CHAPTER-7

HIGHER DIMENSIONAL PERFECT FLUID COSMOLOGICAL MODEL IN GENERAL RELATIVITY WITH QUADRATIC EQUATION OF STATE (EoS)

The work existing in this chapter has been published in *"Journal of Mathematical and Computational Science*", Scopus indexed journal (UGC care listed). ISSN: 1927-5307, Volume-11, Issue-3, pp.3155-3169, (2021).

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Higher Dimensional Perfect Fluid Cosmological Model in General Relativity with Quadratic Equation of State (EoS)

7.1 INTRODUCTION

Bianchi type-V cosmological models are the natural generalisation of the open Friedmann-Robertson-Walker (FRW) model, which plays an important role in the description of the Universe. The spatially homogeneous-anisotropic models play important roles in the initial days of the evolution of Universe. Furthermore, according to various types of literature and findings, the anisotropic model was chosen as one of the possible models to initiate the expansion of the Universe. In recent years, the solution of Einstein's field equation for homogeneous and anisotropic Bianchi type models have been studied by several authors such as Hajj-Boutros(1985), Shri Ram(1989,1990), Pradhan & Kumar(2001) by using different generating techniques. A few models that describe an anisotropic space time and generate particular interest are among Lorenz-Pestzold(1984), Singh & Agrawal(1993), Marsha(2000), Socorro & Medina(2000), Banerjee & Sanyal(1988), Coley(1990) who have considered Bianchi type-V cosmological models with bulk viscosity and heat flow. Some authors such as Maartens & Nel(1978), Pradhan & Rai(2004), Bali & Meena (2004), Kandalkar et al.(2012) also have studied Bianchi type-V models in different contexts.

The equation of state (EoS) in cosmology is the relationship amongst combined matter, temperature, energy density, pressure and energy for any region of space. The quadratic EoS is useful for studying relativistic dynamics and dark energy in various cosmological models. Its most general form is

$$p = \rho_0 + \alpha\rho + \beta\rho^2$$

Here $\alpha = 0$, where ρ_0, α and β are the parameters.

Ananda & Bruni(2006) looked into the general relativistic dynamics of Robertson-Walker models with a non-linear equation of state (EoS) in the form of a quadratic equation, $p = \rho_0 + \alpha\rho + \beta\rho^2$. They demonstrated that the anisotropic behaviour at the singularity obtained in the Brane Scenario can be reproduced in a general relativistic theory setting. Many researchers like Ivanov(2002), Sharma & Maharaj(2007), Thirukkanesh & Maharaj(2008), Varela et al.(2010) etc. studied cosmological models with linear and non-linear equation of states. Various authors like Nojiri & Odintsov(2005, 2004, 2011), Nojiri et al.(2005), Capozziello et al.(2006), Bamba et al.(2012) already discussed dark energy Universe with different equations of state(EoS). Many authors like Chavanis(2013, 2015), Rahaman et al.(2011), Feroze & Siddiqui(2011), Maharaj & Takisa(2012), Reddy et al.(2015), Adhav et al.(2015), M.R. Mollah et al.(2018), Beesham et al.(2020) studied different space-time cosmological models with a quadratic EoS in general and modified theories of relativity.

The study on higher-dimensional space-time gives us an important idea about the Universe that our Universe was much smaller at initial epoch than the Universe observed in these days. Due to these reasons studies in higher dimensions inspired and motivated many researchers to enter into such a field of study to explore the hidden knowledge of the present Universe. Subsequently, many researchers have already investigated various cosmological models in five dimensional space-time with various Bianchi type models in different aspects[V.U.M. Rao et al.(2015), M.R. Mollah & K.P. Singh(2016), G.S. Khadekar et al.(2005), G. Mohanty & G.C. Samanta(2010), G.C. Samanta et al.(2011)]

In this chapter, a higher-dimensional cosmological model in Bianchi type-V space–time with a quadratic EoS of the form, $p = \alpha\rho^2 - \rho$, is investigated, interacting with perfect fluid. The design of the problem is described in Section 7.2 of this chapter. The solutions to the cosmological problems are found in Section 7.3. Some of the most essential physical and geometrical parameters are derived in section 7.4. In section 7.5, the findings are discussed. Finally, in section 7.6, there are some concluding remarks.

7.2 THE METRIC AND FIELD EQUATIONS

A 5 dimensional Bianchi type-V metric is considered as

$$ds^2 = -dt^2 + a^2dx^2 + e^{-2x}(b^2dy^2 + c^2dz^2 + D^2dm^2) \qquad (7.1)$$

Here a, b, c and D are directional scale factors which are functions of time only and m extra dimension which is space-like.

