluid in Lyra geometry and obtained a model of cosmology for vacuum energy type Universe together with matter filled Universe for dust case, Zeldovich fluid and stiff fluid.

Studying five-dimensional LRS Bianchi type-I metric in presence of bulk viscous fluid, Mohanty et al.(2007) constructed a higher dimensional string model of cosmology in Lyra Geometry for displacement field(time dependent) and constant coefficient of bulk viscosity. This model had no initial singularity.

In the Saez and Ballester's scalar tensor theory of gravitation, Mohanty et al.(2007a) demonstrated that a five-dimensional LRS Bianchi type-I models do not sustain for geometric and Takabayasi strings, whereas Barotropic string $\rho = \rho(\lambda)$ survives and degenerates string with $\rho + \lambda = 0$.

Considering five-dimensional plane symmetric metric, Mohanty et al.(2009) attempted to obtain a string cosmological model Universe both in Riemannian geometry and in Lyra geometry. But they had observed that, in both the theories, the string cosmological models were not survived. Accordingly, they had formulated the vacuum models and discussed their properties.

In five-dimensional space-time, K.S. Adhav et al.(2010a, 2011) investigated the Bianchi types-I and V universes with a binary mixture of dark energy and perfect fluid. The ideal fluid obeys the EoS, $p = \gamma\rho$ with $\gamma \epsilon [0,1]$ and dark energy is classified as either the chaplygin gas or the quintessence.

Also B. Mishra & S.K. Biswal(2014) investigated a self consistent system of Bianchi-VI_o universes with a binary mixture of dark energy and perfect fluid in five dimensional space-time. The model predicted that the rate of expansion of Universe would increase with time.

A five dimensional LRS Bianchi type-V string cosmological model has been investigated by D. Trivedi & A.K. Bhabor(2021) within the framework of Brans-Dicke scalar-tensor theory of gravitation. Determinate solutions of field equations have been found using trace free energy momentum tensor, Berman's(1983) special law of variation and relation between shear scalar & scalar expansion. The model represents expanding, non-rotating and anisotropic nature at the early stage of the universe. The average anisotropic parameter and shear scalar seem to have vanished at some point so that model cannot stay anisotropic throughout the progression of universe and become shear free.

Various authors like X.X. Wang(1991), R. Venkateswarlu & K. Pavan Kumar(2004), G.S. Rathore & K. Mandawat(2010), G.C. Samanta & S. Debata (2012), K. Pawar et al.(2013), M.R. Mollah et al.(2015), Ladke et al.(2016), R.K. Jumale et al.(2016), D.R.K. Reddy & K.D. Raju(2019), D.R.K. Reddy & G. Ramesh(2019), J. Daimary & R.R Baruah(2021) studied five dimensional (Higher dimensional) cosmological models in general relativity and modified theories of relativity in different contexts.

1.9 PRINCIPLES AND LAWS

1.9.1 Cosmological Principles

Cosmological principles were developed by the Astronomer Edward Arthur Milne. There are two basic assumptions employed in constructing cosmological models of the Universe. These are

(i) The Universe is homogenized and isotropic, that is, it's the same everyplace on cosmological scale and therefore the same in each direction seen from any location.

(ii) The same laws of physics, that are valid on the earth, are valid for the complete Universe and for all time.

1.9.2 Hubble's law

In 1929, the Astronomer Edwin Hubble discovered that distant galaxies are receding from us with a speed v that is directly proportional to their distances. That means,

Recessional velocity ∝ Distance

∴ Recessional velocity =H. Distance

$$v = Hd \qquad (1.10)$$

That is understood as Hubble's law, wherever d is the right distance of the galaxy from us in MPc. (Mega Parsecs) and H is the constant of proportion referred as Hubble constant measure in km/sec/MPc (Mega Parsecs), v is the observed velocity (in km/sec) of the galaxy from earth. Hubble's law shows that the Universe is expanding by showing that how the distant galaxies are moving away from us (earth), as evidenced by their red shifts.

1.10 COSMOLOGICAL PARAMETERS

The cosmological parameters describe the physical state of the Universe. There are many cosmological parameters used in cosmology. Some of them are discussed below.

