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BULK VISCOUS FLUID BIANCHI TYPE-I STRING COSMOLOGICAL MODEL WITH NEGATIVE CONSTANT DECELERATION PARAMETER

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ABSTRACT. Here we have studied a Bianchi Type-I string cosmological model with bulk viscous fluid and negative constant deceleration parameter in general relativity. To solve the survival field equations here we assumed that the shear scalar and scalar expansion are directly proportional to each other $\sigma \infty \theta$. The geometrical as well as physical features of the model are obtained and discussed. The model universe starts at initial epoch $t = 0$ with 0 volume and then expand with accelerated rate. The model universe obtained here is non shearing. The coefficient of bulk viscosity plays an important role in the cosmological consequences.The tension density diminishes with faster rate than particle density in the evolution of universe which shows that the present day universe is particle dominated.

1. INTRODUCTION

One of the tough problem for the researcher is to obtained the actual physical state of the universe at the very early days of its formation. Strings cosmological models are studied widely in present days due to their major contribution in the study of the evolution of the universe in early stages after the big bang. According to the grand unified theories (Everett [1], Vilenkin [2]), those strings was formed during the transition of phases when the temperature went down

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beneath some critical temperature soon after the explosion of big-bang . Letelier [3] and Stachel [4] are the two prominent authors who initiated to study about the strings. Letelier solved the Einstein's field equation and obtained the solutions for a cloud of strings with the plane, spherical, and cylindrical symmetry. He also solved the same equation for the cloud of massive strings in the year 1983, and constructed the cosmological models in the Bianchi Type-I Space-time.

In the evolution of the universe the bulk viscosity contribute a significance role. It can arise in different circumstances and can lead to constructive mechanism of the formation of galaxies. The amplitude of the bulk viscous stress with respect to the expansion can be determined by means of the coefficients of bulk viscosity. The homogeneous and anisotropic Bianchi type-I cosmological models are considered to understand the evolution in the early stages of the universe. Many authors have tried to find the precise solutions of field equations by the way of considering viscous consequences in general relativity in isotropic as well as anisotropic cosmological model. Misner [5, 6], studied about the consequences of bulk viscosity in the evolution of cosmological model. Nightingale [7] has obtained the importance of viscosity in cosmology within the evolution from the early epoch of the universe. Wang [8] constructed a Bianchi type-III cosmological models with string within the framework of general relativity considering the bulk viscous fluid. The behavior of the Bianchi type-III cosmological models with strings in general relativity with and without bulk viscosity are discussed by Bali and Pradhan [9]. Kandalkar et al. [10] constructed cosmic string in Bianchi type-I cosmological model with bulk viscosity. Also, Kandalkar et al. [11] investigated a Bianchi-V cosmological models in general relativity with constant deceleration parameter and viscous fluid. Humad et al [12] constructed a string cosmological model in Bianchi type-I space time with the help of Bulk viscosity in context of general relativity. Rao et al. [13], Singh [14], Tripathi et al. [15], Pradhan and Jaiswal [16], Dubey et al. [17], Singh and Daimary [18], are some of the prominent authors who have investigated various string cosmology in the Bianchi models with bulk viscosity in different space-time.

Here we have attempted to find a model in cosmology with string with Bianchi type-I space-time by considering the deceleration parameter(q) as a constant quantity in general relativity with bulk viscous fluid. In the first Section of this paper we discussed a brief introduction of Bianchi type string cosmological models, In second Section a Bianchi type-I metric is presented and the field equations in general relativity are derived. We determined the solutions of the survival field equations in the Section 3. In Section 4, physical and geometrical behavior of our model are discussed and then conclusions are given in last Section.

2. Metric and Field equations

Here we take the Bianchi type-I metric as

$$ds^2 = -dt^2 + a^2(dx^2 + dy^2) + b^2 dz^2 \,, \tag{2.1}$$

where a(t) and b(t) are the metric functions of 't'.

The Einstein's field equation $(8\pi G = 1, C = 1)$in general relativity is given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \,. \tag{2.2}$$

For a cloud string with bulk viscous fluid, the energy-momentum tensor is taken as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta(g_{ij} + u_i u_j) \,. \tag{2.3}$$

Here, $\lambda = \rho - \rho_p$ is tension density, ρ is energy density and ρ_p is particle density, u^i is four velocity vector of particles and x^i is unit space-like vector which gives the direction of string, given by

$$u^i = (0, 0, 0, 1) \; and \; x^i = (a^{-1} 0, 0, 0) \tag{2.4}$$

such that,

$$u_i u^j = -1 = -x_i x^j \; and \; u_i x^i = 0 \,. \tag{2.5}$$

The spatial volume, scalar expansion, Hubble parameter, shear scalar and mean anisotropy parameter are respectively given by

$$V = a^2 b = R^3 \tag{2.6}$$

$$\begin{aligned}
\theta &= u^i_{,i} = 2\frac{\dot{a}}{a} + \frac{\dot{b}}{b} \\
H &= \frac{1}{3}(2\frac{\dot{a}}{a} + \frac{\dot{b}}{b}) \\
\sigma^2 &= \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}[2(\frac{\dot{a}}{a})^2 + (\frac{\dot{b}}{b})^2] - \frac{\theta^2}{6} \\
\Delta &= \frac{1}{3}\sum_{i=1}^{3}(\frac{H_i - H}{H})^2 ,
\end{aligned}$$

where, H_i (i=x,y,z)are defined as $H_x = H_y = \frac{\dot{a}}{a}$, and $H_z = \frac{\dot{b}}{b}$ for the metric (2.1).

Using (2.3)-(2.5) in (2.2) yields

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} = \xi\theta \tag{2.7}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = \lambda + \xi\theta \tag{2.8}$$

$$\frac{\dot{a}^2}{a^2} + 2\frac{\dot{a}\dot{b}}{ab} = \rho , \tag{2.9}$$

where the overhead dots denotes the order of derivative w.r.t. time ‘t’.

3. Solution of the Field Equations

We have 3 highly nonlinear independent differential equations (2.7)-(2.9) with five unknowns variables a, b,ρ, λ and ξ. So to find exact solution we must used two extra plausible conditions. So here we used the following assumptions:

The shear scalar and scalar expansion are directly proportional to each other, $\sigma \infty \theta$ leading to the equation:

$$a = b^n , \tag{3.1}$$

where $n \neq 0$ is a constant.

The above assumption is based on observations of velocity and red-shift relation for an extragalactic source which predicted that the Hubble expansion is 30 percent isotropic, which is supported by the works of Thorne [19], Kantowski and Sachs [20], Kristian and Sachs [21]. In particular, it can be said that $\frac{\sigma}{H} \geq 0.30$, where σ and H are respectively shear scalar and Hubble constant. Also, Collins et al. [22] has shown that if the normal to the spatially homogeneous line element is congruent to the homogeneous hyper-surface then $\frac{\sigma}{\theta}$ =constant, θ being the expansion factor.

Also, Berman's [23] suggestion regarding variation of Hubble's parameter H provides us a model universe that expands with constant deceleration parameter. So for the determinate solution, let us take deceleration parameter to be a negative constant-

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = h\ (constant)\,. \tag{3.2}$$

It is well known that when q is negative then the model universe expand with acceleration, and when q is positive then it explains a decelerating(contracting) universe. Although the present observations like CMBR and SNe Ia suggested the negative value of q but it can be remarkably state that they are not able to deny about the decelerating expansion(positive q) of universe.

Solving (3.2), we get

$$R(t) = (lt+m)^{\frac{1}{1+h}}\ h \neq -1\,, \tag{3.3}$$

where l, m, are constants of integration.

Using (2.6), (3.1) and (3.3) we get,

$$a = (lt+m)^{\frac{3n}{(1+h)(2n+1)}},\ b = (lt+m)^{\frac{3}{(1+h)(2n+1)}}\,.$$

With the suitable choice of coordinates and constant we can take (l=1 and m=0) and then

$$a = t^{\frac{3n}{(1+h)(2n+1)}},\ b = t^{\frac{3}{(1+h)(2n+1)}}\,.$$

The metric (2.1) reduces to

$$ds^2 = -dt^2 + t^{\frac{6n}{(1+h)(2n+1)}}(dx^2+dy^2) + t^{\frac{6}{(1+h)(2n+1)}}dz^2\,, \tag{3.4}$$

which gives the geometry of the metric (2.1).

