CHAPTER-8

STRING COSMOLOGICAL MODEL IN 5-DIMENSIONAL SPACE-TIME INTERACTING WITH VISCOUS FLUID

The work existing in this chapter has been communicated to the UGC care listed journal for the publication.

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8.1 INTRODUCTION

Still now it is an interesting location for the cosmologists to study and discover its unknown phenomenon that have yet to be observed to study and explore the hidden information of the Universe. As a result, cosmologists have taken a keen interest in understanding the past, present and future evolution of the Universe. Letelier(1983) and Stachel(1980) pioneered the general relativistic study of strings by developing a classical concept of geometric strings. Because of the importance of strings in describing the early stages of our Universe, many distinguished authors are now interested in cosmic strings within the framework of general relativity [Kibble(1976, 1980)] and it is believed that strings cause density perturbations that lead to the formation of massive scale structures (like galaxies) in the Universe [Zel'dovich(1974, 1980)]. These strings contain stress energy and are divided into geometric strings and massive strings. According to the grand unified theories[Everett(1981), Vilenkin(1981)], those strings arose during the transition of phases when the temperature went down beneath some critical temperature soon after the explosion of Big-Bang.

A higher-dimensional cosmological model plays a very important role in various aspects of the early cosmological evolution of the Universe. The higher dimensional model was introduced by T. Kaluza(1921) and O. Klein(1926) in an effort to unify gravity with electromagnetism. Chakraborty & Debnath(2010), Krori et al.(1994), Rahaman et al.(2003), Mohanty & Samanta (2010), Mohanty & Samanta(2010a), Kandalker et al.(2012), Guth(1981), Alvax & Gavela(1983), Kaiser & Stebbins(1984), Marciano(1984), Banerjee et al.(1990), Chatterjee(1993), Bali et al.(2008), Pawar & Deshmukh(2010), Samanta et al.(2011), Samanta & Debata(2012), Sahoo & Mishra(2015), Singh(2013), Goswami et al.(2016), Khadekar & Vrishali (2005), Khadekar et al.(2007), Ladke(2014), Singh & Mollah(2016), Banik & Bhuyan(2017), Choudhury(2017), Tripathi et al.(2017), Dubey et al.(2018), Tiwari et al.(2019), Mollah et al.(2018) are the some authors who studied different string cosmological models within the general relativity in different context in various space-times.

The early evolution of the Universe was influenced by bulk viscosity. Several researchers have used the well-known concept of general relativity to investigate its impact on the evolution of the Universe. Misner (1968) investigated the effects of bulk viscosity on the cosmological evolution of the Universe. Some of the famous researchers Banerjee & Sanyal(1988), Coley(1990), Pradhan & Rai (2004), X.X. Wang(2005), R. Bali & A. Pradhan(2007), Kandalkar et al.(2011), Kandalkar et al.(2012), V. Humad et al.(2016), who have alredy studied several Bianchi models in the field of general relativity with bulk viscosity.

The preceding discussion inspired us to investigate five-dimensional string cosmological models with particles attached in Bianchi type-I space-time with bulk viscosity in GR to investigate the various possibilities of the Bianchi type model Universe. In addition, the parameters in our model Universe are thoroughly discussed.

8.2 FIELD EQUATIONS AND THEIR SOLUTIONS

For a bulk viscous cloud string, the energy-momentum tensor is given by

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta(u_i u_j + g_{ij}) \tag{8.1}$$

Where, ρ is the energy density and λ the tension density of the string and they are related as, $\rho = \lambda + \rho_p$, where ρ_p is the particle density of matter, ξ is the coefficient of viscosity and θ is the expansion scalar. The co-ordinates are co-moving, x^i is the unit vector (space-like) in the direction of strings and u^i is the five velocity vector which satisfies the conditions-

$$u_i u^i = -1 = -x_i x^i \quad \text{and} \quad u_i x^i = 0, \tag{8.2}$$

A Bianchi type-I metric in 5D space-time is considered in the form of

$$ds^2 = -dt^2 + a^2dx^2 + b^2dy^2 + c^2dz^2 + D^2dm^2 \tag{8.3}$$

Here a, b, c and D are the metric functions of cosmic time t alone and the extra coordinate "m" is considered to be space-like.

