CHAPTER-2

BULK VISCOUS FLUID BIANCHI TYPE-I STRING COSMOLOGICAL MODEL WITH NEGATIVE CONSTANT DECELERATION PARAMETER

The work presented in this chapter has been published in "*Advances in Mathematics: Scientific Journal*", ISSN: 1857-8365 (printed); 1857-8438 (electronic) Scopus indexed journal (UGC care listed), Volume-9, Issue-7, pp. 4907–4916, (2020)

Chapter-2

Bulk Viscous Fluid Bianchi Type-I String Cosmological Model with Negative Constant Deceleration Parameter

2.1 INTRODUCTION

It is a challenging problem for the researcher to determine the exact physical situation at the very early stages of the formation of our Universe. String cosmological models are studied widely in recent times due to their major role in the study of the evolution of Universe in early stages after the Big-Bang explosion. According to the grand unified theories [Everett(1981), Vilenkin(1981)], those strings arose during the transition of phases when the temperature went down beneath some critical temperature soon after the explosion of Big-Bang. The relativistic treatment of string was generally started by Letelier(1979) and Stachel(1980). Letelier solved the Einstein's field equation and obtained the solutions for a cloud of strings with the plane, spherical, and cylindrical symmetry. He also solved Einstein's field equation for the cloud of massive string in the year 1983, and constructed the cosmological models in the Bianchi type-I space-time as well as Kantowski Sachs space-time.

The bulk viscosity plays a great role in the evolution of the Universe in early stage. It could arise in many circumstances and could lead to an effective mechanism of galaxy formation. The magnitude of the bulk viscous stress with respect to the expansion can be determined by means of the coefficients of bulk viscosity. The homogeneous and anisotropic Bianchi type-I cosmological models are considered to understand the evolution in the early stages of the Universe. Due to viscosity, different pictures of the Universe may appear in early cosmological evolution. Several authors used viscous effects in general relativity in both isotropic and anisotropic cosmological model universes to find exact solutions to field equations. Misner (1967, 1968) investigated the effects of bulk viscous fluid on cosmological evolution of Universe. Nightingale (1973) discovered the importance of viscosity in cosmology during the early stages of the evolution of Universe. In the presence of bulk viscosity, Banerjee et al. (1990)

investigated a Bianchi type-I cosmological model. In the presence of a bulk viscous fluid, X. X. Wang (2005) investigated Bianchi type-III cosmological models with string within the framework of general relativity. Bali and Pradhan(2007), Kandalkar et al.(2011) Kandalkar et al.(2012), Varun Humad et al.(2016), Rao et al.(2008), Singh(2013), Tripathi et al.(2017), Pradhan & Jaiswal(2018), Dubey et al.(2018), Singh & Daimary(2019), are some of the most well-known authors who have built various string cosmologies in Bianchi models with and without bulk viscosity in various space-times.

In this chapter, we look at cosmological models with strings in Bianchi type-I space-time with a negative constant deceleration parameter(q) in general relativity and a viscous fluid. In the first section of this chapter a brief introduction of Bianchi type string cosmological models has been discussed. The field equations in general relativity are derived using a Bianchi type-I metric as presented in the second section. Section 2.3 determines the solutions to the survival field equations. Physical and geometrical parameters are determined in section 2.4. The discussion of the results is covered in section 2.5, and the final section contains the conclusions.

2.2 METRIC AND FIELD EQUATIONS

A Bianchi type-I metric is considered in the form of

$$ds^2 = -dt^2 + a^2(dx^2 + dy^2) + b^2 dz^2 \tag{2.1}$$

Where a, and b are the functions of ‘t’ alone.

The Einstein field equations with $8\pi G = 1, C = 1$ in general relativity is given by

$$R_{ij} - \frac{1}{2} g_{ij} R = -T_{ij} \tag{2.2}$$

The energy-momentum tensor for a cloud of string interacting with bulk viscous fluid is taken as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta(g_{ij} + u_i u_j) \tag{2.3}$$

Where, $\lambda = \rho - \rho_p$ is the tension density, the ρ is the energy density, ρ_p is the particle density. Also u^i is the four velocity vector of particles and x^i is the unit space like vector representing the direction of the string given by

$$u^i = (0,0,0,1) \text{ and } x^i = (\frac{1}{a},0,0,0) \tag{2.4}$$

$$u_i u^i = -1 = -x_i x^i \text{ and } \quad u_i x^i = 0 \tag{2.5}$$

The scale factor of the Universe is

$$r(t) = (a^2 b)^{\frac{1}{3}} \tag{2.6}$$

The volume, expansion scalar, Hubble parameter, shear and the mean anisotropy parameter are respectively given by

$$V = a^2 b = r^3 \tag{2.7}$$

$$\theta = u^i_{,i} = 2\frac{\dot{a}}{a} + \frac{\dot{b}}{b} \tag{2.8}$$

$$H = \frac{1}{3}\left(2\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right) \tag{2.9}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left[2\left(\frac{\dot{a}}{a}\right)^2 + \left(\frac{\dot{b}}{b}\right)^2\right] - \frac{\theta^2}{6} \tag{2.10}$$

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{2.11}$$

Using the equations (2.3)-(2.5) in the field equation (2.2) for metric (2.1) yields

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} = \xi\theta \tag{2.12}$$

$$2\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}=\lambda+\xi\theta \tag{2.13}$$

$$\frac{\dot{a}^2}{a^2}+\frac{\dot{a}\dot{b}}{ab}=\rho \tag{2.14}$$

Here, the dots denote the order of differentiation with respect to time `t'.