1.10.1 Scale factor of the Universe

In cosmology, the scale factor of the Universe is a dimensionless parameter. It is a function of cosmic time t and denoted by $r(t)$. It describes how the size of the Universe is changing with respect to its size at the present time. It is the ratio of the proper distance between two heavenly objects at some time t to the proper distance between the two heavenly objects at the present time t_0. Thus, if $d(t)$ is the proper distance between two heavenly objects at some time t and $d(0)$ is their distance at the present time t_0 then the scale factor of the Universe is given by

$$r(t) = \frac{d(t)}{d(0)} \qquad (1.11)$$

Again, if z is the red-shift of a light ray originally emitted by a distant heavenly object at the present time, then

$$r(t) = \frac{1}{1+z} \qquad \text{[Mukhanov(2005)]} \qquad (1.12)$$

The scale factors for different eras are obtained by solving the Friedmann equations in the Friedmann–Lemaître–Robertson–Walker metric. They are

- $r(t) \propto t^{\frac{1}{2}}$, for radiation dominated era.
- $r(t) \propto t^{\frac{2}{3}}$, for matter dominated era.
- $r(t) \propto \exp(Ht)$, for dark energy dominated era, where $H = \sqrt{\frac{\Lambda}{3}}$.

1.10.2 Hubble parameter

From Hubble's law, we have

$$v = Hd \qquad (1.13)$$

Where v is the recession velocity of a galaxy at d km away from us and H is Hubble constant or Hubble parameter. The Hubble parameter H is a function of the cosmic time t. Again, the Hubble parameter H can be written as

$$H = \frac{\dot{r}}{r} = \frac{\dot{d}}{d} \qquad (1.14)$$

The Hubble parameter is used to explain the expansion of the Universe.

1.10.3 Expansion scalar

The expansion scalar measures the relative rate of expansion or contraction of the Universe. It is denoted by θ and is defined by

$$\theta = 3\mathrm{H} = 3\frac{\dot{\mathrm{r}}}{\mathrm{r}} \qquad (1.15)$$

1.10.4 Deceleration parameter

In relativity and cosmology, the dimensionless parameter that is used to measure the acceleration of the expansion of our Universe is known as deceleration parameter, which is generally denoted by q and is defined by

$$q = -\frac{r\ddot{r}}{\dot{r}^2} \tag{1.16}$$

Here, the dots indicate the order of derivatives with respect to cosmic time.

The expansion is accelerating if q is negative or $\ddot{r} > 0$ and the expansion is decelerating if q is positive or $\ddot{r} < 0$.

Again from the Hubble parameter $H = \frac{\dot{r}}{r}$, we have

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{1}{H}\right) = -\frac{\dot{H}}{H^2}$$

$$= \frac{\dot{r}^2 - r\ddot{r}}{\dot{r}^2}$$

$$= 1 - \frac{r\ddot{r}}{\dot{r}^2}$$

Hence, $-\frac{r\ddot{r}}{\dot{r}^2} = \frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{1}{H}\right) - 1$

Therefore in terms of H, q can be expressed as

$$q = \frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{1}{H}\right) - 1 = -\frac{\dot{H}}{H^2} - 1 \tag{1.17}$$

The Universe would exhibit decelerating expansion if $q > 0$, an expansion with constant rate if $q = 0$, accelerating power-law expansion if $-1 < q < 0$, exponential expansion if $q = -1$ (de Sitter expansion) and super-exponential expansion if $q < -1$. Recent observations of type Ia supernovae show that the Universe is currently accelerating and q lies somewhere in the range $-1 < q \leq 0$

The team Riess et al.(1998) was the first to suggest about the accelerating Universe. The Hubble's parameter H should be decreasing so that the expansion of space is always slowing.

From the recent literatures and observational findings, it has been established that our Universe is accelerating at the present epoch instead of slowing down as predicted by the Big-Bang theory [Silk(1989)]. Dark energy is thought to dominate the Universe with positive energy density and negative pressure, causing sufficient acceleration in the Universe's late time evolution.

1.10.5 Anisotropy Parameter

The anisotropy parameter for four dimensional space-time is denoted by Δ or A_m and is defined by

$$\Delta = A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{1.18}$$

Where H_i (i = 1, 2, 3) are directional Hubble parameters in x, y, z directions respectively.

