CHAPTER-4

HIGHER DIMENSIONAL BIANCHI TYPE-I COSMOLOGICAL MODELS WITH MASSIVE STRING IN GENERAL RELATIVITY

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Higher Dimensional Bianchi Type-I Cosmological Models with Massive String in General Relativity

4.1 INTRODUCTION

A higher-dimensional cosmological model plays a very important role in various aspects of the early cosmological evolution of the Universe. The higher dimensional model was introduced by T. Kaluza (1921) and O. Klein (1926) in an effort to unify gravity with electromagnetism. It is not possible to unify the gravitational forces of nature in typical four-dimensional space-times. As a result, higher-dimensional theory may be applicable in the early stages of evolution. The study of higher-dimensional space-time gives us an important idea about the Universe that "Universe was much smaller at the beginning of time than the Universe we see today". There may be nothing inside the equation of general relativity that can restrict the five dimensions from four dimensions. In fact, with the evolution of time the fixed four dimensions x, y, z and t expand on the other hand the extra dimensions contract to the Planckian length, which cannot be detect with the experimental facilities available in present Universe. So, many researchers have been attracted to research the cosmological problems within the area of higher-dimensional cosmic strings and have already studied different five-dimensional space-time with various Bianchi type models in various aspects. Some of them are A.H. Guth(1981), Alvarez & Gavela(1983), Marciano(1984), Chatterjee (1993), Krori et al.(1994), Mohanty et al.(2002), Rahaman et al.(2003), Singh et al.(2004), Samanta & Debata(2012), Mohanty & Samanta(2010), Mohanty & Samanta (2010a), Chakraborty & Debnath(2010), Samanta et al.(2011), Kandalker et al.(2012), Singh & Mollah (2016), Sahoo et al.(2017), Banik & Bhuyan(2017).

From the observational data, we found that our Universe is homogeneous & isotropic on a large scale, however, no physical evidence denies the chance of an anisotropic Universe. In fact, theoretical arguments are presently promoting the existence of an anisotropic phase that approaches the isotropic phase as the time passes as suggested by Charles(1968), Hinshaw et al.(2003), Page et al.(2007). Anisotropy plays a vital role in the early stages of the Universe and so studying homogeneous and anisotropic cosmological models is considered important. Generally, the Bianchi-type models are spatially homogeneous and anisotropic. As time increases, for a suitable choice of the scalars, the Universe which was initially anisotropic starts to become isotropic and finally attains isotropy after some large cosmic time, which agrees with the present-day observational data such as CMB and SNe Ia. Kaiser & Stebbins(1984), Banerjee et al.(1990), Wang(2005), Bali et al.(2007), Pawar & Deshmukh(2010), Sahoo & Mishra (2015), Singh(2013), Goswami et al.(2016), Reddy & Naidu(2007), Khadekar et al.(2005, 2007), Khadekar & Tade(2007), Yadav et al.(2011), Ladke(2014) are some of the authors who studied different string cosmological models in general relativity in different contexts in various space-times. In addition to the above mentioned authors, recently Choudhury(2017), Tripathi et al.(2017), Dubey et al.(2018), Tiwari et al. (2019), Ram & Verma(2019) investigated different string cosmological models in various space times.

The above discussions inspired us to investigate five-dimensional string cosmological models with particles attached in Bianchi type-I space-time in general relativity to investigate the various possibilities of the Bianchi type model Universe that transitions from anisotropic in early evolution to isotropic at a later time . In addition, the parameters in our model Universe are thoroughly discussed.

4.2 THE METRIC AND THE FIELD EQUATIONS

In five-dimensional space-time, a Bianchi type-I metric is considered in the form of

$$ds^2 = -dt^2 + a^2dx^2 + b^2dy^2 + c^2dz^2 + D^2dm^2 \tag{4.1}$$

Here a, b, c, & D are the metric functions of 't' alone. The extra (fifth) coordinate 'm' is assumed to be space-like in this case.

Einstein's field equations are given by

$$R_{ij} - \frac{1}{2}g_{ij}R = -8\pi T_{ij} \tag{4.2}$$

The energy-momentum tensor for a cloud string is taken as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \tag{4.3}$$

Where ρ , λ , ρ_p are the energy density of cloud of strings, tension density, particle density of the string respectively and satisfying the equation $\rho = \rho_p + \lambda$. The co-ordinates are co-moving, x_i is a unit space-like vector towards the direction of strings and u_i is the five velocity vector which satisfies the conditions given bellow.

$$u_i u^i = -1 = -x_i x^i \quad \text{and} \quad u_i x^i = 0 \tag{4.4}$$

$$u^i = (0,0,0,0,1) \ \text{and} \ x^i = (\frac{1}{a},0,0,0,0) \tag{4.5}$$

The spatial volume is given by

$$V = abcD = r^4 \tag{4.6}$$

Where r(t) is the average scale factor of the Universe.

