CHAPTER-6

HIGHER DIMENSIONAL LRS BIANCHI TYPE-I STRING COSMOLOGICAL MODEL WITH BULK VISCOSITY IN GENERAL RELATIVITY

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Higher Dimensional LRS Bianchi Type-I String Cosmological Model with Bulk Viscosity in General Relativity

6.1 INTRODUCTION

Cosmologists are still interested in studying and discovering the unknown phenomena of the Universe that have yet to be observed in order to study and explore its hidden knowledge. As a result, cosmologists have paid close attention to the past, present and future evolution. In the early era, before the formation of particles, strings played a crucial role in the creation and evolution of the Universe. J. Stachel(1980) and P. S. Letelier(1983) pioneered the general relativistic study of strings, developing the classical theory of geometric strings. Because of the importance of strings in describing the early evolution of our Universe, many well-known researchers have become interested in cosmic strings in general relativity in recent years [T.W.B. Kibble(1976, 1980)]. Strings cause density perturbations in the Universe, resulting in the formation of Large Scale Structures (such as galaxies)[Ya.B. Zeldovich et al.(1974), Ya.B. Zel'dovich(1980)], which does not contradict current observational findings of the Universe and results in an anisotropy in space-time, despite the fact that the strings are unobservable in today's Universe. Geometric strings and massive strings are two types of strings that have stress energy. According to grand unified theories, symmetry breaking of the Universe occurred spontaneously during phase transitions in the very early stages of the Universe and these cosmic strings, which are a very important topological defect, arose in the early Universe as the cosmic temperature dropped below some critical point temperatures[A.E. Everett(1981), A. Vilenkin(1981,1981a)]

The bulk viscosity assumes an extraordinary part in the development of the early Universe. There are numerous events inside the development of the Universe wherein the bulk viscosity could emerge. The bulk viscosity coefficients control the size of the viscous stress in relation to the expansion. Bianchi type-I models that are spatially homogeneous and anisotropic are used to try to understand the Universe in its early stages of evolution. The various pictures of the Universe may show up at the beginning phase of the cosmological development because of the dissipative process brought about by viscosity which counteracts the cosmological breakdown (collapse). Using viscous effects in general relativity in both isotropic and anisotropic cosmological models, a few authors attempted to find specific solutions to field equations. A. Banerjee, et al.(1990), Varun Humad et al.(2016), R.K. Dubey & S.K. Srivastava(2018) are a few of them.

Many researchers have been intrigued by the possibility of space-time having more than four dimensions (extra dimensions). In general, T. Kaluza(1921) and O. Klein(1926) proposed the higher-dimensional model to unify electromagnetism and gravity. According to S.K. Banik & K. Bhuyan(2017) a higher-dimensional model can be used to demonstrate the late-time expedited expanding approach. According to G.P. Singh et al.(2004) "Investigation of higher dimensional space-time can be regarded as a task of paramount importance as the Universe might have come across a higher dimensional era during the initial epoch". The detection of time varying fundamental constants, according to W.J. Marciano(1984), can possibly show us the proof for extra dimensions.. According to E. Alvarez & M.B. Gavela(1983) and A.H. Guth(1981), extra dimensions generate huge amount of entropy which gives possible solution to atness and horizon problem. Since we live in a 4D space-time, the hidden extra dimension in 5D is almost certainly linked to the DM and DE[S. Chakraborty & U. Debnath(2010)]. Several authors have investigated various Bianchi type problems in the field of higher dimensional space-time. S. Chatterjee(1993), G. Mohanty & G.C. Samanta(2010), G.C. Samanta et al.(2011), S.P. Kandalkar et al.(2012), S.P. Kandalkar & S. Samdurkar(2015), P.K. Singh & M.R. Mollah(2016), P.S. Singh & K.P. Singh(2020), K.P. Singh & P.S. Singh(2019), C.P. Singh(2013), G.S. Khadekar & P. Vrishali(2005), G.S. Khadekar & S.D. Tade(2007), Khadekar et al.(2007), A.K. Yadav et al.(2011), B.R. Tripathi et al.(2017) are some them.