4. Physical and Geometric Parameters

The tension density, energy density and particle density of the model (3.4) are obtained as-

$$\lambda = \frac{3(2-h)(n-1)}{(1+h)^2(2n+1)t^2},$$
$$\rho = \frac{9n(n+2)}{(1+h)^2(2n+1)^2t^2},$$
$$\rho_p = \frac{3[h(2n^2-n-1)-(n^2-8n-2)]}{(1+h)^2(2n+1)^2t^2}.$$

The spatial volume is

$$V = t^{\frac{3}{(1+h)}}.$$

The expansion scalar is

$$\theta = \frac{3}{(1+h)t}.$$

Hubble parameter is

$$H = \frac{1}{(1+h)t}.$$

The bulk viscosity of the model is

$$\xi = \frac{-h(2n^2+3n+1)+2}{(1+h)(2n+1)^2t}.$$

The shear scalar of the model is

(4.1)
$$\sigma = \frac{\sqrt{3}(n-1)}{(1+h)(2n+1)t}.$$

The mean anisotropy parameter is

(4.2)
$$\Delta = \frac{2(n-1)^2}{(2n+1)^2} = Constant.$$

5. Physical Interpretations

The model given by the equation (3.4) is a Bianchi type-I cosmological model with string in general relativity with constant deceleration parameter(q=constant) and bulk viscosity. The variation of some of the features with time for the model are shown below by taking $n=2, h=-0.5$.

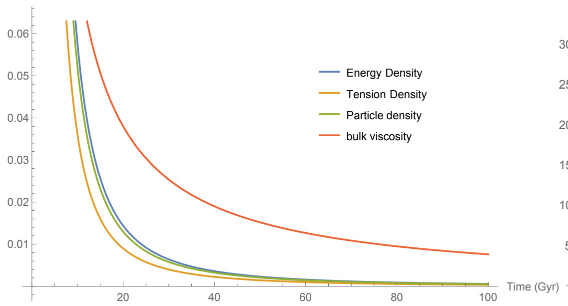

FIGURE 1. Variation of ρ, λ, ρ_p, ξ vs. t.

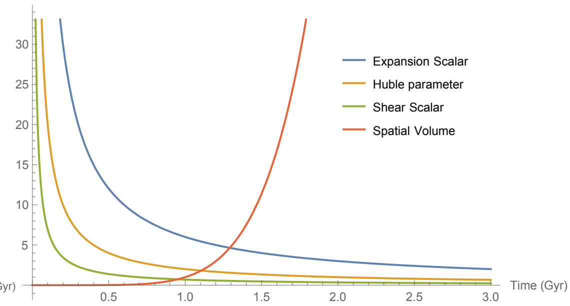

FIGURE 2. Variation of θ, H, σ, V vs. t.

The geometrical and physical behaviors of the model universe for $(1 + h > 0)$ are discussed as

(i) The tension density (λ), energy density (ρ) and particle density (ρ_p) all are infinite as $t = 0$, and are decreasing functions of time t and they all become 0 as $t \to \infty$, Figure 1 , which indicates that the universe starts at $t = 0$ and expand with time. Hence the model admits initial singularity at $t = 0$. This model satisfies the energy density conditions $\rho \geq 0$ and $\rho_p \geq 0$. It is also observed that $\dfrac{\rho_p}{|\lambda|} > 1$ which shows that tension density of string diminishes more quickly than particle density, so the late universe is particle dominated.

(ii) The bulk viscosity $\xi \to \infty$ when t=0 and it decreases with the increasers of time and finally when $t \to \infty$ bulk viscosity ξ vanishes, Figure 1.

(iii) Initially at $t = 0$ the spatial volume is 0 for this model and as time increases the volume also increases, Figure 2. It reaches to infinite value at $t \to \infty$ and so the model represents an expanding universe with respect to time.

(iv) At the initial epoch $t = 0$, the scalar expansion θ as well as Hubble parameter H both are infinite and as the time progresses gradually they decreases and finally they become 0 when $t \to \infty$, Figure 2. Hence the model shows that the universe is expanding with time but the rate of expansion become slow with the increases of time and the expansion end at $t \to \infty$. Since $\dfrac{dH}{dt}$ is negative quantity which also explained that our model universe is expanding with acceleration.

(v) From equation (4.1) and Figure 2 it seen that the value of the shear scalar σ is infinite at initial epoch and decreases with time and become zero at late universe showing that the universe obtained here is shear free in the late time.

(vi) From (4.2) the mean anisotropy parameter $\Delta = constant \neq 0$ for $n \neq 1$ and $\Delta = 0$ for $n = 1$. Also as $t \to \infty$ the value of $\frac{\sigma^2}{\theta^2} = constant \neq 0$ for $n \neq 1$ and $\frac{\sigma^2}{\theta^2} = 0$ for $n = 1$. From both statements we can conclude at late time the universe is anisotropic, when $n \neq 1$ but it is isotropic for $n = 1$ throughout evolution.

6. Conclusion

Here, we have constructed a Bianchi type-I string cosmological models with the help of bulk viscosity and constant deceleration parameter in general relativity. The parameters which are very important in the study of cosmological models are obtained and discussed. The model is expanding,non shearing, anisotropic for $n \neq 1$ and isotropic for $n = 1$. The present universe starts at initial epoch at $t = 0$ with 0 volume and then expand with accelerated rate and the rate of expansion becomes slow with increase of time. The bulk viscosity coefficient plays a significance role in the cosmological consequences. The tension density diminishes with the faster rate than particle density which shows that present day universe is dominated by particles.

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Mathematical analysis on anisotropic Bianchi Type-III inflationary string Cosmological models in Lyra geometry

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Abstract

Objectives: To present a new solution to the field equations obtained for Bianchi type-III universe by using the law of variation of H, which yields constant DP. **Methods**: We study a Bianchi type-III cosmological model with a cloud strings with particles connected to them in Lyra geometry. To find the exact solutions of survival field equations we consider here that the shear scalar and scalar expansion are proportional to each other $(\sigma\alpha\theta)$ that leads to the equation $b = c^m$ and secondly we adopt the assumption considering the Deceleration Parameter q as a negative constant quantity giving the inflationary model. The geometrical and physical properties are studied and compared with the recent observational data. **Findings**: The present model starts at t=0 with 0 volume and as time progresses it expands with accelerated rate and the model shows that the present universe is particle dominated.

Keywords: Bianchi type III metric; inflation; Lyra geometry; cloud string; anisotropic

1 Introduction

It is still an interesting area of research to discover its unknown phenomenon that has yet to observe to study the ultimate fate of the universe. But till today cosmologists cannot make a final and comprehensive conclusion about the origin and evolution of universe with strong evidence. So more and more investigations are required to discover and understand the unknown phenomenon of the universe and many mysterious particles which are to be observed to study the ultimate fate of the universe. The cosmologist or researchers developed the string theory to describe the universe, its early stages and the evolution during the time. So, the study on string cosmology is becoming very interesting area for the cosmologist, because of its significant role in the study of formation and evolution of the universe at the early stages and to understand about the future evolution. In the field of the general relativity the investigation of string was generally initiated by prominent authors, Stachel [1] and Letelier [2,3]. In the recent past years many prominent authors have investigated the cosmic strings in the context of Lyra geometry since it can play a great role in describing the universe in the early stages of evolution (Kibble [4,5]) and they can give rise to density perturbations which can lead to the creation of large scale structure (galaxies) of the universe(Zel'dovich [6,7]).

The strings are crucial topological stable defects occurred due to the phase transition at the early days of the universe, when the temperature is lower than a specific temperature, known as critical temperature. The occurrence of strings inside the universe results in anisotropy within the space-time, though the strings aren't seen in the present epoch. Strings cause no damage to the cosmological models, but they can result in very interesting astrophysical effects. Because of the great position of strings in the description of the evolution of the early universe, nowadays, many prominent authors have significantly studied the string cosmology. Soon, after the big-bang, there was a breaking of symmetry during the time of phase transition and the cosmic temperature went down below some critical temperatures due to which the strings arose, as according to grand unified theories(Everett [8], Vilenkin1 [9,10]).

Though the Einstein general relativity is one of the most acceptable theory in modern era to describe the universe, it is unable to explain some of the strong unknown facts about the universe such as accelerated expansion of universe, reason behind the expansion etc. So the several researchers are trying to solve and explain those aspects of the universe by the help of different modified theories of Einstein General theory of relativity such as Weyl's theory, Brans-Dicke theory, f(R) gravity theory, f(R, T) theory, Lyra geometry, scalar tensor theory etc. Among these theories Lyra geometry is one of the most important modified theory. Inspired by the geometrization of gravitation, Weyl [11] developed a theory by geometrizing electromagnetism and gravitation, known as Weyl's theory. However, this theory was criticized and not accepted due to the condition of non-integrability of length of vector under parallel displacement. To remove this non integrability condition to H. Weyl's geometry, Lyra [12] suggested a modification by introducing a gauge function ϕ_μ into the structureless geometry to Riemannian geometry and this modified Riemannian geometry proposed by Lyra is known as Lyra's Geometry. Halford [13] constructed a theory in cosmology in Lyra geometry, and he showed that in general theory of relativity the constant ϕ_μ performs as cosmological constant term. Bhamra [14], Beesham [15], Singh and Singh [16,17], Rahaman et al. [18], Reddy and Rao [19,20], Yadav et al. [21], Adhav et al. [22], Reddy [23], Rao et al. [24] are the some of the prominent authors who have already constructed various cosmological models in Lyra geometry. Recently, Singh et al. [25], W. D. R. Jesus, and A. F. Santos [26], Singh and Mollah [27], Mollah et al. [28], Yadav and Bhardwaj [29], Maurya and Zia [30], A. K. Yadav [31] have studied various cosmological models in different contexts considering Lyra's geometry.