For the above metric lets

$$x^1 = x\,,\ x^2 = y\,,\ x^3 = z\,,\ x^4 = m \text{ and } x^5 = t \tag{8.4}$$

Here without loss of generality we can take

$$u^i = (0,0,0,0,1) \text{ and } x^i = (a^{-1},0,0,0,0), \tag{8.5}$$

The Einstein's field equation is written as

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \tag{8.6}$$

For the metric (8.3) by using the equations (8.1)-(8.2) and (8.4)-(8.5) in the field equation (8.6) yields

$$\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\ddot{D}}{D} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{b}\dot{D}}{bD} + \frac{\dot{c}\dot{D}}{cD} = \lambda + \xi\theta \tag{8.7}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{c}}{c} + \frac{\ddot{D}}{D} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{c}\dot{D}}{cD} = \xi\theta \tag{8.8}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{D}}{D} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{b}\dot{D}}{bD} = \xi\theta \tag{8.9}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{b}\dot{c}}{bc} = \xi\theta \tag{8.10}$$

$$\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{b}\dot{D}}{bD} + \frac{\dot{c}\dot{D}}{cD} = \rho \tag{8.11}$$

Where an over dot and double over dot denote the first derivative and the second derivative w.r.t. cosmic time `t' respectively.

From equations (8.8),(8.9),(8.10) the following equations are deduced

$$\frac{\dot{b}}{b}-\frac{\dot{c}}{c}=\frac{k_1}{abcD} \tag{8.12}$$

$$\frac{\dot{b}}{b}-\frac{\dot{D}}{D}=\frac{k_2}{abcD} \tag{8.13}$$

$$\frac{\dot{c}}{c}-\frac{\dot{D}}{D}=\frac{k_3}{abcD} \tag{8.14}$$

Here k_1, k_2, and k_3 are the constant of integration. Without loss of generality, let us choose $k_1 = k_2 = k_3$.

Equation (8.12) to (8.14) yields

$$b = c = D \tag{8.15}$$

Using equations (8.15) in the equations (8.7)-(8.11), we get

$$3\frac{\ddot{b}}{b}+3\frac{\dot{b}^2}{b^2}=\lambda+\xi\theta \tag{8.16}$$

$$\frac{\ddot{a}}{a}+2\frac{\ddot{b}}{b}+2\frac{\dot{a}\dot{b}}{ab}+\frac{\dot{b}^2}{b^2}=\xi\theta \tag{8.17}$$

$$3\frac{\dot{a}\dot{b}}{ab}+3\frac{\dot{b}^2}{b^2}=\rho \tag{8.18}$$

The equations (8.16)-(8.18) represents a system of three-independent equations involving five unknowns a, b, ρ, λ and ξ. In order to obtain deterministic solution, two more physical conditions involving these variables are required. Let us consider these two physical conditions as

The shear scalar σ is directly proportional to the expansion scalar θ, so that we may take [Collins et al.(1980), Kiran et al.(2015)].

$$a = b^n \tag{8.19}$$

Where $n \neq 0$ is a constant and it will describe the anisotropy of the space-time.

And the average scale factor is an integrating function of time [Saha et al.(2012)]

$$r(t) = (t^k e^t)^{\frac{1}{l}} \tag{8.20}$$

Using the equations (8.15), (8.19) and (8.20), we obtained

$$b = (t^k e^t)^{\frac{4}{(n+3)l}} \tag{8.21}$$

So the directional scale factors a, b, c and D are obtained as

$$a = (t^k e^t)^{\frac{4n}{(n+3)l}}, \ b = c = D = (t^k e^t)^{\frac{4}{(n+3)l}} \tag{8.22}$$

By the use of the equations (8.22), the metric (8.3) can be written as

$$ds^2 = -dt^2 + (t^k e^t)^{\frac{8n}{(n+3)l}} dx^2 + (t^k e^t)^{\frac{8}{(n+3)l}} (dy^2 + dz^2 + dm^2) \tag{8.23}$$

The equation (8.23) is a five dimensional Bianchi type-I string cosmological Universe.

8.3 DYNAMICAL PARAMETERS OF THE MODEL

The Spatial Volume is

$$V = (t^k e^t)^{\frac{4}{l}} \tag{8.24}$$

The expansion scalar θ which determines the volume behavior of the fluid is given by

$$\theta = \frac{4(t+k)}{lt} \tag{8.25}$$

At the initial epoch $t \to 0$, expansion scalar $\theta \to \infty$ and $\to \frac{4}{l}$, when $t \to \infty$.