If $\Delta = A_m = 0$, then the Universe becomes isotropic otherwise it is anisotropic.

1.10.6 Shear Scalar

The shear scalar is represent by the symbol sigma (σ) and is defined as

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{1}{2}\left[\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right] = \frac{3}{2}\Delta H^2 = \frac{1}{6}\Delta\theta^2 \tag{1.19}$$

If $\sigma = 0$, then the Universe becomes isotropic otherwise it is anisotropic.

1.11 BASIC TERMINOLOGIES

The word 'Terminology' is a universal word for the group of particular words related to a fixed area and their studies. In this section, we have studied some specific terms that are direct related to the topic of the thesis.

1.11.1 Viscous Fluid and Viscosity

Viscosity is a property of a fluid due to which it offers some resistance to sliding motion of a particle pass another particle. It is a measure of the fluid's resistance for flow. The viscosity is also known as a fluid's internal friction. Almost all known fluids have this property. A fluid is said to be perfect or non-viscous if it has no viscosity, that is, it has no resistance to flow. Again, if a fluid has some viscosity, that is, some resistance to flow, it is said to be real or viscous.

The energy momentum tensor for a cloud of string with containing viscous fluid is defined as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta(g_{ij} + u_i u_j) \tag{1.20}$$

Here θ is the expansion scalar and ξ is the coefficient of bulk viscosity

Bulk viscosity has been widely discussed in the literature as a possible mechanism of galaxy formation in the evolution of the Universe and it could occur in a variety circumstances. Padmanabhan & Chitre(1987) discussed how the bulk viscosity in general relativistic FRW models may lead to inflationary-like solutions. According to Johri & Sudharsan(1989), the bulk viscosity causes inflationary solutions in Brans-Dicke theory. Many researchers have been drawn to the study of viscous mechanisms in cosmology because of their importance in describing the high entropy of current Universe [Weinberg(1971, 1972)]. Bulk viscosity in the grand unified theory (phase transition) could lead to inflationary cosmology, which is used to fill several important gaps in standard big bang cosmology. Murphy(1973) demonstrated that the introduction of bulk viscosity can prevent the Big-Bang singularity.

Heller & Suszycki(1974) have studied dust model of FRW Universe with bulk viscosity and shown that initial singularity may be avoided for suitable conditions.

Banerjee et al.(1985) investigated the distribution of Bianchi type-I cosmological viscous fluids in Einstein's theory. They demonstrated how shear and fluid density changed over time.

Johri & Sudharsan(1988) studied the impacts of bulk viscosity in the course of evolution of Friedmann models and discovered that the involvement of a small amount of time independent bulk viscosity will play a crucial part in driving the current Universe into a steady state Universe.

Beesham(1993) considered non flat variable-Lambda cosmological models with bulk viscosity. He obtained its solutions and elucidated their relationship with bulk viscosity solutions. He also derived exponentially expanding solutions and discussed the stability of the models.

Ibotombi Singh & Gokulchandra Singh(1998) established the relativistic cosmological field equations and its solutions for a Robertson-Walker house time interacting with viscous fluid and large scalar field. They additionally studied the character of the scalar field and the viscous fluid and located that the solutions obtained are realistic just for a closed Universe.

Dutta Choudhury & Sil(2006) investigated the evolution of a homogeneous and isotropic Universe with flat geometry stuffed with a viscous fluid in the appearance of a variable cosmological Λ and obtained a non-singular solution of a variable deceleration parameter that reduces to Murphy's solution in the "no Λ limit" and Vishwakarma's solution in the "no viscosity limit".

Mostafapoor & Gron(2011) investigated the impact of bulk viscous fluid in the evolution of the Universe by studying flat Λ cold dark matter models via bulk viscosity. They solved a few cases of bulk viscosity using the dynamical equations for such models. They also obtained the differential equations for the dark matter's energy density and the Hubble parameter.

C.P. Singh(2012) studied the dynamical consequence of the bulk viscosity and particle creation for the early evolution of the Friedmann-Robertson-Walker model in case open thermodynamical systems and thought of bulk viscosity and Particle creation as separate irreversible processes. He obtained the exact solutions of the Einstein field equations by applying the 'gamma-law' equation of state $p = (\gamma - 1)\rho$, wherever the adiabatic parameter γ varies with scale factor of the metric and thought of the cosmological model to review the early parts of the evolution of the Universe because it goes from an inflationary phase to a radiation dominated era within the presence of bulk viscosity and particle creation.

