CHAPTER-5

HIGHER DIMENSIONAL BIANCHI TYPE-III STRING UNIVERSE WITH BULK VISCOUS FLUID AND CONSTANT DECELERATION PARAMETER (DP)

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5.1 INTRODUCTION

From the Big Bang to today, the Universe has expanded at an accelerating rate, as evidenced by observational and theoretical cosmological facts. However, no one can guarantee that the Universe will continue to expand indefinitely because no definitive conclusion about the expansion or contraction of the Universe has been reached as of yet. According to various literatures and opinions, the acceleration of current Universe may be accompanied by a deceleration. However, the precise reason for the expanding Universe is unknown to us, which has inspired all cosmologists and physicists to conduct similar research in this field. Several cosmological models were proposed in last few years by various researchers in attempt to describe the hidden reasons for the acceleration of existing Universe. The string theory is one of most important cosmological theories that investigate the unknown facts of the Universe. Comic strings are topologically stable objects that are thought to have formed during the phase transition or prior to the introduction of particles in the early Universe. Stachel(1980) and Letelier(1979, 1983) pioneered the study of strings in general relativity. The field equation of Einstein for string was studied by Letelier in 1983 with spherical symmetry, cylindrical symmetry and plane symmetry. Bianchi cosmology, which is spatially homogeneous, anisotropic is very useful in describing the large-scale behaviour of the Universe. Furthermore, according to various types of literature and findings, the anisotropic model was chosen as one of the possible models to begin the expansion of the Universe.

Bulk viscosity played a key role in the early cosmological evolution of the Universe. The mechanism of bulk viscosity in cosmology has piqued the interest of many researchers due to its significant role in describing the Universe's high entropy in the modern era. Nightingale(1973) was one of the researchers who investigated the various roles of bulk viscous fluids in cosmology. Misner(1968), Wang(2005), Bali &

Pradhan(2007), Kandalker et al.(2011), Singh(2013), Humad et al.(2016), Kandalker et al.(2012) are among the well-known researchers who have investigated various Bianchi models in the field of general relativity interacting with bulk viscosity.

A higher-dimensional cosmological model plays an important role in various aspects of the early cosmological evolution of Universe. It is not possible to unify the gravitational forces of nature in typical four-dimensional space-times. As a result, higher-dimensional theory may be applicable in the early stages of evolution. The study of higher-dimensional space-time gives us an important idea about the Universe that "Universe was much smaller at the beginning of time than the Universe we see today". There may be nothing inside the equation of general relativity that can restrict the five dimensions from four dimensions. Many researchers motivated to enter in to the theory of higher dimensions to discover the hidden phenomenon of the Universe. In fact, with the evolution of time the fixed four dimensions x, y, z and t expand on the other hand the extra dimensions contract to the Planckian length, which cannot be detect with the experimental facilities available in present Universe. So, many authors have been attracted to research the cosmological problems within the area of higher-dimensional cosmic strings. As a result, many authors have already studied different cosmological models in five-dimensional space-time with various Bianchi type models in various aspects. Some of them are Krori et al.(1994), Khadekar & Vrishali(2005), Reddy & Naidu(2007), Mohanty & Samanta(2010), Samanta et al.(2011), Ladke(2014), Sahoo & Mishra(2015), Jumale et al.(2016).

Inspired by the preceding research, this chapter investigates a higher dimensional bulk viscous cosmology with string in Bianchi type III space–time. Section 5.2 of this chapter describes the formulation of the problem. The solutions to the cosmological problems are given in Section 5.3, and some of the parameters are obtained in Section 5.4. The findings are discussed in Section 5.5. Finally, in the final Section, concluding remarks are provided.

5.2 THE METRIC AND FIELD EQUATIONS

Here Bianchi type-III metric in 5-dimension is considered as

$$ds^2 = -dt^2 + a^2dx^2 + b^2(e^{-2x}dy^2 + dz^2) + c^2dm^2 \tag{5.1}$$

Here a, b and c are the functions of `t' alone and 'm' is the extra dimensions which is space-like.

The EFE in general relativity with $8\pi G = 1$, $C = 1$ is

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \tag{5.2}$$

The energy-momentum tensor with bulk viscosity is

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta(g_{ij} + u_i u_j) \tag{5.3}$$

Here $\rho = \lambda + \rho_p$ is the energy density, λ is the tension density and ρ_p is the particle density, θ is expansion scalar and ξ is the bulk viscosity coefficient. Also u^i is the five velocity vector of particles and x^i represent the unit vector which is space-like and this represents the direction of the strings, given by.