Inspired by the above discussions, here we have studied the string cosmological model with particles connected to them in Bianchi type-III universe considering Lyra geometry. The work done in this paper and findings are somewhat distinct from the earlier findings. In the sec.2, Bianchi type-III metric is presented and the field equations in Lyra geometry are derived; In the sec.3, the determinate solutions of the field equations are determined by using some plausible conditions. Physical and geometrical properties of our model with the help of graph are discussed in sec.4; In sec.5 conclusions of the paper are given.

2 The metric and field equations

We consider the Bianchi type-III metric as

$$ds^2 = a^2dx^2 + b^2e^{-2x}dy^2 + c^2dz^2 - dt^2 \tag{1}$$

Here, a, b and c are the functions of 't' alone. For the above metric let

$$x^1 = x, x^2 = y, x^3 = z \text{ and } x^4 = t \tag{2}$$

The field equations with gauge function and $8\pi G = 1, \mathrm{C} = 1$ in Lyra manifold is

$$R_{ij} - \frac{1}{2}Rg_{ij} + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -T_{ij} \tag{3}$$

Where, ϕ_i is the displacement field vector given by

$$\phi_i = (0,0,0,\beta) \tag{4}$$

Here, β is the function of time.

The energy-momentum tensor for a cosmic string is taken as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \tag{5}$$

Here, $\lambda = \rho - \rho_p$ is the string tension density, ρ is the energy density and ρ_p is the particle density of the string. Also, u^i is the four velocity vector and x^i is the unit space-like vector which represents the direction of strings, and they are given by

$$x^i = \left(0,0,c^{-1},0\right) \text{ and } u^i = (0,0,0,1) \tag{6}$$

$$\text{Such that } u_i u^i = -1 = -x_i x^i \text{ and } u_i x^i = 0 \tag{7}$$

If R is the average scale factor then volume is

$$V = abc = R^3 \tag{8}$$

The expansion scalar is given by

$$\theta = u^i_{,i} = \frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c} \tag{9}$$

Hubble parameter is given by

$$H = \frac{1}{3}\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c}\right) \tag{10}$$

The shear scalar is given by

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left[\left(\frac{\dot{a}}{a}\right)^2 + \left(\frac{\dot{b}}{b}\right)^2 + \left(\frac{\dot{c}}{c}\right)^2 - \frac{\dot{a}\dot{b}}{ab} - \frac{\dot{b}\dot{c}}{bc} - \frac{\dot{c}\dot{a}}{ca}\right] \tag{11}$$

And the mean anisotropy parameter is

$$\Delta = \frac{1}{3}\sum_{r=1}^{3}\left(\frac{H_r - H}{H}\right)^2 \tag{12}$$

Where, $H_r(r = x, y, z)$ denotes the directional Hubble factors, and they are given by $H_x = \frac{\dot{a}}{a}, H_y = \frac{\dot{b}}{b}$ and $H_z = \frac{\dot{c}}{c}$ for the metric (1).

The field Equation (3) with the Equations (4), (5), (6) and (7) for the Equation (1) takes the form

$$\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{b}\dot{c}}{bc} + \frac{3}{4}\beta^2 = 0 \tag{13}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{c}}{c} + \frac{\dot{c}\dot{a}}{ca} + \frac{3}{4}\beta^2 = 0 \tag{14}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} + + \frac{3}{4}\beta^2 - \frac{1}{a^2} = \lambda \tag{15}$$

$$\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{c}\dot{a}}{ca} - \frac{3}{4}\beta^2 - \frac{1}{a^2} = \rho \tag{16}$$

$$\frac{\dot{a}}{a} - \frac{\dot{b}}{b} = 0 \tag{17}$$

Here the overhead dots represent the order of differentiation w. r. t. time 't'.

3 Solutions of the field equations

Solving Equation (17), we have

$$a = rb \tag{18}$$

Here r is the integration constant. With generality, we can take $r = 1$.

And using it, (18) can be written as,

$$a = b \tag{19}$$

Thus using relation (19) the field Equations (13), (14), (15) and (16) reduces to

$$\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{b}\dot{c}}{bc} + \frac{3}{4}\beta^2 = 0 \tag{20}$$

$$2\frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{3}{4}\beta^2 - \frac{1}{b^2} = \lambda \tag{21}$$

$$\frac{\dot{b}^2}{b^2} + 2\frac{\dot{b}\dot{c}}{bc} - \frac{3}{4}\beta^2 - \frac{1}{b^2} = \rho \tag{22}$$

We have 3 highly nonlinear independent differential Equations (20), (21) and (22) with variables b, c, λ, β and ρ which are unknown. So to obtain the exact solutions of above equations we must have two extra conditions. So here we used the following two physically plausible conditions:

Here, we take the assumption that the shear scalar and expansion scalar are proportional to each other $(\sigma \alpha \theta)$ which leads to the equation

$$b = c^m \tag{23}$$

Here $m \neq 0$ is a constant.

This is based on observations of velocity and red-shift relation for an extragalactic source which predicted that the Hubble expansion is 30 percent isotropic, which is supported by the works of Thorne [32], Kantowski and Sachs [33], Kristian and Sachs [34]. In particular, it can be said that $\frac{\sigma}{H} \geq 0.30$, where σ and H are respectively shear scalar and Hubble constant. Also, Collins et al. [35] has shown that if the normal to the spatially homogeneous line element is congruent to the homogeneous hyper-surface then $\frac{\sigma}{\theta} =$ constant , θ being the expansion factor.

Secondly we adopt the assumption proposed by Berman [36] about the variation of hubble's parameter H, which gives constant DP in the model as

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = (\text{constant}) \tag{24}$$

When h is negative then the model universe expand with acceleration and when q is positive then the model universe contract with deceleration. Although the present observations like CMBR and SNe Ia suggested the negative value of q but it can be remarkably state that they are not able to deny about the decelerating expansion (positive q) of universe. This is the most suitable condition to explore the physically meaningful solutions of the above field equations.

The scale factor R admits the solution-

$$R = (ht + k)^{\frac{1}{1+q}}, \quad q \neq -1 \tag{25}$$

Here $h \neq 0$ and k are integration constants.

Using the Equations (8), (19), (23) and (25), we get,

$$a = b = (ht + k)^{\frac{3m}{(1+q)(2m+1)}} \tag{26}$$

$$c = (ht + k)^{\frac{3}{(1+q)(2m+1)}} \tag{27}$$

Without loss of generality we take h=1 and k=0 then (26), (27) becomes

$$a = b = t^{\frac{3m}{(1+q)(2m+1)}}, \quad c = t^{\frac{3}{(1+q)(2m+1)}} \tag{28}$$

Using (28) the metric (1) can be reduced to

$$ds^2 = t^{\frac{6m}{(1+q)(2m+1)}} \left(dx^2 + e^{-2x}dy^2\right) + t^{\frac{6}{(1+q)(2m+1)}} dz^2 - dt^2 \tag{29}$$

This gives the geometry of the metric (1).

4 Physical and geometrical parameters

We obtained some of the important physical and geometrical parameters that are useful for the discussion on the evolution of the universe.