Hubble's parameter (H) is given by

$$H = \frac{(t+k)}{lt} \tag{8.26}$$

Using equation (8.18), the energy density is obtained as

$$\rho = \frac{48(n+1)(t+k)^2}{l^2(n+3)^2 t^2} \tag{8. 27}$$

When $\xi = \xi_0$ =constant, the tension density and the particle density of the string are obtained as

$$\lambda = \frac{96(t+k)^2}{l^2(n+3)^2t^2} - \frac{12k}{l(n+3)t^2} - \xi_o\frac{4(t+k)}{lt} \tag{8.28}$$

$$\rho_p = \frac{6(n-1)^2(t+k)^2}{l^2(n+3)^2t^2} + \frac{12k}{l(n+3)t^2} + \xi_o\frac{4(t+k)}{lt} \tag{8. 29}$$

When $\xi = \xi(t)$, then

$$\xi = \frac{4(n^2+2n+2)(t+k)}{(n+3)^2lt} - \frac{(n+2)k}{(n+3)t(t+k)} \tag{8. 30}$$

For this case the tension density and the particle density of the string are obtained as

$$\lambda = \frac{(64-16\mathrm{n}^2-32\mathrm{n})(t+k)^2}{l^2(n+3)^2t^2} + \frac{4k(n-1)}{l(n+3)t^2} \tag{8.31}$$

$$\rho_p = \frac{(22\mathrm{n}^2+20\mathrm{n}+38)(t+k)^2}{l^2(n+3)^2t^2} - \frac{4k(n-1)}{l(n+3)t^2} \tag{8. 32}$$

The expression of deceleration parameter is obtained as

$$q = -1 + \frac{kl}{(t+k)^2} \tag{8.33}$$

Shear scalar is given by

$$\sigma^2 = \frac{6(n-1)(t+k)^2}{l^2(n+3)^2t^2} \tag{8.34}$$

From (8.25) and (8.34), we obtain

$$\lim_{t\to\infty}\frac{\sigma^2}{\theta^2} = \frac{3(n-1)^2}{2(n+3)^2} = constant \neq 0, \text{ for } n \neq 1 \tag{8. 35}$$

Where n can never be -3. Therefore, the model does not approach isotropy for large value of t for $n \neq 1$ [Asgar & Ansari(2014)] but it approaches to isotropy for $n = 1$.

$$\Delta = \frac{3(n-1)^2}{2(n+3)^2} = constant \neq 0, \text{ for } n \neq 1. \tag{8.36}$$

As a result for $n \neq 1$, the model (8.23) has a constant anisotropy parameter throughout the evolution and it approaches isotropy for $n = 1$ [Asgar & Ansari (2014)].

8.4 PHYSICAL INTERPRETATIONS OF THE SOLUTIONS

A Bianchi type-I string cosmological model in general relativity in five-dimensional space-time with bulk viscosity, given by the equation (8.23) has been constructed. Taking $k = l = 1; n = 2; \xi_o = 1$, the variation of the parameters of model are shown by the Figures. The physical and geometrical behavior of model can be discussed as

From the Eqn.(8.24), it is observed that the spatial volume V, is an increasing function of cosmic time t. It increases exponentially and evolves with zero volume at $t = 0$ and it becomes infinite as $t \to \infty$. Figure 8.1 depicts this behavior of volume V verses time t. Also, when $t \to \infty$, scale factors a, b, c and D are found to be infinite.

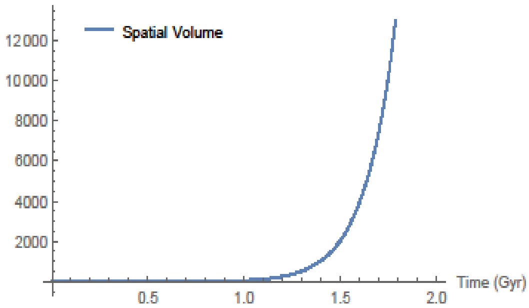

Figure 8.1: Variation of Volume V vs. time t.

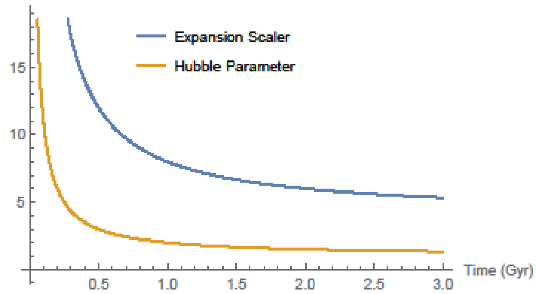

Figure 8.2: Variation of θ, H vs. time t.