As a result of the above discussion and investigations, here we investigated the five-dimensional LRS Bianchi type-I string model in cosmology with particles attached in general relativity. The chapter is organized as follows: A brief overview of strings, bulk viscosity and their significance are discussed in section 6.1. The metric is introduced and the field equations are determined in section 6.2. The determinate solutions are obtained in section 6.3 by making some simplifying assumptions. Graphs were used to examine the geometrical and physical properties of our model Universe in section 6.4. Conclusions are presented in section 6.5.

6.2 THE METRIC AND FIELD EQUATIONS

A 5-dimensional LRS Bianchi type-I metric is considered as

$$ds^2 = -dt^2 + a^2 dx^2 + b^2 (dy^2 + dz^2) + c^2 dm^2 \tag{6.1}$$

Here a, b and c are the metric functions of cosmic time t alone and the extra coordinate "m" is taken to be space-like.

For the above metric let

$$x^1 = x\,,\; x^2 = y\,,\; x^3 = z\,,\; x^4 = m \;\text{ and }\; x^5 = t$$

In general relativity the Einstein's field equation is written as

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \tag{6.2}$$

For a cloud string the energy-momentum tensor is

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta(u_i u_j + g_{ij}) \tag{6.3}$$

Where ρ is the energy density, λ is the tension density of the string and they are related as $\rho = \lambda + \rho_p$, where ρ_p is the particle density of matter, ξ is the coefficient of viscosity and θ is the expansion scalar. The co-ordinates are co-moving, x^i is the unit space-like vector along the direction of strings and u^i is the five velocity vector which satisfies the conditions-

$$u_i u^i = -1 = -x_i x^i \quad \text{and} \quad u_i x^i = 0, \tag{6.4}$$

Here without loss of generality we can take

$$u^i = (0,0,0,0,1) \text{ and } x^i = (a^{-1},0,0,0,0), \tag{6.5}$$

The spatial volume is given by

$$V = ab^2c = r^4 \tag{6.6}$$

Where $r(t)$ is the average scale factor of the Universe.

The Scalar Expansion is given by

$$\theta = \frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} + \frac{\dot{c}}{c} \tag{6.7}$$

Hubble Parameter is given by

$$H = \frac{1}{4}\left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} + \frac{\dot{c}}{c}\right) \tag{6.8}$$

Deceleration parameter is

$$q = -\frac{r\ddot{r}}{\dot{r}^2} \tag{6.9}$$

The shear scalar is given by

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left[\left(\frac{\dot{a}}{a}\right)^2 + 2\left(\frac{\dot{b}}{b}\right)^2 + \left(\frac{\dot{c}}{c}\right)^2 - \frac{\theta^2}{4}\right] \tag{6.10}$$

And the mean anisotropy parameter is given by

$$\Delta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2 \tag{6.11}$$

Where H_i $(i = 1,2,3,4)$ represents the directional Hubble Parameters in the directions of x, y, z and m axes and are defined as $H_1 = \frac{\dot{a}}{a}$, $H_2 = H_3 = \frac{\dot{b}}{b}$ and $H_4 = \frac{\dot{c}}{c}$ for the metric (6.1).

Using the equations (6.3)-(6.5), the field equation (6.2) takes the form

$$2\frac{\ddot{b}}{b}+\frac{\ddot{c}}{c}+\frac{\dot{b}^2}{b^2}+2\frac{\dot{b}\dot{c}}{bc}=\lambda+\xi\theta \tag{6.12}$$

$$\frac{\ddot{a}}{a}+\frac{\ddot{b}}{b}+\frac{\ddot{c}}{c}+\frac{\dot{a}\dot{b}}{ab}+\frac{\dot{a}\dot{c}}{ac}+\frac{\dot{b}\dot{c}}{bc}=\xi\theta \tag{6.13}$$

$$\frac{\ddot{a}}{a}+2\frac{\ddot{b}}{b}+\frac{\dot{b}^2}{b^2}+2\frac{\dot{a}\dot{b}}{ab}=\xi\theta \tag{6.14}$$

$$2\frac{\dot{a}\dot{b}}{ab}+\frac{\dot{a}\dot{c}}{ac}+2\frac{\dot{b}\dot{c}}{bc}+\frac{\dot{b}^2}{b^2}=\rho \tag{6.15}$$

Here the overhead dots mean differentiation with time `t'.