Using (28) in (22) we obtained ρ as

$$\rho = \frac{3(m+1)(2-q)}{(1+q)^2(3m+1)t^2} - t^{-\frac{6m}{(1+q)(2m+1)}} \tag{30}$$

From (19) and (20) using (28) we obtained

$$\lambda = \frac{3(m-1)(2-q)}{(1+q)^2(3m+1)t^2} - t^{-\frac{6m}{(1+q)(2m+1)}} \tag{31}$$

From (30), (31) we obtained the ρ_p as

$$\rho_p = \frac{6(2-q)}{(1+q)^2(3m+1)t^2} \tag{32}$$

The gauge function β is obtained as

$$\beta^2 = \frac{4\left[(m+1)(2m+1)(1+q) - 3\left(m^2+m+1\right)\right]}{(1+q)^2(3m+1)^2t^2} \tag{33}$$

The spatial volume, scalar expansion, Hubble parameter, shear scalar and mean anisotropy parameter of the model are

$$V = t^{\frac{3}{1+h}} \tag{34}$$

$$\theta = \frac{3}{(1+h)t} \tag{35}$$

$$H = \frac{1}{(1+h)t} \tag{36}$$

$$\sigma = \frac{\sqrt{3}(m-1)}{(1+h)(2m+1)t} \tag{37}$$

$$\Delta = \frac{2(m-1)^2}{(2m+1)^2} = \text{Const.} \tag{38}$$

5 Interpretations of the solutions

The Equation (29) represents the Bianchi type-III anisotropic cosmological model with strings in Lyra geometry. The physical and geometrical behavior of model for $-1 < h < 0$ are discussed as

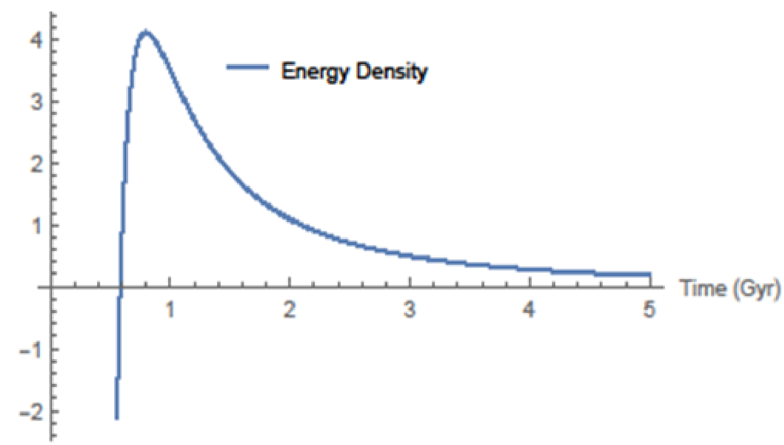

Fig 1. Energy density Vs. Time

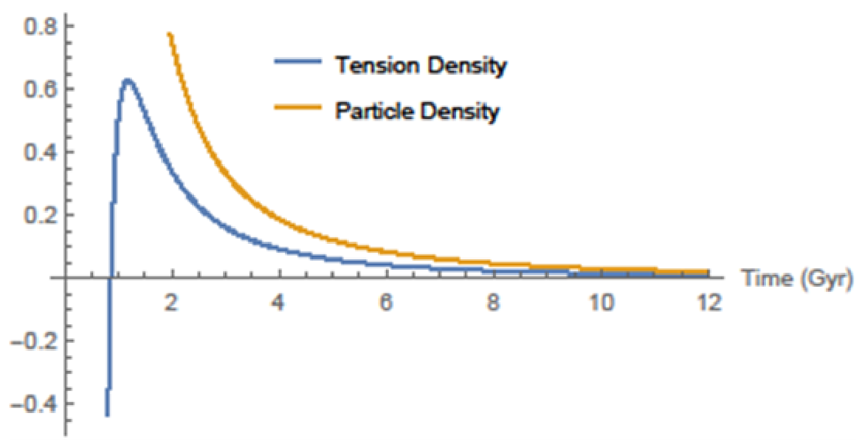

Fig 2. Tension density, Particle density Vs. Time

- From the expressions of energy density ρ and tension density λ given by Equations (30) and (31), we have observed that both of them are negative at the initial epoch of time but as the time progresses they changes sign from negative to positive and then decreases gradually and finally become zero when $t \to \infty$. Figure 1 presents the variations of energy density with time t, which clearly indicate that at infinite time, $\rho \to 0$. Again, the nature of the variations of tension density λversus time t is shown by Figure 2. From this we can conclude that initially when $t \to 0$, $\lambda$is negative but with the passage of cosmic time it changes sign from negative to positive and finally at infinite time it becomes zero, which is supported by Letelier[2,3]
- For the model universe, the expression of particle density ρ_p is found as the Equation (32) and its variations versus cosmic time is shown in Figure 2. Which shows that $\rho_p$is always positive which decreases from $\rho_p = \infty$ as t = 0 to $\rho_p = 0$ whenever $t \to \infty$. Also, Figure 2 depicts that the tension density diminishes more quickly than the particle density, therefore with the passage of time string will disappear leaving the particles only. Hence, our model is realistic one. And it is also seen that $\frac{\rho_p}{|\lambda|} > 1$, that shows that tension density of string diminishes faster than particle density. This tells us that the late

universe is particle dominated.

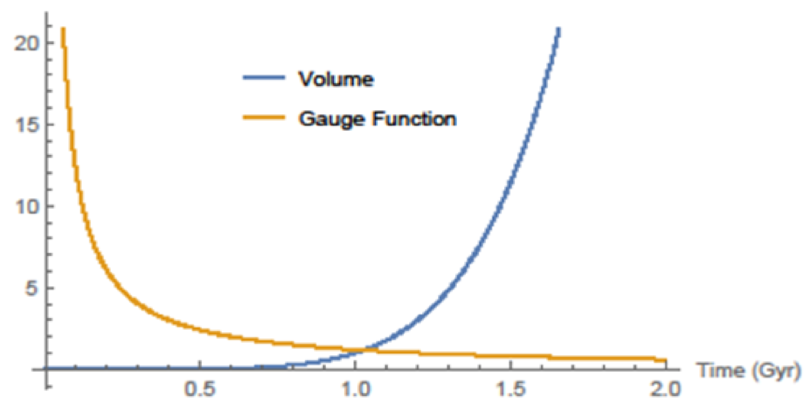

Fig 3. Volume, Gauge Function Vs. Time

- In this model universe, at the initial epoch of time, the gauge function β^2 given by Equation (33) is found to be infinite and it decreases with the increase of time. Finally, the gauge function $\beta^2 \to 0 \; when \; t \to \infty$.
- The volume for this model increases as time increases. The expression of volume V as obtained in Equation (34) shows that the model universe begin with initial singularity at t = 0 from V = 0 i.e. our model universe starts from zero volume at t = 0 and as time increases it expands and also when $t \to \infty, \; V \to \infty$. So, for $1+q>0$ the model shows that the universe is expanding with accelerated rate.

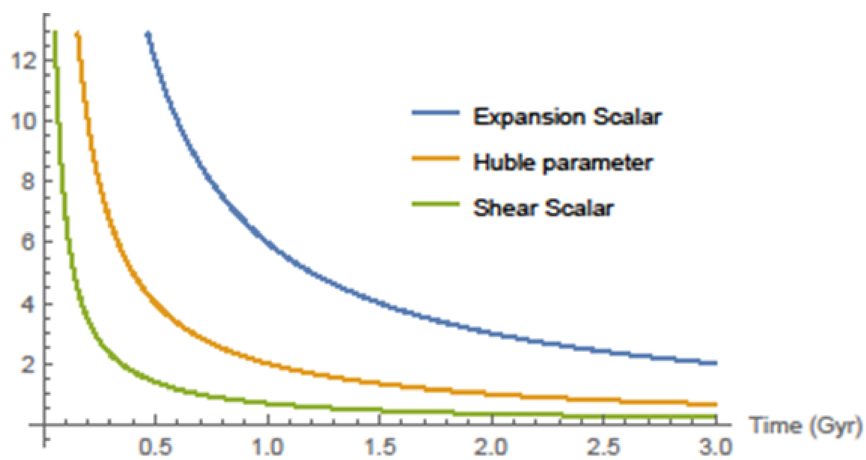

Fig 4. Expansion scalar, Hubble parameter, Shear scalar Vs. Time

- From the expansion scalar and Hubble parameter for the model (29), at $t=0$, the θ and H both are infinite and as the time progresses gradually they decrease and finally θ and H become 0 when t is infinite. Hence, the model shows that the universe expands with time but the rate of expansion slower as the increases of time and the expansion stops at $t \to \infty$. Again, it is seen that $\frac{dH}{dt} = -\frac{1}{(1+h)t^2} = 0$, when t approaches infinity and this implies the greatest value of Hubble's Parameter and accelerated expansion of the universe. These behaviors of the model are presented in Figure 4
- In Equation (37) and Figure 4. It is seen that the value of the shear scalar $\sigma \to \infty$ at initial epoch and it decreases as the time increases and become zero at late universe showing that the universe obtained here is shear free in the late time.

- From Equation (38) the mean anisotropy parameter $\Delta = \text{constant}(\neq 0)$ for $m \neq 1$ and $\Delta = 0$ for $m = 1$. Also as $t \to \infty$ the value of $\frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{3(2m+1)^2} = \text{constant}(\neq 0)$ for $m \neq 1$ and $\frac{\sigma^2}{\theta^2} = 0$ for $m = 1$ From both statements we can conclude that this model is anisotropic for large value of t when $m \neq 1$ but it is isotropic for m=1.