L. Poonia & S. Sharma(2021) studied about bulk viscous inflationary model with flat potential under framework of LRS Bianchi type II metric in general relativity where they obtained that the bulk viscosity coefficient leads to cosmic inflation in present scenario.

N.P. Gaikwad et al.(2021) have deduced that bulk viscous Bianchi type I barotropic fluid cosmological model with varying Λ and functional relation on Hubble parameter by solving the field equations bimetric theory of gravitation. The cosmological term Λ does not appear at the beginning of the universe but finally it remains constant at late epoch of time.

In modified f(Q) gravity, R. Solanki et al.(2021) explored the role of bulk viscosity in studying the accelerated expansion of the universe. In this theory, the gravitational action has the form f (Q), where Q is the non-metricity scalar. Out of two sets of cases arose on bulk viscosity parameters ξ0, ξ1, ξ2 and model parameter α, one limiting condition supports the current structure of cosmic acceleration with transition of a phase and refers a Big-Bang origin of the universe.

Various authors like Mohanty & Pradhan(1990, 1991), Mohanty & Pattanaik(1991), Reddy & Rao(2001), Wang(2006), Singh et al.(2007), Pradhan et al.(2007), Khadekar et al.(2007), R. Bali et al.(2008), R.P. Singh & L. Yadav(2009), R. Bali & Swati Singh(2014), R. Venkateswarlu & J. Satish(2014a), Soni & Shrimali(2015), Halife Çağlar & Sezgin Aygün(2016), R.K. Dubey et al.(2017), K. R. Borgade et al.(2021), G. Gadbail et al.(2021), L.K. Tiwari & A. Kumar(2021), S. Jokweni et al.(2021) studied bulk viscous fluid cosmological models in general relativity and modified theories of relativity in different contexts.

1.11.2 Perfect Fluid and Energy-Momentum Tensor

A perfect fluid is a frictionless, homogeneous and incompressible fluid that cannot sustain tangential stress (action) in the form of shear, but the normal force acts between adjacent layers of fluid. Whether the fluid is at rest or moving, the pressure at every point is equal in all directions. It can be completely described by its rest frame energy stresses, viscosity and heat conduction. The energy-momentum tensors describing matter is given by

$$T_{ij} = \rho u^i u^j + S^{ij} \tag{1.21}$$

Here S^{ij} is the stress tensor where the speed of the light $C = 1$. If the matter contains perfect fluid, the stress tensor can be expressed as

$$S^{ij} = p(u^i u^j - g^{ij}) \tag{1.22}$$

Where p is the isotropic pressure.

Thus the energy-momentum tensors becomes

$$T_{ij} = (\rho + p)u^i u^j - pg^{ij} \tag{1.23}$$

The isotropic pressure p is the only stress they can sustain. Except in the early Universe, it is generally agreed that the pressure of the sources can be ignored. Dust is the technical term for a perfect fluid with zero pressure that is still on the substratum because any random motion would be a source of pressure. However, uniform radiation is thought to have predominated in the early Universe. There is pressure in this and its equation of state is

$$p = \frac{\rho}{3} \tag{1.24}$$

A few relevant works done by different authors connected to the Bianchi type metric in general relativity interacting with prefect fluid has been discussed.

Interacting with perfect fluid, R.K. Tiwari (2010) studied the Einstein's field equations for the cosmological constants and variable gravitational for Bianchi type-I in which the cosmological term is considered to be proportional to the R^(-m), R being scale factor of the Universe. This model provides a unified illustration of the evolution of the Universe beginning with the early decelerating Universe and ending with the current accelerating Universe.

Also, R.K. Tiwari(2011) studied the Einstein's field equations for the gravitational and cosmological constants in presence of perfect fluid in a Robertson-Walker Universe by considering the cosmological term to be reciprocally proportional to the scale factor of Universe. He additionally presented varied solutions of the field equation and analyzed the physical significance of the models.

Zeyauddin et al.(2020) obtained an exact solution for scale covariant theory in perfect fluid medium with Bianchi type-V as the line element. The model shows the transition from early decelerating to present accelerating Universe and the model represents a Big-Bang singularity.