The energy-momentum tensor for the perfect fluid is

$$T_{ij} = (p + \rho)u_i u_j + p g_{ij} \tag{7.2}$$

Here ρ and p are the energy density and pressure respectively and $u^i = (0,0,0,0,1)$ is the five velocity vector of particles.

Also $$g_{ij}u^i u^j = 1 \tag{7.3}$$

In this chapter, we take the quadratic (EoS) as

$$p = \alpha\rho^2 - \rho \tag{7.4}$$

Here $\alpha \neq 0$ is arbitrary constant.

In general relativity, the Einstein's field equation is given by

$$R_{ij} - \frac{1}{2}g_{ij}R = -T_{ij} \tag{7.5}$$

The spatial volume is given by

$$V = abcD = r^4 \tag{7.6}$$

Where r(t) is average scale factor, given by

$$r(t) = (abcD)^{\frac{1}{4}} \tag{7.7}$$

The expansion scalar is defined as

$$\theta = \frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c} + \frac{\dot{D}}{D} \tag{7.8}$$

The Hubble's parameter is defined as

$$H = \frac{\dot{r}}{r} = \frac{1}{4}\theta = \frac{1}{4}\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c} + \frac{\dot{D}}{D}\right) \tag{7.9}$$

Also we have

$$H = \frac{1}{4}(H_1 + H_2 + H_3 + H_4) \tag{7.10}$$

Here $H_1 = \frac{\dot{a}}{a}$, $H_2 = \frac{\dot{b}}{b}$, $H_3 = \frac{\dot{c}}{c}$ and $H_4 = \frac{\dot{D}}{D}$ are directional Hubble's factors in x, y, z and m directions respectively.

Deceleration parameter q is defined a

$$q = -\frac{r\ddot{r}}{r^2} \tag{7.11}$$

The mean anisotropy parameter is defined as

$$\Delta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2 \tag{7.12}$$

The shear scalar is defined as

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left(\frac{\dot{a}^2}{a^2} + \frac{\dot{b}^2}{b^2} + \frac{\dot{c}^2}{c^2} + \frac{\dot{D}^2}{D^2} - \frac{\theta^2}{4}\right) \tag{7.13}$$

Using the equations (7.1)-(7.3) and (7.5), we obtain

$$\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\ddot{D}}{D} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{b}\dot{D}}{bD} + \frac{\dot{c}\dot{D}}{cD} - \frac{3}{a^2} = -p \tag{7.14}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{c}}{c} + \frac{\ddot{D}}{D} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{c}\dot{D}}{cD} - \frac{3}{a^2} = -p \tag{7.15}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{D}}{D} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{b}\dot{D}}{bD} - \frac{3}{a^2} = -p \tag{7.16}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{b}\dot{c}}{bc} - \frac{3}{a^2} = -p \tag{7.17}$$

$$\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{b}\dot{D}}{bD} + \frac{\dot{c}\dot{D}}{cD} - \frac{6}{a^2} = \rho \tag{7.18}$$

$$3\frac{\dot{a}}{a} = \frac{\dot{b}}{b} + \frac{\dot{c}}{c} + \frac{\dot{D}}{D} \tag{7.19}$$

The overhead dots represent the order of differentiation with respect to time 't'.

The conservation law of energy momentum tensor is

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c} + \frac{\dot{D}}{D}\right) = 0 \tag{7.20}$$

7.3 COSMOLOGICAL SOLUTIONS

From Equation (7.19) we get

$$a^3 = kbcD \tag{7.21}$$

Here k is the constant of integration. Without loss of generality, we can choose $k = 1$ and so we get

$$a^3 = bcD \tag{7.22}$$

Subtracting (7.14) from (7.15), we get

$$\frac{\ddot{a}}{a} - \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{c}}{ac} - \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{a}\dot{D}}{aD} - \frac{\dot{b}\dot{D}}{bD} = 0 \tag{7.23}$$

Subtracting (7.15) from (7.16), we get

$$\frac{\ddot{b}}{b} - \frac{\ddot{c}}{c} + \frac{\dot{a}\dot{b}}{ab} - \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{b}\dot{D}}{bD} - \frac{\dot{c}\dot{D}}{cD} = 0 \tag{7.24}$$

Subtracting (7.16) from (7.17), we get

$$\frac{\ddot{c}}{c} - \frac{\ddot{D}}{D} + \frac{\dot{a}\dot{c}}{ac} - \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{b}\dot{c}}{bc} - \frac{\dot{b}\dot{D}}{bD} = 0 \tag{7.25}$$