6 Conclusions

In this article, we have attempted to present a new solution to the field equations obtained for Bianchi type-III universe in Lyra geometry by using the law of variation of Hubble's parameter H which yields constant DP. This variational law for H in Equation (24) explicitly determine the values of the average scale factors(R). So here we have constructed a Bianchi type-III cosmological model attached to strings in Lyra geometry, which is an anisotropic and inflationary model. The physical and geometrical parameters which are very important in the description of cosmological models have been obtained and discussed. The model starts at t = 0 with volume 0 and it expand with acceleration in which the strings disappear leaving the particles only in the late universe giving particle dominated universe which agrees with the present observational data. The model is expanding, anisotropic for $m \neq 1$ at late universe, accelerating, non-shearing and admits initial singularity at t=0, that also agree with the present day observational data. Through this study, we hope to present a better knowledge of the cosmological evolution of the present universe with the help of Bianchi type-III universe in Lyra geometry.

Financial disclosure/conflict of interest

The authors declare that there was no financial aid received and no conflict of interest associated with this research work.

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Higher Dimensional Bianchi Type-I Cosmological Models With Massive String in General Relativity

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Here we studied Bianchi type-I cosmological models with massive strings in general relativity in five dimensional space time. Out of the two different cases obtained here, one case leads to a five dimensional Bianchi type-I string cosmological model in general relativity while the other yields the vacuum Universe in general relativity in five dimensional space time. The physical and geometrical properties of the model Universe are studied and compared with the present day's observational findings. It is observed that our model is anisotropic, expanding, shearing, and decelerates at an early stage and then accelerates at a later time. The model expands along x, y, and z axes and the extra dimension contracts and becomes unobservable at $t \to \infty$. We also observed that the sum of the energy density (ρ) and the string tension density (λ) vanishes ($\rho + \lambda = 0$).

Keywords: five dimension, cloud string, bianchi type-I space-time, general relativity, anisotropic

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1 INTRODUCTION

Nowadays, there has been drastic interest in string cosmology because of its important role in the study of the origin of the Universe and its very early phases before the formation of particles. It is an interesting concept for cosmologists to study and discover its unknown phenomena that have yet to be observed to study and explore the hidden information of the Universe. So cosmologists have taken an enormous interest to understand the past evolution, present state, and future evolution of the Universe. The general relativistic study of string was started by Letelier (Letelier, 1983) and Stachel (Stachel, 1980), who developed a classical concept of the geometric strings. Due to the key role of strings in describing the evolution of the early stage of our Universe, these days, many distinguished authors are inquisitive about cosmic strings within the framework of general relativity (Kibble (Kibble, 1976; Kibble, 1980)) and it is believed that strings give rise to density perturbations leading to the creation of the massive scale structures (like galaxies) of the Universe (Zel'dovich (Zel'dovich et al., 1974; Zel'dovich, 1980)). These strings have stress-energy and they are classified as geometric strings and massive strings. The occurrence of strings in the Universe results in anisotropy in space-time, though the strings cannot be seen in the latest epoch. These strings are not harmful to the cosmological models, alternatively they can result in plenty of very interesting astrophysical outcomes. There was a spontaneous symmetry breaking of the Universe during the phase transition in the early stage of the Universe after the big-bang explosion and those cosmic strings which are very important topological defects arose in the early epoch as the cosmic temperature went down below a few critical temperature points which are consistent with grand unified theories (Everett (Everett, 1981), Vilenkin (Vilenkin, 1981a; Vilenkin, 1981b)).

The study of cosmological models in higher dimensional space-time provides us with an idea that our present Universe is much greater than the Universe at the early stage of evolution due to the accelerated expansion of the Universe. So, nowadays it is becoming very interesting to study string cosmology in higher-dimensional space-time in the framework of general relativity. The possibility of space-time possessing more than 4D (Higher Dimensions) has attracted many authors to study higher dimensional models to study cosmology. The higher dimensional model was introduced by Kaluza (Kaluza, 1921)and Klein (Klein, 1926) in an effort to unify gravity with electromagnetism. Higher-dimensional models can be regarded as a tool to illustrate the late time expedited expanding paradigm (Banik and Bhuyan (Banik and Bhuyan, 2017)). Investigation of higher-dimensional space-time can be regarded as a task of paramount importance as the Universe might have come across a higher dimensional era during the initial epoch (Singh et al.(Singh et al., 2004)). Marciano (Marciano, 1984) asserts that the detection of a time-varying fundamental constant can possibly show us the proof for extra dimensions. According to Alvax and Gavela (Alvarez and Gavela, 1983) and Guth (Guth, 1981), extra dimensions generate a huge amount of entropy which gives a possible solution to the fitness and horizon problems. Since we are living in a 4D space-time, the hidden extra dimension in 5D is highly likely to be associated with the invisible Dark Matter and Dark Energy (Chakraborty and Debnath (Chakraborty and Debnath, 2010)). Many researchers have investigated various Bianchi type models in the field of five -dimensional space-time to explore the hidden information of the Universe. Chatterjee (Chatterjee, 1993) constructed a cosmological model in higher dimensional inhomogeneous space-time with massive strings.

From the observational data, we found that our Universe is homogeneous and isotropic on a large scale, however, no physical evidence denies the chance of an anisotropic Universe. In fact, theoretical arguments are presently promoting the existence of an anisotropic phase of the Universe that approaches the isotropic phase as suggested by Charles (Charles, 1968), Hinshaw et al. (Hinshaw et al., 2003), Page et al. (Page et al., 2007). Anisotropy plays a vital role in the early phase of the evolution of the Universe and so studying homogeneous and anisotropic cosmological models is considered important. Generally, the Bianchi-type models are spatially homogeneous and are in general anisotropic. The simplicity of the field equations made Bianchi type space-time useful in constructing models which are spatially homogeneous and isotropic. For the Bianchi type I cosmological models the corresponding anisotropy parameters are time-dependent. As time increases, for a suitable choice of the scalars, the Universe which was initially anisotropic starts to become isotropic and finally attains isotropy after some large cosmic time, which agrees with the present-day observational data such as cosmic microwave background (CMB) and type Ia supernovae.

We can find fascinating studies considering the anisotropic Universe in Bianchi type-I space-time by Mohanty et al. (Mohanty et al., 2002), Sahoo et al. (Sahoo et al., 2017). A Bianchi type-I cosmological model in higher dimensional space-time with string was investigated by Krori et al. (Krori et al., 1994) where they found that the strings and matter coexist throughout the evolution of the Universe. Rahaman et al.(Rahaman et al., 2003) obtained the exact solutions of the field equations of a five-dimensional space-time within the framework of Lyra manifold with massive string as a source of gravitational field. Mohanty and Samanta (Mohanty and Samanta, 2010a) constructed an LRS Bianchi type-I inflationary string cosmological model in five-dimensional space-time with massive scalar field in general relativity and obtained that the sum of energy density and tension density is zero. Also, Mohanty and Samanta (Mohanty and Samanta, 2010b) constructed string cosmological models with massive scalar field in five dimensional space-time considering Lyra manifold and obtained that the models avoid the initial singularity. Bianchi type-III cosmological models in five-dimensional space-time in general relativity with massive string as an origin of gravitational field were constructed by Samanta et al. (Samanta et al., 2011). Bianchi type-III string cosmological models in general relativity in presence of magnetic field were investigated by Kandalker et al. (Kandalkar et al., 2012), where they solved the field equations by using the Reddy string condition ($\rho + \lambda = 0$). Samanta and Debata (Samanta and Debata, 2011) constructed a five dimension Bianchi type-I string cosmological model in the framework of Lyra manifold. Singh and Mollah (Sing and Mollah, 2016) studied an LRS Bianchi type-I cosmological model with perfect fluidity in the framework of Lyra geometry in five dimensional space-time by using constant deceleration parameter. Kaiser (Kaiser and Stebbins, 1984), Banerjee (Banerjee et al., 1990), Wang (Wang, 2005), Bali et al. (Bali et al., 2007), Power and Deshmukh (Pawar and Deshmukh, 2010), Sahoo and Mishra (Sahoo and Mishra, 2015), Singh (Singh, 2013), Goswami (Goswami et al., 2016), Reddy (Reddy and Naidu, 2007), Khadekar (Khadekar et al., 2005; Khadekar et al., 2007; Khadekar and Tade, 2007), Yadav (Yadav et al., 2011), Ladke (Ladke, 2014), Singh and Baro (Singh and Baro, 2020), Baro and Singh (Baro and Singh, 2020) are some of the authors who studied different string cosmological models within the general relativity in a different contexts in various space-times. In addition to the above mentioned authors, recently Choudhury (Choudhury, 2017), Tripathi (Tripathi et al., 2017), Dubey et al. (Dubey et al., 2018), Tiwari et al. (Tiwari et al., 2019), Ram et al. (Ram and Verma, 2019), Mollah et al. (Mollah et al., 2019), and Baro et al.(Baro et al., 2021) investigated different string cosmological models in various space times.

The above discussion motivated us to investigate here the five-dimensional string cosmological models with particles attached to them in Bianchi type-I space-time in general relativity to investigate the different possibilities of the Bianchi type model Universe which transitions from anisotropic in early evolution to isotropic at a later time point where the survival field equations are solved by making some simplifying assumptions. Also, the physical and geometrical properties of some parameters of our model Universe are discussed in detail.