6.3 SOLUTIONS OF THE FIELD EQUATIONS

In this part, the solutions of the four highly non-linear independent equations (6.12)-(6.15) with 6 unknown variables a, b, c, ξ, λ and ρ are solved. The following physical plausible conditions are considered for deterministic solutions:

The shear scalar and expansion scalar are considered to be proportional, which leads to

$$c=b^n, \tag{6.16}$$

Here, $n\neq 0$ is a constant.

The reason of assuming the above condition depends on observations of the velocity red-shift relation for extragalactic sources recommended that the Hubble expansion of the Universe is isotropic today to within 30% [K.S. Thorne(1967), R. Kantowski & R.K. Sachs(1966), J. Kristian & R.K. Sachs (1966)]. If H is Hubble constant and σ is the shear then the red-shift studies limit $\frac{\sigma}{H}\leq 0.30$. If θ is expansion scalar, the normal to the spatially homogeneous metric is congruence to the homogeneous hyper-surface that satisfies the condition $\frac{\sigma}{\theta}$ is constant[C. B. Collins (1980)].

From (6.13), (6.14) by using (6.15), we get

$$\frac{\dot{b}}{b}\left[\frac{\ddot{b}}{b}(n+1)\frac{\dot{b}}{b}+\frac{\dot{a}}{a}\right]=0 \qquad (6.17)$$

Using (6.17) in (6.14) and comparing with (6.15), we get n=1.

The equation (6.16) yields

$$b=c \qquad (6.18)$$

The equation (6.17) yields the following cases:

$$\text{CaseI: } \frac{\ddot{b}}{b}+2\frac{\dot{b}}{b}+\frac{\dot{a}}{a}=0, \quad \text{and} \quad \text{CaseII: } \frac{\dot{b}}{b}=0 \qquad (6.19)$$

We intend to determine the cosmological models for the above two cases separately.

6.3.1 CaseI: $\frac{\ddot{b}}{b}+2\frac{\dot{b}}{b}+\frac{\dot{a}}{a}=0$

Solving we get

$$b(t)=\left[3\left(\int\frac{K}{a(t)}dt+K_1\right)\right]^{\frac{1}{3}} \qquad (6.20)$$

Clearly, the solution are not unique because b(t) can be obtained for any given a(t). So for further studies here we consider [G. Mohanty & K.L. Mahanta (2007)].

$$\frac{\ddot{b}}{b}+2\frac{\dot{b}}{b}=-\frac{\dot{a}}{a}=k \text{ (Constant)} \qquad (6.21)$$

Now solving (6.21) and using (6.18) a, b, and c are obtained as

$$a=k_1e^{-kt} \qquad (6.22)$$

$$b=\left[3\left(\frac{k_2}{k}e^{kt}+k_3\right)\right]^{\frac{1}{3}} \qquad (6.23)$$

$$c=\left[3\left(\frac{k_2}{k}e^{kt}+k_3\right)\right]^{\frac{1}{3}} \qquad (6.24)$$

Here $k_1(\neq 0)$, k_2 and k_3 are constants of integrations.

The following metric describes the geometry of the model

$$ds^2 = -dt^2 + k_1^2 e^{-2kt} dx^2 + \left[3\left(\frac{k_2}{k} e^{kt} + k_3\right)\right]^{\frac{2}{3}} (dy^2 + dz^2 + dm^2) \tag{6.25}$$

The tension density of the string is obtained as

$$\lambda = \frac{k k_2 e^{kt}}{(\frac{k_2}{k} e^{kt} + k_3)} - k^2, \tag{6.26}$$

The energy density of the string is

$$\rho = \frac{k_2^2 e^{2kt}}{3(\frac{k_2}{k} e^{kt} + k_3)^2} - \frac{k k_2 e^{kt}}{(\frac{k_2}{k} e^{kt} + k_3)}, \tag{6.27}$$