Subtracting (7.17) from (7.14), we get

$$\frac{\ddot{D}}{D} - \frac{\ddot{a}}{a} + \frac{\dot{b}\dot{D}}{bD} - \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{c}\dot{D}}{cD} - \frac{\dot{a}\dot{c}}{ac} = 0 \tag{7.26}$$

From the equations (7.23)-(7.26) the following equations are obtained

$$\frac{\dot{a}}{a} - \frac{\dot{b}}{b} = \frac{k_1}{abcD} \tag{7.27}$$

$$\frac{\dot{b}}{b} - \frac{\dot{c}}{c} = \frac{k_2}{abcD} \tag{7.28}$$

$$\frac{\dot{c}}{c} - \frac{\dot{D}}{D} = \frac{k_3}{abcD} \tag{7.29}$$

$$\frac{\dot{D}}{D} - \frac{\dot{a}}{a} = \frac{k_4}{abcD} \tag{7.30}$$

Here k_1, k_2, k_3 and k_4 are the constant of integration. Without loss of generality, we can choose $k_1 = k_2 = k_3 = k_4 = k$(say)

The Equations (7.27)-(7.30) yields

$$a = b = c = D \tag{7.31}$$

We consider negative constant deceleration parameter model defined by

$$q = -\frac{r\ddot{r}}{\dot{r}^2} = \text{constant} \tag{7.32}$$

The solution of above equation is

$$r = (Lt + M)^{\frac{1}{1+q}} \tag{7.33}$$

Where $L \neq 0, M$ are constants and $q + 1 \neq 0$

Using (7.7), (7.22), (7.31) and (7.33), we obtain

$$a = b = c = D = (Lt + M)^{\frac{1}{1+q}} \tag{7.34}$$

The following metric describe the model

$$ds^2 = -dt^2 + (Lt + M)^{\frac{2}{1+q}} dx^2 + e^{-2x}(Lt + M)^{\frac{2}{1+q}}(dy^2 + dz^2 + dm^2) \tag{7.35}$$

7.4 COSMOLOGICAL PARAMETERS

Some of the important Physical and Geometrical Parameters are obtained bellow:

Volume element of the model is

$$V = (Lt + M)^{\frac{4}{1+q}} \tag{7.36}$$

The Hubble's Parameter is given by

$$H = \frac{L}{(q+1)(Lt+M)} \tag{7.37}$$

The expansion scalar (θ) is given by

$$\theta = \frac{4L}{(q+1)(Lt+M)} \tag{7.38}$$

Using equation (7.4) & (7.34), the equation (7.20) yields

$$\rho = \frac{q+1}{4\alpha \log(Lt+M) + (q+1)k_5} \tag{7.39}$$

Where k_5 is arbitrary constant.

From equations (7.4) and (7.39), p is obtained as

$$p = -\frac{(q+1)[3\alpha \log(Lt+M) - (q+1)(k_5+\alpha)]}{[3\alpha \log(Lt+M) + (q+1)k_5]^2} \tag{7.40}$$

The shear scalar and the average anisotropy parameter respectively are

$$\sigma^2 = 0 \tag{7.41}$$

$$\Delta = 0 \tag{7.42}$$

7.5 DISCUSSION

Taking $q = -0.5$, $L = M = 1$, $k_5 = 2$ and $\alpha = 1$, the variation of some of the important cosmological parameters (Physical and Geometrical) with respect to time for the model are shown below:

When time $t = 0$, the spatial volume V is finite, as shown by equation (7.36). As time passes, the spatial volume V expands, eventually becoming infinitely large as $t \to \infty$, as shown in Figure-7.1. This indicates that the Universe expands as time increase.

From the equations (7.37) and (7.38), it is observed that the quantities like Hubble's parameter H and expansion scalar θ are decreasing functions of time and they are finite when $t = 0$ and are vanishing for infinitely large value of $'t'$ which are shown in the Figure-7.2 bellow. Also, since $\frac{dH}{dt} < 0$, which also tells us that our present Universe is in the accelerated expanding mode.

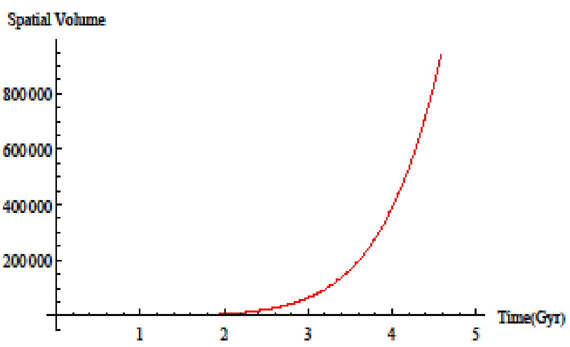

Figure 7.1.Variation of Volume ‘V’ Vs. Time t.