2 THE METRIC AND FIELD EQUATIONS

We consider the Bianchi type-I metric in five dimensional space-time in the form of

$$ds^2 = -dt^2 + a^2dx^2 + b^2dy^2 + c^2dz^2 + D^2dm^2 \tag{1}$$

where a, b, c and D are the metric functions of cosmic time 't' only. Here the extra (fifth) coordinate 'm' is taken to be space-like.

The Einstein's field equation in general relativity is given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi T_{ij} \tag{2}$$

The energy-momentum tensor for a cloud string is taken as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \tag{3}$$

where ρ, λ, ρ_p are the energy density of cloud of strings, tension density, particle density of the string respectively and they satisfy the equation $\rho = \rho_p + \lambda$. The co-ordinates are co-moving, x_i is a unit space-like vector towards the direction of strings and u_i is the five velocity vector which satisfies the conditions given below.

$$u^i u_i = -x^i x_i = -1 \tag{4}$$

$$and \quad u^i x_i = 0 \tag{5}$$

$$u^i = (0,0,0,0,1) \quad and \quad x^i = (a^{-1},0,0,0,0) \tag{6}$$

For the metric 1 by using **Equations 3–6** in the field **Equation 2** yields

$$\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\ddot{D}}{D} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{b}\dot{D}}{bD} + \frac{\dot{c}\dot{D}}{cD} = 8\pi\lambda \tag{7}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{c}}{c} + \frac{\ddot{D}}{D} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{c}\dot{D}}{cD} = 0 \tag{8}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{D}}{D} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{b}\dot{D}}{bD} = 0 \tag{9}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ab} + \frac{\dot{b}\dot{c}}{bc} = 0 \tag{10}$$

$$\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{b}\dot{D}}{bD} + \frac{\dot{c}\dot{D}}{cD} = 8\pi\rho \tag{11}$$

where an over dot and double over dot denote the first derivative and the second derivative w.r.t. cosmic time 't' respectively.

3 SOLUTION OF THE FIELD EQUATIONS

In this section we find physically meaningful solutions of the set of field **Equations 7–11** by taking some simplifying assumptions.

3.1 Case-I(Isotropic Model)

Let us consider the Isotropic Model as

$$a = b = c = t^{l_1} \quad and \quad D = t^{l_2} \tag{12}$$

where l_1 and l_2 are arbitrary constants.

By using **Eq. 12** in **Equations 7–11**, we get

$$\frac{1}{t^2}\left(3l_1^2 + 2l_1l_2 - 2l_1 - l_2 + l_2^2\right) = 8\pi\lambda \tag{13}$$

$$\frac{1}{t^2}\left(3l_1^2 + 2l_1l_2 - 2l_1 - l_2 + l_2^2\right) = 0 \tag{14}$$

$$\frac{1}{t^2}\left(2l_1^2 - l_1\right) = 0 \tag{15}$$

$$\frac{1}{t^2}\left(3l_1^2 + 3l_1l_2\right) = 8\pi\rho \tag{16}$$

Now from **Equation 15** we get.

$l_1 = 0 \;\; or \;\; l_1 = \frac{1}{2}$

For $l_1 = 0$, from **(14)** we obtained

For $l_1 = \frac{1}{2}$, from **(14)** we obtained.

$l_2 = \frac{1}{2} \quad or \quad l_2 = -\frac{1}{2}$

Eq. 12 shows that, with the increases of time t the Universe expands indefinitely if $l_1 > 0$ and the extra dimension "m" contract to a Planckian length as $t \to \infty$ if $l_2 < 0$. The string cosmological model will be physically realistic only if we take $l_1 = \frac{1}{2} > 0$ and $l_2 = -\frac{1}{2} < 0$.

In this case the geometry of the model is described by the metric

$$ds^2 = -dt^2 + t\left(dx^2 + dy^2 + dz^2\right) + t^{-1}dm^2 \tag{17}$$

Using $l_1 = \frac{1}{2}$ and $l_2 = -\frac{1}{2}$ in **Equation 13**, the string tension density is obtained as

$$\lambda = 0 \tag{18}$$

Using $l_1 = \frac{1}{2}$ and $l_2 = -\frac{1}{2}$ in **Equation 16**, the energy density is obtained as

$$\rho = 0 \tag{19}$$

And using **18** and **19**, the particle density is obtained as

$$\rho_p = 0 \tag{20}$$

This shows that the five-dimensional isotropic Bianchi type-I model in general relativity with strings do not survive and so it results in the five-dimensional vacuum Universe in the context of the general theory of relativity.

3.2 Case-II(Anisotropic Model)

The models with the anisotropic background are the most suitable models to describe the early stages of the Universe. Bianchi type-I models are among the simplest models with the anisotropic backgrounds.

In this case, let us consider

$$a = t^{k_1}, b = t^{k_2}, c = t^{k_3} \quad and \quad D = t^{k_4} \tag{21}$$

Where k_1, k_2, k_3 *and* k_4 are arbitrary constants.

Now from **Equations 7**, **11**, by the use of **21**, we find

$$\lambda = \frac{1}{8\pi t^2}\left[k_2^2 + k_3^2 + k_4^2 + k_2k_3 + k_3k_4 + k_4k_2 - (k_2 + k_3 + k_4)\right] \tag{22}$$

$$\rho = \frac{1}{8\pi t^2}\left[k_1(k_2 + k_3 + k_4) + k_2k_3 + k_3k_4 + k_4k_2\right] \tag{23}$$

and particle density is

$$\rho_p = \frac{1}{8\pi t^2}\left[(k_1 + 1)(k_2 + k_3 + k_4) - \left(k_2^2 + k_3^2 + k_4^2\right)\right] \tag{24}$$

We observed that the anisotropic three space will expand as $t \to \infty$ when k_1, k_2 and k_3 are all positive and the extra dimension will contract as $t \to \infty$ if $k_4 < 0$.

The Geometry of the model is described by the metric

$$ds^2 = -dt^2 + t^{2k_1}dx^2 + t^{2k_2}dy^2 + t^{2k_3}dz^2 + t^{2k_4}dm^2 \tag{25}$$

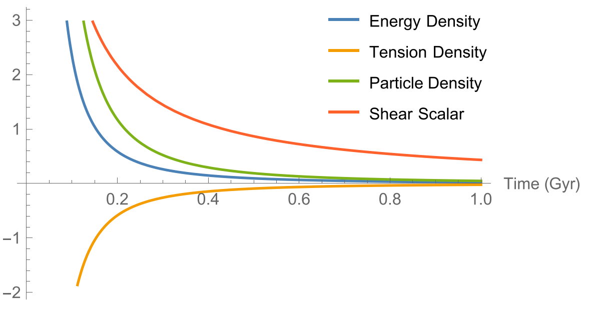

FIGURE 1 | Variation of ρ, λ, ρ_p, σ vs time t.

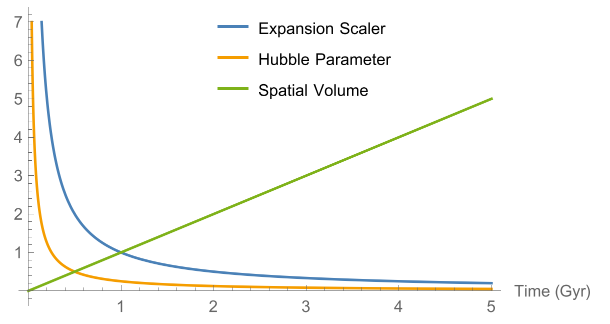

FIGURE 2 | Variation of θ, H, V vs time t.

The Scalar Expansion θ for model 25 is given by

$$\theta = \frac{l}{t} \tag{26}$$

Where, $l = k_1 + k_2 + k_3 + k_4$

The Hubble Parameter is given by

$$H = \frac{l}{4t} \tag{27}$$

The Spatial Volume of the Universe is obtained as

$$V = t^l \tag{28}$$

The Shear Scalar is obtained as

$$\sigma^2 = \frac{1}{2t^2}\left[k_1^2 + k_2^2 + k_3^2 + k_4^2 - \frac{1}{4}\right] \tag{29}$$

Deceleration Parameter q is given by

$$q = \frac{4}{l} - 1 \tag{30}$$

In this case, it is observed that the value of the deceleration parameter q is a positive constant when $l < 4$ which implies that our model Universe 25 decelerates in the standard way and the value of the deceleration parameter q is a negative constant when $l > 4$ which implies that our model Universe accelerates in the standard way. However, in the early stage of the evolution of the Universe the Bianchi type models represent the cosmos and though the Universe decelerates in the standard way in the early Universe, it will accelerate in finite time because of cosmic recollapse where the Universe in turns inflates "decelerates and then accelerates" (Kandalkar and Samdurkar (Kandalkar and Samdurkar, 2015)).