The particle density is given by

$$\rho_p = \frac{k_2^2 e^{2kt}}{3(\frac{k_2}{k} e^{kt} + k_3)^2} - \frac{2k k_2 e^{kt}}{(\frac{k_2}{k} e^{kt} + k_3)} + k^2, \tag{6.28}$$

From equation (6.14) we get

$$\xi\theta = k^2 - \frac{k_2^2 e^{2kt}}{3(\frac{k_2}{k} e^{kt} + k_3)^2}, \tag{6.29}$$

The spatial volume is obtained as

$$V = 3k_1 e^{-kt} (\frac{k_2}{k} e^{kt} + k_3), \tag{6.30}$$

The scalar expansion is obtained as

$$\theta = \frac{k_2 e^{kt}}{(\frac{k_2}{k} e^{kt} + k_3)} - k, \tag{6.31}$$

Using (6.31) in (6.29), we get

$$\xi = \frac{-k_2^2 e^{2kt}}{3kk_3(\frac{k_2}{k} e^{kt} + k_3)^2} - \frac{k_2}{k_3} e^{kt} - k, \tag{6.32}$$

The Hubble parameter is obtained as

$$H = \frac{k_2 e^{kt}}{4(\frac{k_2}{k} e^{kt} + k_3)} - \frac{k}{4}, \tag{6.33}$$

The deceleration parameter is

$$q = -\left[\frac{4k_2 e^{kt}}{kk_3} + 1\right], \tag{6.34}$$

The shear scalar of the model is

$$\sigma^2 = \frac{1}{8}\left[3k^2 + \frac{k_2^2 e^{2kt}}{3(\frac{k_2}{k} e^{kt} + k_3)^2} + \frac{2kk_2 e^{kt}}{(\frac{k_2}{k} e^{kt} + k_3)}\right], \tag{6.35}$$

6.3.2 CaseII: $\frac{\dot{b}}{b} = 0$

Solving it, we get

$$b = k_4 \tag{6.36}$$

Here k_4 is constant of integration.

From the equation (6.18) together with (6.36), we have

$$c = k_4 \tag{6.37}$$

Now using equations (6.36)-(6.37) in the field equations (6.12)-(6.15) we get,

$$\lambda + \xi\theta = 0 \tag{6.38}$$

$$\frac{\ddot{a}}{a} = \xi\theta \tag{6. 39}$$

$$\rho = 0 \tag{6.40}$$

Here ξ, λ, ρ and a are four unknowns involved in three equation (6.38)-(6.40). To obtained a determinate solution we have assumed following different plausible conditions of equations of state as

$$\rho = \lambda \text{ (Geometric String)} \tag{6.41}$$

And $\rho = (1+\omega)\lambda$ (p-string) (6.42)

Where $\omega > 0$ is a constant.

6.3.2.1: $\rho = \lambda$ (Geometric String)

This yields,

$$\rho = \lambda = 0 \tag{6.43}$$

$$\xi = 0 \tag{6.44}$$

and $a = lt + m$ (6.45)

Where, $l \neq 0$ and m are integrating constants.

This case leads to the five-dimensional LRS Bianchi type-I vacuum model Universe in Einstein's theory of relativity. The following metric described the geometry of this model-

$$ds^2 = -dt^2 + (lt+m)^2 dx^2 + k_4^2(dy^2 + dz^2 + dm^2) \quad (6.46)$$

6.3.2.2: $\rho = (1+\omega)\lambda$ (p-string)

Using (6.40) in (6.42) we get,

$$(1+\omega)\lambda = 0 \quad (6.47)$$

Which yields either $\omega = -1$ or $\lambda = 0$. But $\omega = -1$ is not acceptable as $\omega > 0$.

Since, $\lambda = 0$ the model in this case also reduces to the model already obtained in above case.