2.3 SOLUTIONS OF THE FIELD EQUATIONS

There are 3 highly nonlinear independent equations (2.12)-(2.14) with five unknown variables a, b, λ, ρ and ξ. So, to find determinate solutions of the field equations, we must use two physically plausible conditions. So, the following two physically plausible conditions are considered here.

The shear scalar and the scalar expansion are proportional to one another, ($\sigma \propto \theta$), resulting in the equation.

$$a=b^n \tag{2.15}$$

Where, $n \neq 0$ is a constant.

The above assumption is based on observations of the velocity and red-shift relationship for an extragalactic source, which predicted that the Hubble expansion is 30% isotropic, as supported by the works of Thorne(1967), Kantowski & Sachs(1966), Kristian & Sachs(1966), Collins et al.(1980).

Furthermore, Berman's (1983) suggestion regarding the variation of Hubble's parameter H gives us a model Universe that expands with a constant deceleration parameter. Assume that the deceleration parameter is a negative constant for the determinate solution-

$$q=-\frac{r\ddot{r}}{\dot{r}^2}=(constant) \tag{2.16}$$

It is recognized that when q is negative, the model Universe expands with acceleration, whereas when q is positive, the model Universe contracts (decelerates). Despite the fact that recent observations such as CMBR and SNe Ia suggested a negative value of q, it is remarkable that they are unable to deny the Universe's slowing expansion (positive q).

Solving (2.16), we get

$$r = (Lt + M)^{\frac{1}{1+q}}, \ -1 < q < 0 \tag{2.17}$$

Where, L and M are constants of integration.

Using (2.6), (2.15) and 2.17) we get

$$a = (Lt + M)^{\frac{3n}{(1+q)(2n+1)}} \tag{2.18}$$

$$b = (Lt + M)^{\frac{3}{(1+q)(2n+1)}} \tag{2.19}$$

With the suitable choice of coordinates and constant, we can take $L = 1$, $M = 0$ and then

$$a = t^{\frac{3n}{(1+q)(2n+1)}}, \ b = t^{\frac{3}{(1+q)(2n+1)}} \tag{2.20}$$

The geometry of the model is given by the metric

$$ds^2 = -dt^2 + t^{\frac{6n}{(1+q)(2n+1)}}(dx^2 + dy^2) + t^{\frac{6}{(1+q)(2n+1)}}dz^2 \tag{2.21}$$

2.4 PHYSICAL AND GEOMETRICAL PARAMETERS

The energy density ρ, tension density λ and particle density ρ_p for the model (2.21) are obtained as

$$\rho = \frac{9n(n+2)}{(1+q)^2(2n+1)^2t^2} \tag{2.22}$$

$$\lambda = \frac{3(2-q)(n-1)}{(1+q)^2(2n+1)t^2} \tag{2.23}$$

$$\rho_p = \frac{3[q(2n^2-n-1)-(n^2-8n-2)]}{(1+q)^2(2n+1)^2t^2} \tag{2.24}$$

The spatial volume V of the model is

$$V = t^{\frac{3}{1+q}} \tag{2.25}$$

The scalar expansion θ is

$$\theta = \frac{3}{(1+q)t} \tag{2.26}$$

The Hubble Parameter is given by

$$H = \frac{1}{(1+q)t} \tag{2.27}$$

The bulk viscosity is given by

$$\xi = \frac{-q(2n^2+3n+1)+2}{(1+q)(2n+1)^2t} \tag{2.28}$$

The shear scalar is obtained as

$$\sigma = \frac{\sqrt{3}(n-1)}{(1+q)(2n+1)t} \tag{2.29}$$

The mean anisotropy parameter Δ is

$$\Delta = \frac{2(n-1)^2}{(2n+1)^2} = \text{Constant} \tag{2.30}$$

2.5 PHYSICAL INTERPRETATIONS

The model given by equation (2.21) represents a Bianchi type-I cosmological model Universe with string in the presence of bulk viscosity in GR with constant DP ($q = constant$). The variation of parameters with time(Gyr) for the model are shown below by taking $n = 2, q = -0.5$