4 PHYSICAL INTERPRETATIONS OF THE SOLUTIONS

Case I: From the case I, it is observed that $\rho = \lambda = \rho_p = 0$, which results in the five-dimensional vacuum Universe in general relativity. So, the isotropic Bianchi type-I five dimensional cosmic strings Universe do not survive in general Relativity.

Case II: In case II, we have constructed the anisotropic Bianchi type-I string cosmological model in general relativity in five-dimensional space-time given by **Equation 25**. Taking $k_1 = \frac{2}{5}$, $k_2 = \frac{3}{5}$, $k_3 = \frac{1}{4}$ and $k_4 = \frac{-1}{4}$, the variation of the parameters of model 25 are shown by **Figures 1, 2.** The physical and geometrical behavior of the model can be discussed as.

1) We observed that our model expanded along x, y, and z axes as $t \to \infty$ when, k_1, k_2 and k_3 are all positive and the extra dimension contracts and becomes unobservable at $t \to \infty$, when $k_4 < 0$.
2) It is observed that at the initial epoch, i.e., $t = 0$, the energy density $\rho \to \infty$ and $\rho \to 0$ as $t \to \infty$(shown by **Figure 1**) and it satisfies the reality condition when, $k_1 + k_2k_3 + k_3k_4 + k_4k_2 > k_1^2$
3) It is also observed that at the initial epoch, i.e., $t = 0$, the string tension density $\lambda \to -\infty$ and $\lambda \to 0$ as $t \to \infty$ (**Figure 1**). From **Eqs. 22, 23**, we obtained an equation of state $\rho + \lambda = 0$, which occurs naturally in our case.
4) Also from **Figure 1**, it is observed that the particle density (ρ_p) is infinite when $t = 0$ and as time increases it decreases and finally it becomes 0 as $t \to \infty$. It satisfies the reality condition when, $(k_1^2 + k_2^2 + k_3^2 + k_4^2) < 1$
5) The spatial volume V in this model is 0 at initial epoch $t = 0$, and it increases w. r.t time which shows that our model Universe is expanding with the evolution of time, which is clearly shown in **Figure 2**.
6) The expansion scalar $\theta \to \infty$ at initial epoch $t = 0$, and as the time progresses gradually it decreases and finally it becomes 0 when $t \to \infty$(as shown in **Figure 2**). Hence the model shows that the Universe is expanding with the increase of time but the rate of expansion is slower as time increases and the expansion stops at $t \to \infty$.
7) It is observed that the value of deceleration parameter q is positive when $l < 4$ which implies that our model Universe decelerates for an instant. It is also observed that the value of deceleration parameter q is negative when $l > 4$, which implies that our model Universe accelerates in the standard way which is in accordance with the present-day observational scenario of accelerating Universe. Here, for our proposed model it may be noted that Bianchi type models represent the cosmos in its initial stage of evolution and there may be some possibilities to have an anisotropic Universe for some finite duration but the initial anisotropy of the Bianchi type -I Universe quickly dies away and the Universe turns to an isotropic one in the late

Universe. However, though the Universe decelerates in the standard way for an instant, it will accelerate in finite time because of cosmic recollapse where the Universe in turns inflates "decelerates and then accelerates" (Kandalkar and Samdurkar (Kandalkar and Samdurkar, 2015)). The decelerating behavior of the expansion in the early stage and the accelerating behavior of the expansion of the present Universe has been indicated by many cosmological observations such as cosmic microwave background (CMB), clusters of galaxies, and type Ia supernovae, etc. which have suggested that the reason for this transition from deceleration to acceleration may be due to the presence of an anti-self attraction of matter. This shows that the Universe attains isotropy at late times and transits to the accelerating Universe which is consistent with the present day observational data such as cosmic microwave background (CMB) and type Ia supernovae. We may note that according to CMB and Planck results, our Universe is homogeneous and isotropic on a large scale, however, no physical evidence denies the chances of an anisotropic Universe. In fact, theoretical arguments are presently promoting the existence of an anisotropic phase of the Universe that approaches the isotropic phase as suggested by Charles (Charles, 1968), Hinshaw et al. (Hinshaw et al., 2003), Page et al. (Page et al., 2007).

5 CONCLUSION

Here we investigated an anisotropic five dimensional Bianchi type-I string cosmological model in the context of the general theory of relativity. The model represents an expanding Universe that starts at the time $t = 0$ with a volume $V = 0$ and expands with acceleration after an epoch of deceleration. Our model Universe satisfies the energy conditions $\rho \geq 0$ and $\rho_p \geq 0$. The model Universe can represent a stage of evolution from deceleration to acceleration. The deceleration parameter "q" is decelerating at the initial stage of the evolution of the Universe and then accelerates after some finite time because of the cosmic recollapse, indicating inflation in the model after an epoch of deceleration which is in accordance with the present-day observational scenario of the accelerated expansion of our Universe as claimed by type Ia supernovae [Riess et al. (Riess et al., 1998) and Perlmutter et al. (Perlmutter et al., 1999)]. It is observed that our model Universe is anisotropic, expanding, shearing, and the sum of the energy density (ρ) and the string tension density (λ) vanishes ($\rho + \lambda = 0$) for this model.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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HIGHER DIMENSIONAL BIANCHI TYPE-III STRING UNIVERSE WITH BULK VISCOUS FLUID AND CONSTANT DECELERATION PARAMETER(DP)

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ABSTRACT. Here, we have studied a Bianchi type-III string cosmologicl model with bulk viscous fluid and negative constant DP in general relativity considering five dimensional space-time. To get the exact solutions of the survival field equations, we assume that (i) DP is a constant and negative quantity and (ii) the shear scalar and expansion scalar are proportional. Some of the most important parameters of the model are obtained and their behaviors are studied. The model universe obtained here is expanding, shear free throughout the evolution, anisotropic at late time when $n \neq 1$ and the late universe is dominated by the particles.

1. INTRODUCTION

It is now almost proved from observational and theoretical fact that the universe is expanding with acceleration from the big-bang till today. However, no one can guarantee for forever expansion because there is no final conclusion about the expansion or contraction of universe till today. From various literatures and opinions it can be belief that the acceleration of present universe may be accompanied by way of deceleration. But the precise motive of this expanding universe is not known to us till today which inspired all the cosmologists and physicists for similarly research within the area of studies on this field. In the recent past years, several models in cosmology has been proposed by different

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authors in order to explain the hidden reasons of expansion of the existing universe with the acceleration in the framework of string theory. The string theory is one of the most important theories in cosmology that study about the unknown facts of the universe. In latest years, the string cosmological problem has attracted huge interest in the field of research because of their great position in the evolution of the universe in early era. Cosmic strings are topologically stable defects, which are probably formed at some stage of the phase transition or earlier the introduction of particles in the early universe. In the field of general relativity, Stachel [1] and Letelier [2] initiated the study on strings. Spatially homogeneous and anisotropic Bianchi type cosmological model plays a great function to describe the large-scale behavior of the universe. Furthermore, from several kinds of literature and findings one can actually locate that the anisotropic model had been taken as possible models to initiate the expansion of the universe.

Bulk viscosity performed a great role in the evolution of the early universe. Its impact on the evolution of universe has been studied by means of several researchers in the frame work of well known concept of general relativity. Misner [3] studied about the consequences of bulk viscosity in the cosmological evolution of the universe. Some of the famous researchers [4-8], who have studied several Bianchi models in the field of general relativity with bulk viscosity.

A cosmological model in higher-dimensions performs a crucial role in different aspects of the early phases of the cosmological evolution of the universe. It is not possible to unify the gravitational forces in nature in typical four-dimensional space-times. So the theory in higher dimensions may be applicable in the early evolution. The study on higher-dimensional space-time gives us an important idea about the universe that our universe was much more smaller at initial epoch than the universe observed in these days. Many researchers motivated to enter into the theory of higher dimensions to discover the hidden phenomenon of the universe. Subsequently, many researchers have already investigated various cosmological models in five dimensional space-time with various Bianchi type models in different aspects [9-12].

Inspired by the above studies, here in this article we have investigated the higher dimensional bulk viscous cosmological model with string in Bianchi type

III space–time considering constant DP. In this paper, Sec. 2 describes the formulation of problem. Sec.3. gives the solutions of the cosmological problems. Some of the important physical and geometrical parameters are derived in sec.4. The results found are discussed in Sec.5. Finally, in last Sec., concluding points are provided.