In the context of self–creation theory of gravitation, P. Advani(2021) presented the Bianchi Type-V barotropic perfect fluid cosmology in Lyra geometry. The model begins with a big bang at $T = 0$, and the model's expansion decreases as time passes, with a point type singularity.

Various authors like J.P. Singh et al.(2007), R. Bali & U.K. Pareek(2008), A. Pradhan et al.(2011), P.V. Tretyakov(2021) studied perfect fluid cosmological models in general relativity and modified theories of relativity in different contexts.

1.12 SOME CANDIDATES OF DARK ENERGY

There are lots of candidates for the dark energy proposed by the researchers to describe the nature of the dark energy that represent the expansion history of the Universe. Some of them have discussed below.

1.12.1 Cosmological Constant

The cosmological constant, denoted by lambda Λ in cosmology, represents the vacuum energy or energy density of space. It was firstly introduced by Albert Einstein in 1917 in his field equations of the general relativity to describe 'Static Universe', because at that time it was believed that the Universe was static. But, in 1929 when Edwin Hubble discovered that the Universe is expanding, then Einstein remarked that the inclusion of the cosmological term in the field equations was the biggest mistake of his life. During 1930-1990, most of the cosmologists proposed that the value of Λ is zero. It became the main point of discussion for the cosmologists. In 1998 it has been confirmed from the observations of distant SNe Ia that the Universe is expanding with acceleration. As a result, Λ could have a nonzero positive value. So, the value of Λ may be zero, positive or negative. The cosmological constant is considered to be the simplest candidate for the dark energy which is responsible for the acceleration of the Universe, because the energy density derived from the positive cosmological constant is associated with negative pressure which is similar to the dark energy. But, due to its non-evolving nature, the cosmological constant faces some problems. Firstly, its EoS parameter is $\omega = \frac{p}{\rho} = -1$ but, most of the observations indicate that ω is close to -1 not necessarily to -1. Secondly, cosmological constant is to be fine-tuned to have the observed small value [Weinberg(1989)].

The present observed value of Λ is

$$\Lambda = 1.1056 \times 10^{-52} m^{-2}$$

$$= 2.888 \times 10^{-122} \text{ in Planck units}$$

$$= 4.33 \times 10^{-66}\ eV^2 \text{ in natural units.}$$

1.12.2 Scalar Fields

A scalar field on a region is a real valued function or complex valued function defined at each point of the region. This region may be a subspace of the Euclidean or Minkowski space. The scalar fields are widely used in the particle physics and in the string theory. These can be formulated in such a way that they can act as a candidate for the dark energy. There are many scalar field models have been proposed by the researchers to explain the expansion of the Universe with accelerated motion. Some of the popular scalar fields are Phantom, K-essence, Quintessence, Tachyon, Quintom, Dilaton etc.

1.13 OBJECTIVES OF THE RESEARCH WORK

(i) To study various string cosmological models with the help of Bianchi type-I, type-III, type-V cosmological models for different contexts with perfect fluid, bulk viscous fluid, quadratic EoS etc.

(ii) To study the standard model of cosmology, this is based on some assumption that includes the validity of general theory of relativity and large scale homogeneity and isotropy of our Universe.

(iii) To study the problem of some five-dimensional Bianchi type models with the perfect fluid and bulk viscous fluid.

(iv) To study some higher dimensional Bianchi type-I and type III string models in Lyra geometry.

(v) To establish a correspondence between the viscous fluid model and other dynamical models.

1.14 METHODOLOGY AND TOOLS

All the data used for this research work are secondary data with reference to Journals, books, magazines and Published materials in this field which are collected from the various sources like Internet, library, university or other institutes or other Research centers like The Inter-University Centre for Astronomy and Astrophysics (IUCAA), The Tata Institute of Fundamental Research (TIFR) etc. The problems are solved manually considering the different cases and compared the solutions obtained with the observational data to check our solutions are realistic one or not. Some mathematical softwares like Mathematica, Mathe type, Maple, Matlab and Reduce-algebra, C language, C++ language etc. are applied for solving some of the mathematical problems and used to draw the required graphs in this research work.