6.4 INTERPRETATIONS OF THE RESULTS

In case I, a 5-dimensional LRS Bianchi type-I cosmological model Universe with string in general relativity is obtained which given by (6.25). The variation of parameters with time for this model are shown below by taking $k = -1$, $k_1 = k_2 = k_3 = 1$ and the physical and geometrical behaviours of the model are discussed in detail as

At initial epoch $t = 0$, the model (6.25) becomes flat. It is seen that at time $t = 0$ the evolution of energy density ρ is infinite and it decreases gradually as the time t increases and become constant after some finite time(Figure-6.1). This model satisfies the conditions $\rho \geq 0$ and $\rho_p \geq 0$ (Known as energy density conditions).

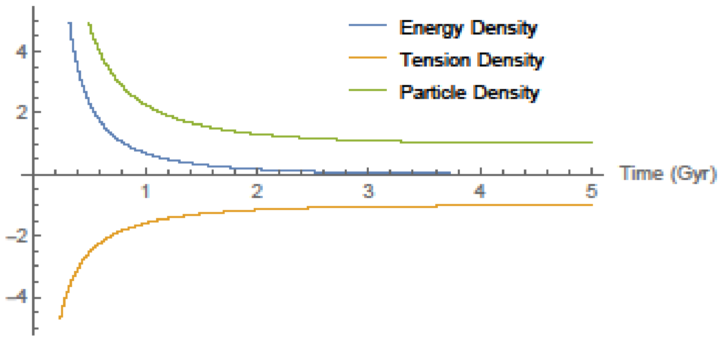

Figure- 6.1. Variation of ρ, λ, ρ_p Vs. time t.

It is also seen from Figure-6.1 that $\rho > 0$, $\lambda < 0$ and $\rho_p > 0$ showing that at early era particles exists with positive ρ_p but strings exists with negative λ. During the Big Bang when $t = 0$, the particle density ρ_p has a large value and as the evolution of time it decreases and moves to a finite value (constant) at $t \to \infty$, which corresponds to a total number of particles in the late Universe that is constant and finite. This may correspond to the matter dominated era. From the comparative study of tension density λ and particle density ρ_p with time as shown in Figure-6.1, it is seen that $|\lambda| < |\rho_p|$ which describes that in the late time the string vanishes, leaving particles only.

Initially at $t = 0$, the expansion scalar θ is very large finite and as the time progresses gradually it decreases and it becomes a small constant after some finite time (as shown in the Figure-6.2) explaining the Big-Bang scenario of the Universe. So, this model shows that the Universe is expanding with the increase of time. However, as time passes, the rate of expansion slows and the expansion scalar becomes small finite at $t \to \infty$.

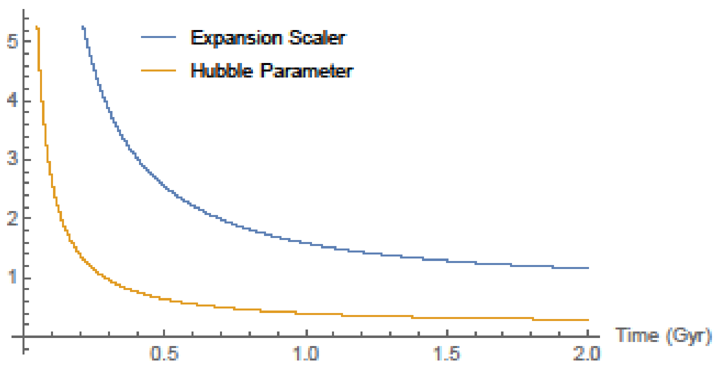

Figure-6.2. Variation of θ, H Vs. time t.

The Hubble parameter H is a decreasing function of time. It is large constant when $t \to 0$ but as the time progresses gradually it also decreases and reached to a small constant value after some finite time as shown by Figure-6.2 and at $t \to \infty$, the value of $\frac{dH}{dt} \to 0$, which also shows the expanding Universe of our model.

Since, $\frac{\sigma^2}{\theta^2} = \text{Constant}(\neq 0)$ as $t \to \infty$ and hence our model Universe obtained here is anisotropic one. Though the anisotropy is included, it does not make any contradiction with the present day observational findings that the Universe is isotropic one. This is due to the fact that, as our Universe evolves, the initial anisotropy fades after a certain epoch and approaches isotropy in the late time Universe.