From Eqns. (8.25) and (8.26) and their respective graphical presentations (Figure-8.2), it is seen that at the initial epoch of cosmic time $t = 0$, both the expansion scalar θ and the Hubble's parameter H are infinite and are decreasing functions of time t approaching to finite values at $t \to \infty$.

The expression for energy density ρ is given by Eqn.(8.27). It is a decreasing function of cosmic time t and satisfies the energy condition $\rho \geq 0$ for all $n \geq -1$. Its variation against cosmic time is presented in Figure-8.3, which also shows that ρ decreases with time t and initially when $t \to 0$, then $\rho \to \infty$. So it has an initial singularity.

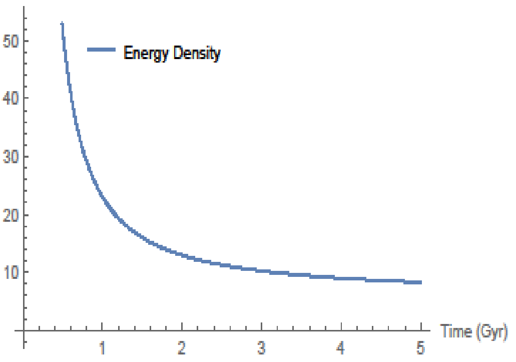

Figure 8.3: Variation of energy density ρ vs. time

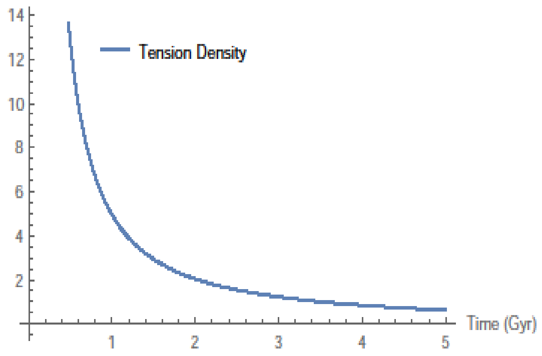

Figure 8.4: Variation of tension density λ vs. time

Eqn. (8.28) shows that the string tension density is positive $\lambda \geq 0$ for all values of cosmic time t when bulk viscosity is constant. Initially at $t \to 0$, λ is very large(attains the peak value) and just after that it becomes a decreasing function of cosmic time t and finally tends to a very small positive quantity. The variation of string tension density λ against cosmic time t is graphically presented in Figure 8.4.

Initially at $t \to 0$, the value of particle density ρ_p is very large (attains the peak value) when the bulk viscosity is constant. The value of ρ_p decreases with the increase of time and approaches to a constant value at infinite time, which shows that there will remain a finite number of particles in our Universe. This may corresponds to the matter dominated era.

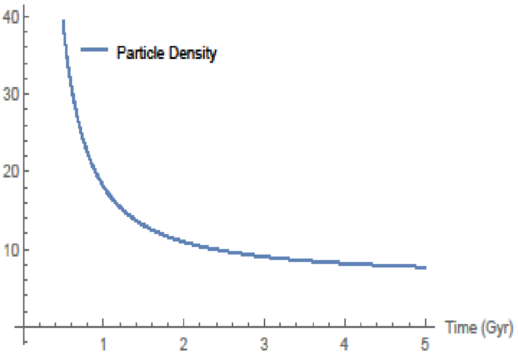

Figure8.5:Variation of particle density ρ_p vs. t.

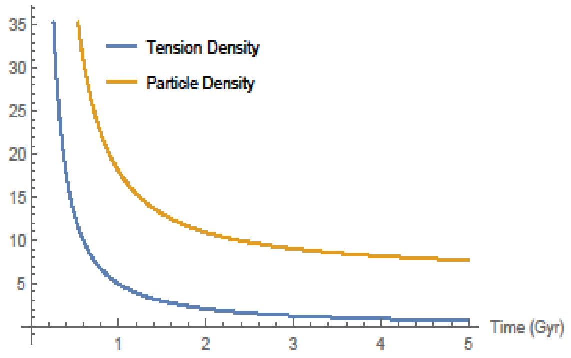

Figure 8.6: Variation of ρ, ρ_p vs. time t.

When $\xi = \xi(t)$, then ξ is a decreasing function of time t, decreases from $\xi \to \infty$ to a small finite value. In this case also both λ and ρ_p are very large and decrease to small finite value with the passes of time.