The Scalar Expansion is given by

$$\theta = \frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c} + \frac{\dot{D}}{D} \tag{4.7}$$

Hubble Parameter is given by

$$H = \frac{1}{4}\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c} + \frac{\dot{D}}{D}\right) \tag{4.8}$$

Deceleration parameter is

$$q = -\frac{r\ddot{r}}{\dot{r}^2} \tag{4.9}$$

The shear scalar is given by

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left[\left(\frac{\dot{a}}{a}\right)^2 + \left(\frac{\dot{b}}{b}\right)^2 + \left(\frac{\dot{c}}{c}\right)^2 + \left(\frac{\dot{D}}{D}\right)^2 - \frac{\theta^2}{4}\right] \tag{4.10}$$

The mean anisotropy parameter is

$$\Delta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2, i = 1,2,3,4 \tag{4.11}$$

Where H_i represents the directional Hubble Parameters in the directions of x, y, z and m axes and are defined as $H_1 = \frac{\dot{a}}{a}$, $H_2 = \frac{\dot{b}}{b}$, $H_3 = \frac{\dot{c}}{c}$ and $H_4 = \frac{\dot{D}}{D}$.

For the metric (4.1) by using the equations (4.3)-(8.5, the field equation (4.2) yields

$$\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\ddot{D}}{D} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{b}\dot{D}}{bD} + \frac{\dot{c}\dot{D}}{cD} = 8\pi\lambda \tag{4.12}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{c}}{c} + \frac{\ddot{D}}{D} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{c}\dot{D}}{cD} = 0 \tag{4.13}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{D}}{D} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{b}\dot{D}}{bD} = 0 \tag{4.14}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{b}\dot{c}}{bc} = 0 \tag{4.15}$$

$$\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{b}\dot{D}}{bD} + \frac{\dot{c}\dot{D}}{cD} = 8\pi\rho \tag{4.16}$$

Where an over dot and double over dot denote the first and second derivatives w.r.t. cosmic time `t' respectively.

4.3 SOLUTIONS OF THE FIELD EQUATIONS

Using some simplifying assumptions, we look for physically meaningful solutions to the set of field equations (4.12)-(4.16), in this section.

4.3.1 Case-I:(Isotropic Model)

Let us consider Isotropic Model as

$$a = b = c = t^{l_1} \text{ and } \quad D = t^{l_2} \tag{4.17}$$

Where l_1 and l_2 are arbitrary constants.

By using equation (4.17) in the equations (4.12)-(4.16), we get

$$\frac{1}{t^2}(3l_1^2 + 2l_1 l_2 - 2l_1 - l_2 + l_2^2) = 8\pi\lambda \tag{4.18}$$

$$\frac{1}{t^2}(3l_1^2 + 2l_1 l_2 - 2l_1 - l_2 + l_2^2) = 0 \tag{4.19}$$

$$\frac{1}{t^2}(2l_1^2 - l_1) = 0 \tag{4.20}$$

$$\frac{1}{t^2}(3l_1^2 + 3l_1 l_2) = 8\pi\rho \tag{4.21}$$

Now from equation (4.20), we get $l_1 = 0$ or $l_1 = \frac{1}{2}$

For $l_1 = 0$ we get $l_2 = 0$ or $l_2 = 1$

And for $l_1 = \frac{1}{2}$ we get $l_2 = \frac{1}{2}$ or $l_2 = -\frac{1}{2}$

Equation (4.17) shows that the universe expands indefinitely with the increases of time t if $l_1 > 0$ and as t approaches to infinity the extra dimension 'm' contract to a Planckian length if $l_2 < 0$. So, the string cosmological model will be physically realistic only if we take $l_1 = \frac{1}{2} > 0$ and $l_2 = -\frac{1}{2} < 0$

In this case the geometry of the model is described by the metric

$$ds^2 = -dt^2 + t(dx^2 + dy^2 + dz^2) + t^{-1}dm^2 \tag{4.22}$$

Using $l_1 = \frac{1}{2}$ and $l_2 = -\frac{1}{2}$ in the equations (4.18), (4.21), the string tension density and energy density are obtained as

$$\lambda = 0 \quad (4.23)$$

$$\rho = 0 \quad (4.24)$$

And using (4.23) and (4.24), the particle density is obtained as

$$\rho_p = 0 \quad (4.25)$$

This shows that the five dimensional isotropic Bianchi type-I model in GR with strings don't survive and so it results in the five-dimensional vacuum universe in the context of general theory of relativity.