$$u^i = (0,0,0,0,1) \text{ and } \quad x^i = (0,0,c^{-1},0,0) \tag{5.4}$$

$$u_i u^i = -1 = -x_i x^i \text{ and } \quad u_i x^i = 0 \tag{5.5}$$

If $r(t)$ is the scale factor then volume is given by

$$V = ab^2c = r^4 \tag{5.6}$$

The scalar expansion is given by

$$\theta = \frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} + \frac{\dot{c}}{c} \tag{5.7}$$

Hubble parameter is given by

$$H = \frac{\theta}{4} = \frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} + \frac{\dot{c}}{c} \tag{5.8}$$

The shear scalar is given by

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left[\left(\frac{\dot{a}}{a}\right)^2 + 2\left(\frac{\dot{b}}{b}\right)^2 + \left(\frac{\dot{c}}{c}\right)^2\right] - \frac{\theta^2}{8} \tag{5.9}$$

The anisotropy parameter is

$$\Delta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2 \tag{5.10}$$

Where $H_i\ (i = x, y, z, m)$ represents the directional Hubble factors and they are given by $H_x = \frac{\dot{a}}{a}$, $H_y = H_z = \frac{\dot{b}}{b}$ and $H_m = \frac{\dot{c}}{c}$.

Using the equations (5.1)-(5.5), we obtain

$$2\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{b}^2}{b^2} + 2\frac{\dot{b}\dot{c}}{bc} = \xi\theta \tag{5.11}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{a}\dot{c}}{ac} = \xi\theta \tag{5.12}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{a}\dot{c}}{ac} - \frac{1}{a^2} = \lambda + \xi\theta \tag{5.13}$$

$$\frac{\ddot{a}}{a} + 2\frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + 2\frac{\dot{a}\dot{b}}{ab} - \frac{1}{a^2} = \xi\theta \tag{5.14}$$

$$2\frac{\dot{a}\dot{b}}{ab} + 2\frac{\dot{b}\dot{c}}{bc} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{b}^2}{b^2} - \frac{1}{a^2} = \rho \tag{5.15}$$

$$\frac{\dot{a}}{a} - \frac{\dot{b}}{b} = 0 \tag{5.16}$$

The overhead dots here denotes the order of derivative w.r.t. time `t

5.3 SOLUTIONS OF THE MODEL

Equation (5.16) yields

$$a = hb \tag{5.17}$$

Here h is integration constant and $h \neq 0$. We can take $h = 1$ then

$$a = b \tag{5.18}$$

By the use of (5.18) in the equations (5.11)-(5.15), we get

$$2\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{b}^2}{b^2} + 2\frac{\dot{b}\dot{c}}{bc} = \xi\theta \tag{5.19}$$

$$2\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{b}^2}{b^2} + 2\frac{\dot{b}\dot{c}}{bc} - \frac{1}{b^2} = \lambda + \xi\theta \tag{5.20}$$

$$3\frac{\ddot{b}}{b} + 3\frac{\dot{b}^2}{b^2} - \frac{1}{b^2} = \xi\theta \tag{5.21}$$

$$3\frac{\dot{b}\dot{c}}{bc} + 3\frac{\dot{b}^2}{b^2} - \frac{1}{b^2} = \rho \tag{5.22}$$

The variables b, c, λ, ρ, ξ and θ are unknown in four highly nonlinear independent differential equations (5.19)-(5.22). As a result, two additional conditions must be met in order to obtain exact solutions to the equations above. In this case, we used following two physically plausible conditions:

Berman's(1983) suggestion regarding variation of Hubble's parameter H provides us a model Universe that expands with constant deceleration parameter. So for the determinate solution, let us take DP to be a negative constant-

$$q = -\frac{r\ddot{r}}{\dot{r}^2} = (constant) \tag{5.23}$$

The model Universe expands with acceleration when q is negative and contracts with deceleration when q is positive. Although recent observations such as CMBR and SNe Ia have suggested a negative value for q, they cannot deny the decelerating expansion of Universe.

The shear scalar and expansion scalar are proportional $\sigma \propto \theta$, this leads to the equation

$$b = c^n \ , \quad n \neq 0 \text{ is a constant} \tag{5.24}$$

It is based on velocity and red-shift observations for an extragalactic source, which anticipated that the Hubble expansion is 30% isotropic, which is represented by Thorne's work (1967). In specific, it can be stated that $\frac{\sigma}{H} \geq 0.30$, where σ & H stand for shear scalar & Hubble constant respectively. Collins et al. (1980) also demonstrated that if the normal to the spatially homogeneous line element is compatible to the homogeneous hyper-surface, then $\frac{\sigma}{\theta} =$ constant, where θ is expansion scalar.

Solving (5.23), we get

$$r = (kt + l)^{\frac{1}{1+q}}, \; q \neq -1 \tag{5.25}$$

Here k and l are integration constants.