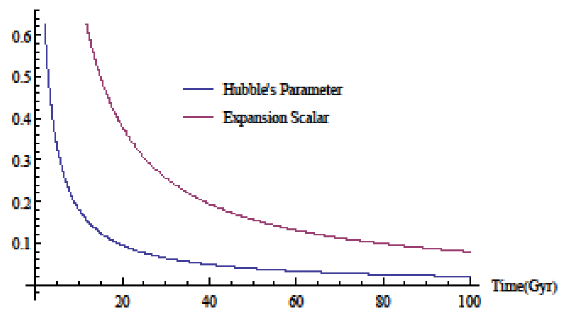

Figure 7.2.Variation of H, θ Vs. Time ‘t’.

Figure 7.3 or equation (7.39) shows that the evolution of the energy density ρ is large and constant at time $t = 0$, then decreases with the increases of time until it reaches zero at time $t \to \infty$. This demonstrates that the Universe is expanding with the passage of time.

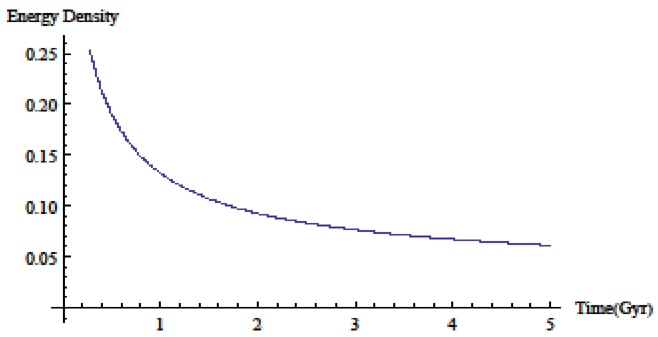

Figure 7.3.Variation of energy density ‘ρ’ Vs. Time ‘t’.

According to the equation (7.40), the isotropic pressure p is a small positive constant at the initial epoch $t = 0$ and it decreases with time and changes sign from positive to negative as shown in Figure-7.4. As a result, the current model Universe is transitioning from a matter-dominated to an inflationary era.

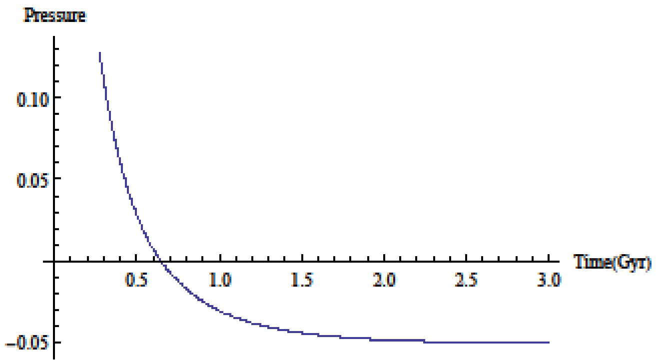

Figure 7.4.Variation of p Vs. Time t.

As shown in equation (7.41), $\sigma \to 0$ throughout the evolution, indicating that the model is shear free. The ratio $\frac{\sigma^2}{\theta^2} \to 0$ as time approaches infinity, indicating that the model approaches isotropy in the late time Universe, which is consistent with Collins & Hawking (1973).

According to equation (7.42), the average anisotropy parameter $\Delta = 0$, indicating isotropic model and this model also depicts the late time acceleration of Universe.

At the initial epoch $t = 0$, the spatial volume, all three scale factors and all other physical and kinematical parameters are constant. This demonstrates that the current model is free of the initial singularity.

7.6 CONCLUSIONS

In this chapter, we look at a Bianchi type-V cosmology with quadratic EoS direct interaction with perfect fluid in 5-dimensional space-time. The general solutions of EFE were obtained under the assumption of quadratic EoS $p = \alpha\rho^2 - \rho$, where $\alpha \neq 0$ is an arbitrary constant treating DP as a constant quantity. The resulting model expands with accelerated motion, which corresponds to the most recent observational data. The model remains isotropic, non-sharing and free of the initial singularity throughout its evolution. We observe from eqn. (7.39) that the evolution of the energy density ρ is constant large finite at the time $t = 0$ and at time $t \to \infty$ the energy density ρ decreases to a small finite constant value. While eqn.(7.40) indicates that the isotropic pressure (p) is a negative quantity and it decreases when time $t \to \infty$. Such type of solution is consistent with recent observational data like SNe1a. The negative pressure may be a possible cause of the accelerated expansion of the Universe. Also, generally the negative pressure can resists the attractive gravity of matter.

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