4.3.2 Case-II: (Anisotropic Model)

The models with anisotropic backgrounds are best suited to describe the initial stages of the Universe. Bianchi type-I models are the simplest models with anisotropic backgrounds. In this case let us consider

$$a = t^{k_1}, b = t^{k_2}, c = t^{k_3} \text{ and } D = t^{k_4} \quad (4.26)$$

Where k_1, k_2, k_3 and k_4 are arbitrary constants.

Now from the equations (4.12) and (4.16), by the use of (4.26), we find

$$\lambda = \frac{1}{8\pi t^2}[(k_2^2 + k_3^2 + k_4^2 + k_2k_3 + k_3k_4 + k_4k_2) - (k_2 + k_3 + k_4)] \quad (4.27)$$

$$\rho = \frac{1}{8\pi t^2}[k_1(k_2 + k_3 + k_4) + k_2k_3 + k_3k_4 + k_4k_2] \quad (4.28)$$

And particle density is

$$\rho_p = \frac{1}{8\pi t^2}[(k_1+1)(k_2+k_3+k_4)-(k_2^2+k_3^2+k_4^2)] \tag{4.29}$$

It is observed that the anisotropic 3-space x, y, z expands as $t \to \infty$ when k_1, k_2 and k_3 are all ≥ 0 and the extra dimension "m" contracts as $t \to \infty$ if $k_4 < 0$.

The geometry of the model is described by the metric

$$ds^2 = -dt^2 + t^{2k_1}dx^2 + t^{2k_2}dy^2 + t^{2k_3}dz^2 + t^{2k_4}dm^2 \tag{4.30}$$

The scalar expansion θ for model (4.30) is given by

$$\theta = \frac{l}{t}, \text{ where } l = k_1 + k_2 + k_3 + k_4 \tag{4.31}$$

The Hubble parameter is given by

$$H = \frac{l}{4t} \tag{4.32}$$

The spatial volume of the universe is obtained as

$$V = t^l \tag{4.33}$$

The shear scalar is obtained as

$$\sigma^2 = \frac{1}{2t^2}[(k_1^2+k_2^2+k_3^2+k_4^2)-\frac{1}{4}] \tag{4.34}$$

Deceleration Parameter q is given by

$$q = \frac{4}{l} - 1 \tag{4.35}$$

4.4 INTERPRETATIONS OF THE SOLUTIONS

CaseI: From the case I, it is observed that, $\rho = \lambda = \rho_p = 0$, which results in the five-dimensional vacuum universe in general relativity. So, the five-dimensional isotropic Bianchi type-I cosmic strings don't survive in general relativity.

Case II: In this case, an anisotropic Bianchi type-I string cosmological model in five-dimensional space-time in general relativity has been investigated, as given by the equation (4.30). Taking $k_1 = \frac{2}{5}$, $k_2 = \frac{3}{5}$, $k_3 = \frac{1}{4}$ and $k_4 = -\frac{1}{4}$ the variation of the parameters of model (4.30) are shown and the physical and geometrical behavior of model can be discussed as

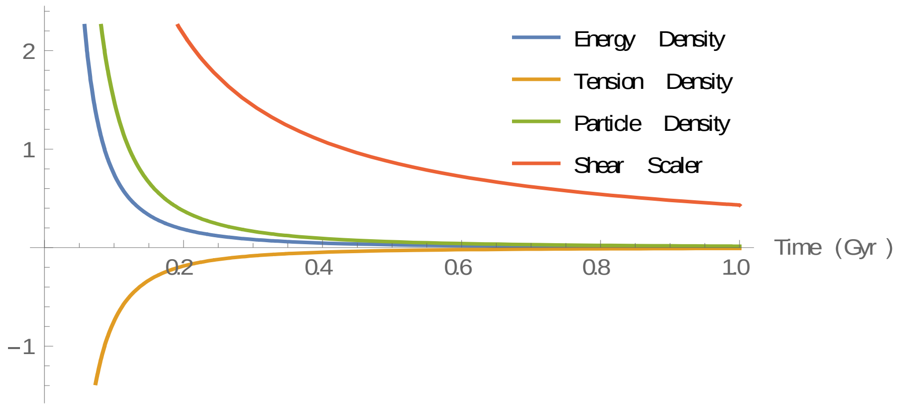

Figure.4.1.Variation of Energy density, Tension density, Particle density and Shear Scalar vs.Time.

It is observed that the anisotropic 3-space x, y, z expands as $t \to \infty$ when k_1, k_2 and k_3 are all ≥ 0 and the extra (5th) dimension "m" contracts as $t \to \infty$ if $k_4 < 0$.