Using (5.6), (5.18), (5.24) and (5.25), we get

$$a = b = (kt + l)^{\frac{4n}{(1+q)(3n+1)}}, \quad c = (kt + l)^{\frac{4}{(1+q)(3n+1)}} \tag{5.26}$$

With the suitable choice of coordinates and constant we can take $k = 1$ and $l = 0$ and then

$$a = b = t^{\frac{4n}{(1+q)(3n+1)}}, \; c = t^{\frac{4}{(1+q)(3n+1)}} \tag{5.27}$$

The metric (5.1) reduces to

$$ds^2 = t^{\frac{8n}{(1+q)(3n+1)}}(dx^2 + e^{-2x}dy^2 + dz^2) + t^{\frac{8}{(1+q)(3n+1)}}dm^2 - dt^2 \tag{5.28}$$

This gives the geometry of the metric (5.1).

5.4 PHYSICAL AND GEOMETRICAL PARAMETERS

Some of the most important physical and geometrical parameters for discussing the evolution of the Universe are obtained in this section.

ρ is obtained by using (5.27) in (5.22) as

$$\rho = \frac{48n(n+1)}{(1+q)^2(3n+1)^2t^2} - t^{-\frac{8n}{(1+q)(3n+1)}} \tag{5.29}$$

From (5.19) and (5.20) using (5.27), we obtained

$$\lambda = -t^{-\frac{8n}{(1+q)(3n+1)}} \tag{5.30}$$

From (5.29), (5.30), ρ_p can be obtained as

$$\rho_p = \frac{48n(n+1)}{(1+q)^2(3n+1)^2t^2} \tag{5.31}$$

The spatial volume, expansion scalar, Hubble parameter, bulk viscosity, shear scalar, and mean anisotropy parameter of the model are

$$V = t^{\frac{4}{1+q}} \tag{5.32}$$

$$\theta = \frac{4}{(1+q)t} \tag{5.33}$$

$$H = \frac{1}{(1+q)t} \tag{5.34}$$

$$\xi = \frac{(2n+1)(3n^2-3nq-q)+3}{(1+q)(2n+1)^2t} \tag{5.35}$$

$$\sigma = \frac{\sqrt{6}(n-1)}{(1+q)(2n+1)t} \tag{5.36}$$

$$\Delta = \frac{3(n-1)^2}{(3n+1)^2} = \text{Constant.} \tag{5.37}$$

5.5 INTERPRETATIONS OF THE RESULT

A Bianchi type-III cosmological model with string in general relativity with constant DP (q=constant) in presence of bulk viscosity in 5-D space-time is represented by the equation (5.28). The variation of parameters with time for the model are shown below by taking $n = \frac{1}{2}$ and $q = -\frac{1}{2}$.

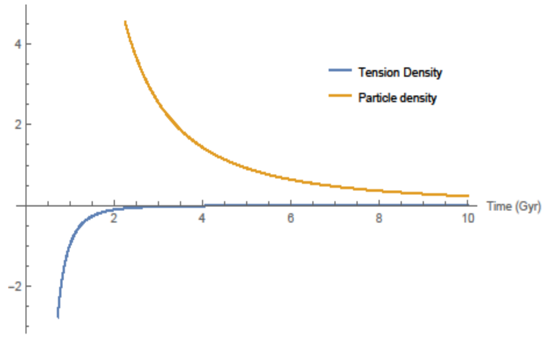

Figure 5.1.Variation of λ and ρ_p Vs. time.

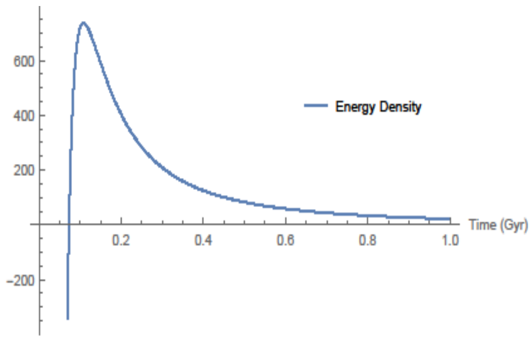

Figure 5.2.Variation of ρ Vs.time.

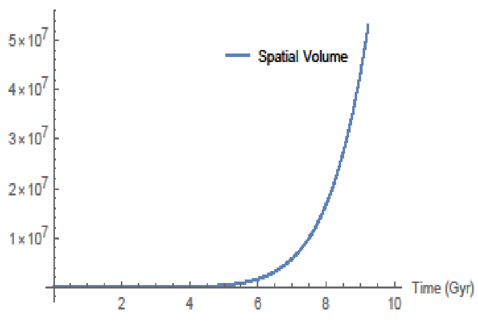

Figure-5.3. Variation of V (volume) Vs. time.