Figure-8.6 depicts the comparative variation of ρ_p and λ against cosmic time, it can be seen that | λ |<$|\rho_p|$ and so string tension density λ vanishes more rapidly than the particle density ρ_p, describing that the string vanishes in the late time, leaving particles only. As a result, our model is realistic.

The expression in Eqn. (8.33) shows that the DP(q) is a decreasing function of time t. Initially at $t = 0$ the DP is negative $(-1 \leq q \leq 0)$ for all $k \geq l$ and then it decreases with the increase of time and at infinite time it tend to -1. It can be confirmed from the graph of q versus cosmic time t presented in Figure-8.7. It means that this model is found to be expanding with time for all $k \geq l$. And since $H > 0$, $q < 0$ for $0 < t < \infty$, our model Universe obtained here shows the expanding and accelerating Universe. As a result, our model represents an incredibly interesting model Universe that should be investigated for a desirable feature of a meaningful string model.

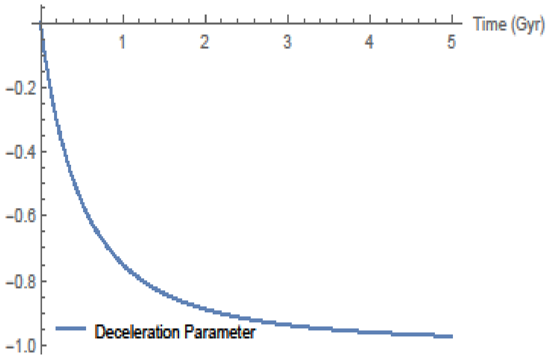

Figure 8.7: Variation of q vs. time t.

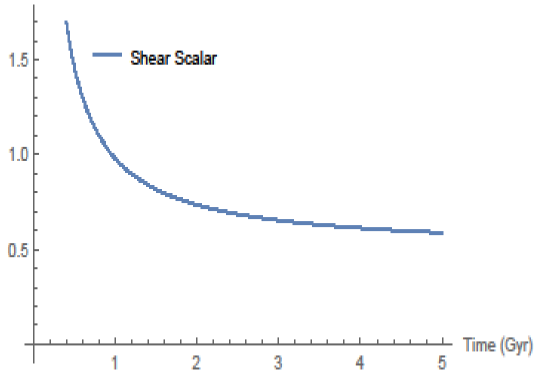

Figure 8.8: Variation of σ vs. time t.

Eq. (8.34) and Figure 8.8 gives us an idea about shear scalar σ which is infinite at initial epoch and it approaches to zero at $t = \infty$, for $n \neq 1$, explaining a shearing model Universe throughout its evolution.

Since, $\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} = \frac{3(n-1)^2}{2(n+3)^2} = constant \neq 0$, for $n \neq 1$, so the model does not approach isotropy for large value of time t [Asgar & Ansari (2014)]. Also, from the expression of Hubble's expansion factor Eqn. (8.26), we found that $\frac{dH}{dt}$ is negative which indicates that our model corresponds to an expanding Universe, which starts evolving at $t = 0$ and is expanding with an accelerated rate. But the model approaches to isotropy for $n = 1$. The anisotropy parameter $\Delta = \frac{3(n-1)^2}{2(n+3)^2} = constant \neq 0$, for

$n \neq 1$ but $\Delta = \frac{3(n-1)^2}{2(n+3)^2} = 0$, for $n = 1$. Thus the model Universe is anisotropic one throughout its evolution for $n \neq 1$ but approaches to small isotropy whenever $n = 1$.

8.5 CONCLUSIONS

In this chapter, the Einstein's field equation in five dimensional Bianchi type-I model in the context of general theory of relativity has been obtained and solved by the use of certain physical assumptions, which are agreeing with the present observational findings by considering bulk viscosity as (i) constant quantity and (ii) functions of cosmic time. In both cases, the model represents an exponentially expanding and accelerating Universe that starts with volume 0 and stops with infinite volume. The model has an initial singularity and will eventually approach the de-Sitter phase ($q = -1$). It also satisfies the energy conditions $\rho \geq 0$ and $\rho_p \geq 0$. Although the tension density and particle density are comparable, the tension density disappears faster than the particle density, leaving only the particles. So, our model represents a matter-dominated Universe that agrees with current observational data. The model is anisotropic one and shearing throughout its evolution for $n \neq 1$ but approaches to small isotropy whenever $n = 1$.

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