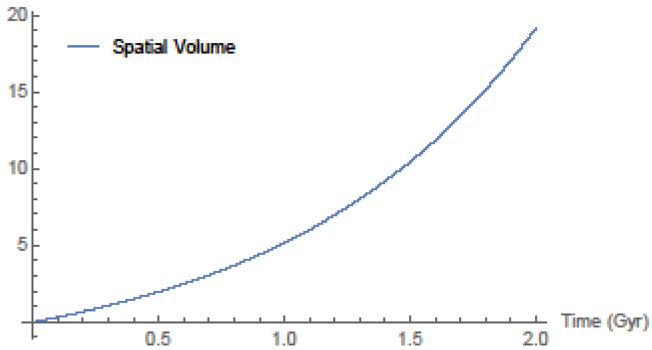

Figure 6.3. Variation of Volume (V) Vs. time t.

From the Figure-6.3 it is seen that the spatial volume V is finite at initial epoch $t = 0$, and it increases exponentially as t increases and becomes infinite as $t \to \infty$**,** which shows the expanding Universe of our model.

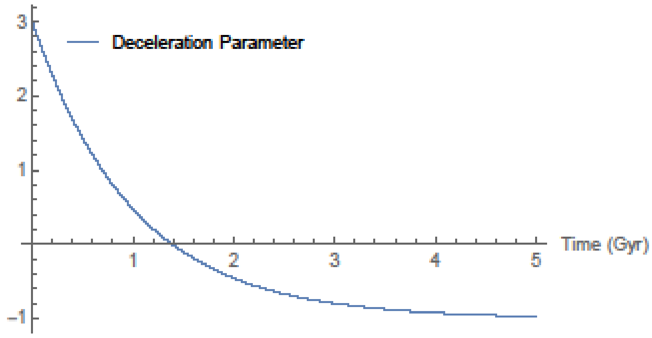

Figure 6.4. Variation of DP (q) Vs. time t.

The DP "q" in the model is decelerating $q > 0$ at $t = 0$ and it decreases as time increases and becomes accelerating $q < 0$ after some finite time as shown in Figure-6.4. Also $q \to -1$ at $t \to \infty$, which indicates that the present model Universe has a transition from decelerating phase to accelerating phase, indicating the inflation in the model after an epoch of deceleration.

For this model, the bulk viscosity decreases as time passes. The function of bulk viscosity is to retard the expansion of the Universe and as bulk viscosity decreases with time, retardness decreases as well, allowing the Universe to expand at a faster rate in the late time Universe. According to the preceding discussion, bulk viscosity plays a significant role in the evolution of the Universe.

The case II, leads to LRS Bianchi type-I vacuum model in Einstein's theory of relativity in five-dimensional space-time represented by (6.46) which is not realistic because strings don't survive for this model.

6.5 CONCLUSIONS

In the present chapter, a 5-dimensional LRS Bianchi type-I String cosmological model in GTR in presence of bulk viscous fluid has been investigated, which is an inflationary model. The model Universe obtained in this chapter is anisotropic, accelerating and expanding. The DP “q” obtained here is decelerating at initial stage and accelerates after some finite time, indicating inflation in the model after an epoch of deceleration which is in accordance with the present day observational scenario of the accelerated expansion of our Universe as type la supernovae[A. Riess et al.(1998), S. Perlmutter et al.(1999)]. The model Universe is anisotropic one in the early epoch. During the inflation, shear decreases with time and then it turns into an isotropic Universe with very small shear. As expected our model satisfies the energy conditions $\rho \geq 0$ and $\rho_p \geq 0$ which in turn imply that our derived models are physically realistic as the present day observational data. The tension density and the particle density are comparable and the model represents a matter dominated Universe in the late time which agrees with the present day observational findings. Also, the model represents an exponentially expanding Universe that starts with Big-Bang at cosmic time $t = 0$ giving inflationary model. The bulk viscosity is the decreasing function of time (t). Thus, our model Universe has cosmological importance since it is clarifying the early Universe and it should be a sensible representation of the Universe at early age.

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