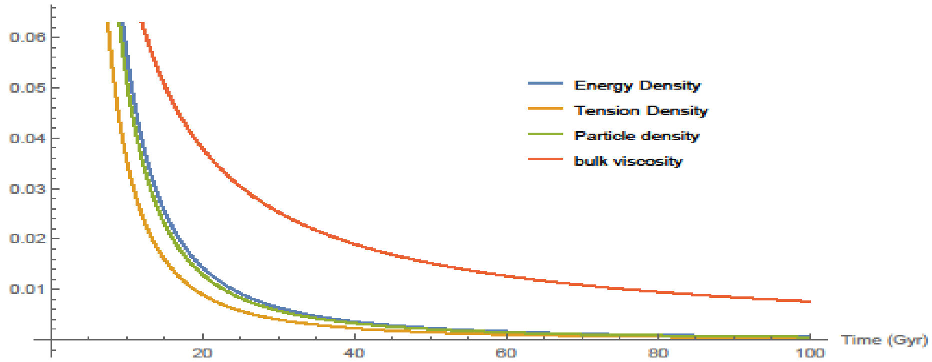

Figure 2.1.Variation of Energy density, Tension density, Particle density & bulk Viscosity Vs.Time.

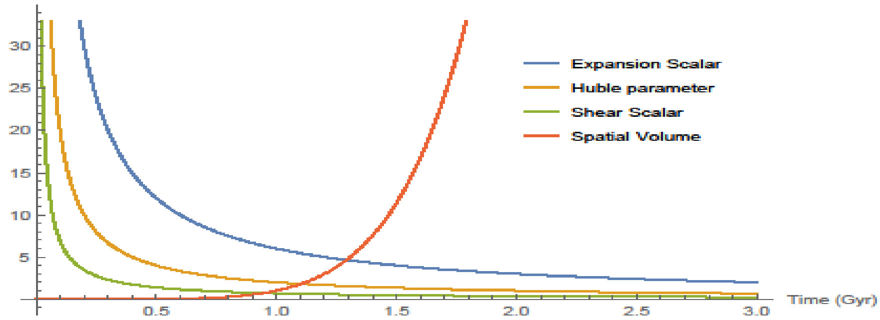

Figure 2.2.Variation of Expansion Scalar, Hubble Parameter, Shear Scalar & Spatial Volume Vs.Time.

The physical and geometrical behaviors for $(1 + q > 0)$ for the model Universe are discussed as follows:

Physical and geometrical quantities such as energy density ρ, tension density λ and particle density ρ_p are all infinite at $t = 0$. They are decreasing functions of time t, and vanish as $t \to \infty$ (Figure 2.1), indicating that the Universe begins at $t = 0$. So, at $t = 0$ the model admits initial singularity. The energy density conditions $\rho \geq 0$ and $\rho_p \geq 0$ are satisfied by this model. It is also worth noting that the string tension density decreases faster than the particle density. This demonstrates that the late Universe is dominated by particles.

The bulk viscosity, $\xi \to \infty$ when $t = 0$ and it decreases with the increasers of time and finally when $t \to \infty$, bulk viscosity ξ vanishes (Figure 2.1). So, it plays the role of accelerating the Universe.

The spatial volume V in this model is zero at initial epoch $t = 0$ and increases as time increases (Figure 2.2). It becomes infinite when $t \to \infty$ and so the model shows the expansion of the Universe with respect to time.

The expansion scalar θ and Hubble parameter H are both infinite at the time $t = 0$ and gradually decrease as time passes, eventually becoming 0 when t is infinite (Figure 2.2). As a result, the model shows that the Universe expands with the passage of time, but at a slower rate as time passes, finally the expansion stops at $t \to \infty$. Since, $\frac{dH}{dt}$ is a negative quantity, it also implies that our Universe is expanding.

From equation (2.29) and Figure 2.2 it seen that the value of the shear scalar σ is infinite at $t = 0$ and decreases with time and reaches zero in the late Universe, indicating that the Universe obtained here is shear free in late time.

From (2.30) the mean anisotropy parameter $\Delta = \text{Constant}(\neq 0)$ for $n \neq 1$ and $\Delta = 0$ for $n = 1$. Also as $t \to \infty$ the value of $\frac{\sigma^2}{\theta^2} = \text{Constant}(\neq 0)$ for $n \neq 1$ and $\frac{\sigma^2}{\theta^2} = 0$ for $n = 1$. Based on the two statements mention above, we can draw the conclusion that the Universe is anisotropic for large values of t for $n \neq 1$ throughout the evolution, but it is isotropic for $n = 1$.

2.6 CONCLUSIONS

This chapter concludes with the construction of a Bianchi type-I string cosmological model in GR with bulk viscosity and a constant DP. The parameters that are essential in the study of cosmological models have been procured and analyzed. The model is expanding, non-shearing, anisotropic for $n \neq 1$ and isotropic for $n = 1$. The existing Universe originates with Big-Bang at initial epoch $t = 0$ with zero volume and then expands with acceleration rather the rate of expansion of the Universe slackens with increase of time. The impact of the bulk viscosity coefficient in the cosmological results that lead to the early accelerated expansion of Universe has been discussed. The tension density decreases faster than the particle density, indicating that particles dominate the current Universe.

****