At initial epoch, i.e., $t = 0$, the energy density $\rho \to \infty$ and $\rho \to 0$ as $t \to \infty$ as shown by Figure 4.1 and it satisfies the reality condition when,

$$k_1 + k_2 k_3 + k_3 k_4 + k_4 k_2 > k_1^2$$

Also at initial epoch, i.e. $t = 0$, the string tension density $\lambda \to -\infty$ and $\lambda \to 0$ as $t \to \infty$ (Figure 4.1), which shows that at early era strings exist with negative λ.

Also from Figure 4.1, it is observed that the particle density ρ_p is infinite when $t = 0$ and as time increases it decreases and finally it becomes 0 as $t \to \infty$. It satisfies the reality condition when

$$(k_1^2 + k_2^2 + k_3^2 + k_4^2) < 1$$

The spatial volume V in this model is 0 at initial epoch $t = 0$ and it increases w.r.t. time t which shows that our model universe is expanding with the evolution of time, which is clearly shown in Figure 4.2.

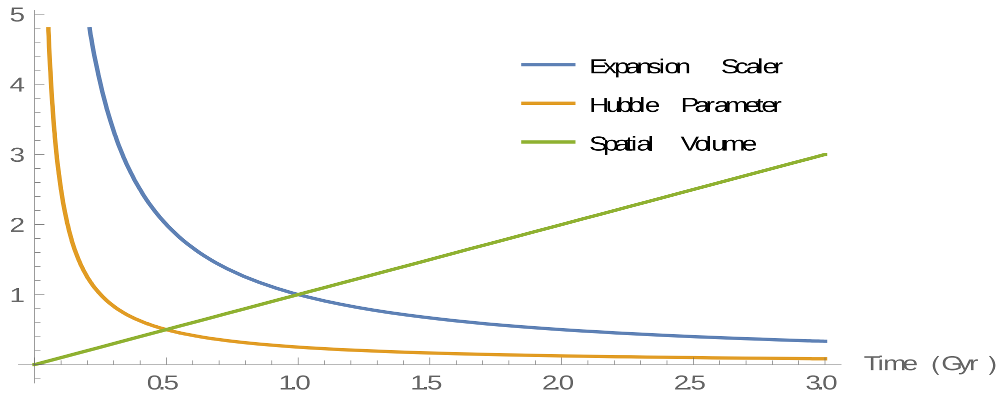

Figure.4.2.Variation of Expansion Scalar, Hubble Parameter and Spatial Volume vs.Time.

The expansion scalar $\theta \to \infty$ at initial epoch $t = 0$ and as the time progresses gradually it decreases and finally it becomes 0 when $t \to \infty$ (as shown in the Figure 4.2). As a result, the model shows that the universe expands with the passage of time, but at a slower rate as time passes, eventually coming to a halt at $t \to \infty$.

It is observed that $q > 0$ when $l < 4$ which implies that our model Universe decelerates for an instant. Again $q < 0$ when $l > 4$, which implies that our model Universe accelerates in the standard way which is in accordance with the present-day observational scenario of accelerating Universe. Here, for our proposed model it may be noted that Bianchi type models represent the cosmos in its initial stage of evolution and there may be some possibilities to have an anisotropic Universe for some finite duration but the initial anisotropy of the Bianchi type-I Universe quickly fades away and the Universe turns to an isotropic one in the late Universe. However, though the Universe decelerates in the standard way for an instant, it will accelerate in finite time because of cosmic recollapse where the Universe in turns inflates “decelerates and then accelerates”[Kandalkar & Samdurkar(2015)]. The decelerating behavior of the expansion in the early stage and the accelerating behavior of the expansion of the present Universe has been indicated by many cosmological observations such as SNe

Ia, CMB, clusters of galaxies etc. which have suggested that the reason for this transition from deceleration to acceleration is because of the presence of an anti-self attraction of matter.

4.5 CONCLUSIONS

In this chapter a five dimensional Bianchi type-I string cosmological model in the context of the general theory of relativity has been investigated. A five-dimensional isotropic Bianchi type-I cosmic strings don't survive in general relativity but a five-dimensional anisotropic Bianchi type-I cosmic strings survive in general relativity, which represents an expanding Universe that starts at the time $t = 0$ with a volume $V = 0$ and expands with acceleration after an epoch of deceleration. Our model Universe satisfies the energy conditions $\rho \geq 0$ and $\rho_p \geq 0$. The model Universe can represent a stage of evolution from decelerating to accelerating. The DP "q" is decelerating at the initial stage of the evolution and then accelerates after some finite time because of the cosmic recollapse, indicating inflation in the model after an epoch of deceleration which is in accordance with the present-day observational scenario of Universe as claimed by SNe Ia [Riess et al.(1998) & Perlmutter et al.(1999)]. It is observed that our model Universe is anisotropic, shearing.

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