Tensor algebra, ordinary and partial differential equations, integration of special functions and the Mathematical softwares are the main tools for solving the problems in this research work. Finally, the Latex documents/MS word offices are used as preparation system for the preparation of the documents of this research work.

1.15 SUMMARY OF THE RESEARCH WORK

This PhD Thesis entitled "**A Study on Some Bianchi Type Cosmological Models in General Theory of Relativity**" consists of nine chapters as mentioned bellow:

In Chapter-1, the definition and history of cosmology, general relativity and Einstein's field equation, some topic-related principles and laws, different cosmological parameters, topic-related basic terminologies, some candidates for dark energy and so on are highlighted. In addition, the objectives, methodology, and summary of the research work are presented. Also, several authors' previously completed research works on related topics have been reviewed.

In Chapter-2, the natures of interaction of strings with bulk viscous fluid is examined by considering four dimensional Bianchi type-I cosmological models with constant deceleration parameter in general relativity. The role of bulk viscosity coefficient in the cosmological results leading accelerated expansion of the early Universe has been discussed. The model is expanding, non-shearing, anisotropic for $n \neq 1$ and isotropic for $n = 1$. The existing Universe originates with Big-Bang at $t = 0$ with volume 0 and with the passes of time, it expands with acceleration, rather the rate of expansion of the Universe slackens with increase of time. The tension density decreases faster than the particle density, indicating that the current Universe is dominated by particles.

In Chapter-3, a new solution to the field equations derived for the Bianchi type-III Universe has been investigated in Lyra geometry using Hubble's law of variation of parameter, which yields constant DP. The model begins at t = 0 with spatial volume of zero and with the passes of time it expands with acceleration until the strings vanish, leaving particles only, resulting in a particle-dominated Universe. The model is expanding, anisotropic for $m \neq 1$ the late Universe, accelerating, inflationary, non-shearing and admits an initial singularity at $t = 0$.

In Chapter-4, a Bianchi type-I model with massive string in five-dimensional space time in GR has been investigated. The isotropic string model doesn't survive in general relativity but anisotropic string model survive. Thus the model represents an anisotropic, shearing and expanding Universe that starts at the time $t = 0$ with a volume $V = 0$ and expands with acceleration after an epoch of deceleration. The model Universe satisfies the energy conditions $\rho \geq 0$ and $\rho_p \geq 0$. The deceleration parameter "q" is decelerating at the initial stage of the evolution of the Universe and then accelerates after some finite time, indicating inflation in the model.

In Chapter-5, the exact solution of five dimensional Bianchi type-III string cosmological models with bulk viscous fluid has been obtained. The model may be free from initial singularity at $t = 0$ and we have the expectation that spatial volume may increase with time supporting the accelerated expansion of the Universe. The strings may disappear during the evolution of the Universe, leaving only the particles, which describes the accelerated expansion of the Universe.

In Chapter-6, a higher-dimensional LRS Bianchi type-I string Universe with bulk viscosity in GR is presented. The model is anisotropic, expanding and decelerating at beginning, then accelerating in the late Universe, resulting the inflationary model Universe.

In Chapter-7, a higher-dimensional Bianchi type-V cosmological model with quadratic equation of state(EoS) $p = \alpha\rho^2 - \rho$, interacting with perfect fluid in general relativity has been obtained, where $\alpha \neq 0$ is arbitrary constant. The model developed in this chapter is expanding with accelerated motion. Throughout its evolution, the model is isotropic, non-sharing and free of the initial singularity. At time $t = 0$, the energy density has a maximum value and at time $t \to \infty$, the energy density decreases to a finite constant value. Isotropic pressure (p) is a negative quantity that decreases as time t passes.

In Chapter-8, by considering bulk viscosity as (i) constant quantity and (ii) functions of cosmic time, a five dimensional Bianchi type-I model in the context of general theory of relativity has been investigated. The model represents an exponentially expanding, matter-dominated and accelerating Universe that starts with volume 0 and stops with infinite volume. The model has an initial singularity and will eventually approach the de-Sitter phase ($q = -1$). The model is anisotropic one and shearing throughout its evolution for $n \neq 1$ but approaches to small isotropy, whenever $n = 1$.

In Chapter-9, findings and Suggestions are included**.**

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