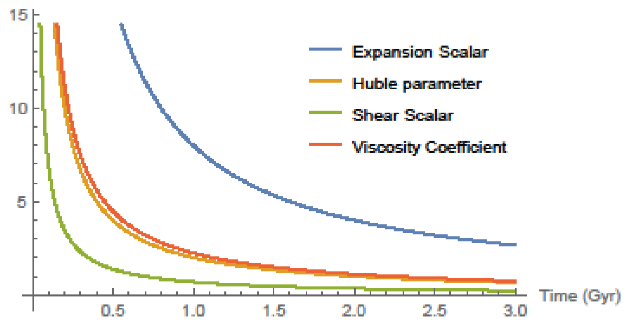

Figure-5.4. Variation of θ, H, σ, ξ Vs time

The properties of the model Universe for $-1 < q < 0$ are discussed as

From the Eqn.(5.29), it is observed that the energy density ρ is negative at the initial epoch of time and it changes sign from negative to positive after some finite time and finally becomes zero when $t \to \infty$(Figure-5.2). Figure-5.1 presents the variation of tension density with respect to time t where we have seen that the tension density λ of string is negative. It is mentioned by Letelier that λ can be <0 or >0. The phase of string disappears when $\lambda < 0$. The strong energy conditions $\rho \geq 0$, $\lambda < 0$ as given by Hawking & Ellis(1974) are satisfied for the model in the late time Universe.

From (5.31), it is noted that $\rho_p \geq 0$ for all time t and ρ_p decreases with time which is shown in the Figure-4.1. It is also seen that $\frac{\rho_p}{|\lambda|} > 1$, indicating that string tension density decreases faster than particle density. This indicates that the late Universe is dominated by particles.

The bulk viscosity $\xi \to \infty$ when $t = 0$ and it decreases with the increasers of t and finally when $t \to \infty$ bulk viscosity ξ demises (Figure-5.4). The function of the bulk viscosity is to retard the expansion of the Universe and since bulk viscosity deceases with the time so retardness also decreases which supports in the expansion in faster rate in the late time Universe. From the above discussion it can be seen that the bulk viscosity plays a great function in the evolution of the Universe.

The volume $V = 0$ at initial epoch $t = 0$ and the volume increases with the increases of the t and it become infinite when time $t \to \infty$, which is shown by the Figure-5.3. So the Universe expands with time. The size of the Universe was very small just after the big bang exploitation then the size continuously increasing till now and it will increase late time also.

At $t = 0$, the Hubble parameter H and scalar expansion θ, both are infinite and as time passes, they gradually decrease until they reach zero when t is infinite (Figure-5.4). Hence the model shows that with the increases of time the Universe expands but the expansion rate becomes slower as time increases and at $t \to \infty$, the expansion stops. And since, $\frac{dH}{dt} < 0$, this also tells us that our present Universe is in the accelerated expanding mode.

At the initial epoch, it is seen that the value of the shear scalar $\sigma \to \infty$, which is seen in equation (5.36) and Figure-5.4 and decreases as time passes until it reaches zero at the late Universe, indicating that the Universe obtained here is shear free in late time.

From (5.37) the mean anisotropy parameter $\Delta = \text{constant}(\neq 0)$ for $n \neq 1$ and $\Delta = 0$ for $n = 1$. Also as $t \to \infty$ the value of $\frac{\sigma^2}{\theta^2} = \text{constant}(\neq 0)$ for $n \neq 1$ and $\frac{\sigma^2}{\theta^2} = 0$ for $n = 1$.

It can be deduce from both the statements that this model is anisotropic for large values of t, when $n \neq 1$ but isotropic for $n = 1$.

5.6 CONCLUSIONS

In this chapter, the field equations for Bianchi type-III line element with bulk viscosity have been obtained and solved using the law of variation of Hubble parameter, which yields constant DP. The average scale factor "r" is explicitly determined by this variational law for H. As per the literature, the bulk viscosity can be treated as an applicable candidate for providing a cosmological or theoretical explanation for the early expansion of the Universe, present state and future expected expansion of the Universe. It plays a great role in the cosmological consequences. So, in this chapter we have constructed a 5D Bianchi type-III string Universe with bulk viscosity and constant DP in GR by the use of certain physically plausible conditions. The geometrical and physical characteristics that are crucial in the study of cosmological evolution are determined and discussed. The model is expanding, non-shearing, anisotropic for $n \neq 1$ throughout and isotropic for $n = 1$ in the late Universe which is also in accordance to the present day observational data made by WMAP and COBE. The present model starts with 0 volumes at initial epoch $t = 0$ and then expands with accelerated motion and the expansion rate slows down with increase of time. The tension density is negative quantity showing that the string phase disappears and present day Universe is particle dominated that agrees with the present day observational data.

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