2. Metric and Field equations

Here Bianchi type-III metric in 5-dimension is considered as

$$ds^2 = a^2dx^2 + b^2(e^{-2x}dy^2 + dz^2) + c^2dm^2 - dt^2. \tag{2.1}$$

Here a, b and c are the functions of t and 'm' is the extra dimensions(space-like). The EFE in general relativity with $8\pi G = 1, C = 1$ is

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij}. \tag{2.2}$$

The energy-momentum tensor with bulk viscosity is

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta(g_{ij} + u_i u_j). \tag{2.3}$$

Here, $\rho = \lambda + \rho_p$ is the energy density, λ is the tension density and ρ_p is the particle density, θ is expansion scalar and ξ is the bulk viscosity coefficient. Also $u^i = (0, 0, 0, 0, 1)$ is the five velocity vector of particles and $x^i = (0, 0, c^{-1}, 0, 0)$ represent the unit vector which is space-like and this represents the direction of the strings such that $u_iu^j = -1 = -x_ix^j$ *and* $u_ix^i = 0$.

If R(t)be the average scale factor then the spatial volume is

$$V = ab^2c = R^4. \tag{2.4}$$

Using the equations (2.1)-(2.3) we obtain

$$2\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{b}^2}{b^2} + 2\frac{\dot{b}\dot{c}}{bc} = \xi\theta, \tag{2.5}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{b}\dot{c}}{bc} = \xi\theta, \tag{2.6}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{b}\dot{c}}{bc} - \frac{1}{a^2} = \lambda + \xi\theta, \tag{2.7}$$

$$\frac{\ddot{a}}{a} + 2\frac{\ddot{b}}{b} + 2\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2} - \frac{1}{a^2} = \xi\theta, \tag{2.8}$$

$$2\frac{\dot{a}\dot{b}}{ab}+\frac{\dot{a}\dot{c}}{ac}+2\frac{\dot{b}\dot{c}}{bc}+\frac{\dot{b}^2}{b^2}-\frac{1}{a^2}=\rho, \tag{2.9}$$

$$\frac{\dot{a}}{a}=\frac{\dot{b}}{b}. \tag{2.10}$$

The overhead dots here denotes the order of derivative w.r.t. time ‘t’.

3. Solution of the Field Equations

Eqn.(2.10) yields $a=lb$, l is integration constant. We take $l=1$, then

$$a=b \tag{3.1}$$

By the use of (3.1) in the equations (2.5)-(2.9) we get

$$2\frac{\ddot{b}}{b}+\frac{\ddot{c}}{c}+\frac{\dot{b}^2}{b^2}+2\frac{\dot{b}\dot{c}}{bc}=\xi\theta, \tag{3.2}$$

$$2\frac{\ddot{b}}{b}+\frac{\ddot{c}}{c}+\frac{\dot{b}^2}{b^2}+2\frac{\dot{b}\dot{c}}{bc}-\frac{1}{b^2}=\lambda+\xi\theta, \tag{3.3}$$

$$3\frac{\ddot{b}}{b}+3\frac{\dot{b}^2}{b^2}-\frac{1}{b^2}=\xi\theta, \tag{3.4}$$

$$3\frac{\dot{b}\dot{c}}{bc}+3\frac{\dot{b}^2}{b^2}-\frac{1}{b^2}=\rho. \tag{3.5}$$

We have 4 highly nonlinear independent differential equations (3.2)-(3.5) with variables b, c, λ, ρ, ξ and θ which are unknown. So to get the exact solutions of above equations we must have two extra conditions.

Berman’s [13] suggestion regarding variation of Hubble’s parameter H provides us a model universe that expands with constant DP. So for the determinate solution, let us take DP to be a negative constant,

$$q=-\frac{R\ddot{R}}{\dot{R}^2}=h \text{ (constant)}. \tag{3.6}$$

The shear and expansion scalar are proportional ($\sigma \propto \theta$) [Thorne [14], Collins et al. [15]]. This leads to the equation

$$b=c^n, \text{where}, n\neq 0 \text{ is a constant}. \tag{3.7}$$

Solving (3.6), we get

$$R=(\alpha t+\beta)^{\frac{1}{1+h}}\ h\neq -1,\ \alpha \text{ and } \beta \text{ are constants of integration}.$$

Using (2.4), (3.1) and (3.7) we get,

$$a = b = (\alpha t + \beta)^{\frac{4n}{(1+h)(3n+1)}}, \; c = (\alpha t + \beta)^{\frac{4}{(1+h)(3n+1)}}.$$

With the suitable choice of coordinates and constant we take $(\alpha = 1, \; \beta = 0)$

$$a = b = t^{\frac{4n}{(1+h)(3n+1)}}, \; c = t^{\frac{4}{(1+h)(3n+1)}}$$

Which gives the geometry of the metric (2.1)as

$$ds^2 = t^{\frac{8n}{(1+h)(3n+1)}}(dx^2 + e^{-2x}dy^2 + dz^2) + t^{\frac{8}{(1+h)(3n+1)}}dm^2 - dt^2. \tag{3.8}$$

4. Physical and Geometric Parameters

We obtained some of the important physical and geometrical parameters which are useful for the discussion on the evolution of the universe.

$$\rho = \frac{48n(n+1)}{(1+h)^2(3n+1)^2t^2} - t^{-\frac{8n}{(1+h)(3n+1)}}, \tag{4.1}$$

$$\lambda = -t^{-\frac{8n}{(1+h)(3n+1)}},$$

$$\rho_p = \frac{48n(n+1)}{(1+h)^2(3n+1)^2t^2}, \tag{4.2}$$

$$V = t^{\frac{4}{(1+h)}}, \quad \theta = \frac{4}{(1+h)t},$$

$$H = \frac{1}{(1+h)t},$$

$$\xi = \frac{(2n+1)(3n^2 - 3nq - q) + 3}{(1+h)(3n+1)^2t},$$

$$\sigma = \frac{\sqrt{6}(n-1)}{(1+h)(3n+1)t}. \tag{4.3}$$

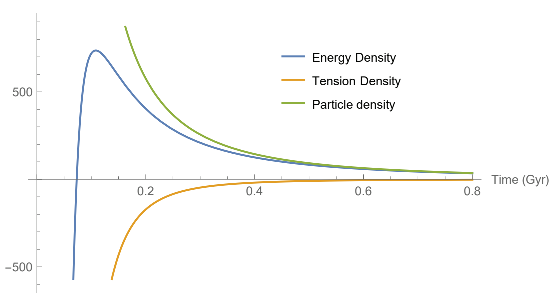

FIGURE 1. Variation of ρ, λ, ρ_p vs. t.

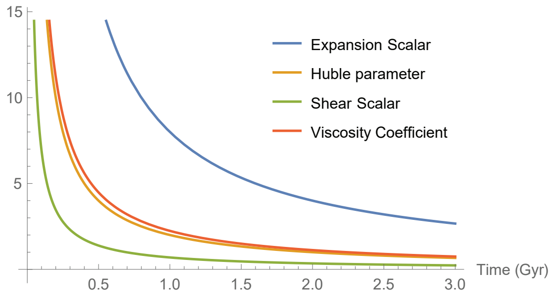

FIGURE 2. Variation of θ, H, σ, V vs. t.

5. Physical Interpretations

The equation (3.8) here represents a Bianchi type-III cosmological model with string in general relativity with constant DP (q=constant) in presence of bulk viscosity in 5-D space-time. The variation of parameters with time for the model are shown above by taking $n = \frac{1}{2}, h = -\frac{1}{2}$.

i. From Fig. 1. and Eqn.(4.1) we have observed that the energy density ρ is negative at the initial epoch and it changes sign from negative to positive after some finite time and finally becomes 0 when $t \to \infty$. Also it is seen that the tension density λ of string is negative(Fig1). It is mentioned by Letelier [2] that λ can be < 0 or > 0. The phase of string disappears when $\lambda < 0$. The strong energy condition $\rho \geq 0$, $\lambda < 0$ as given by Hawking and Ellis [16] are satisfied for the model in the late time universe.

ii. From (4.2), we note $\rho_p \geq 0$ for all time t and ρ_p decreases with time (Fig1). It is also observed that $\frac{\rho_p}{|\lambda|} > 1$ which shows that λ diminishes more faster than ρ_p. This tells us that the late universe is particle dominated.

iii. The bulk viscosity $\xi \to \infty$ when t=0 and it decreases with the increasers of t and finally when $t \to \infty$ bulk viscosity ξ demises (Fig. 2). The function of the bulk viscosity is to retard the expansion of the universe and since bulk viscosity deceases with the time so retardness also decreases which supports in the expansion in faster rate in the late time universe. From the above discussion it can be seen that the bulk viscosity plays a great function in the evolution of the universe.

iv. The volume V=0 at initial epoch t=0 and the volume increases with the increases of the t and it become ∞ when time $t \to \infty$. So the universe is expanding with time. The size of the universe was very small just after the big bang exploitation then the size continuously increasing till now and it will increase late time also.

v. At $t = 0$, the Hubble parameter H and scalar expansion θ, both are infinite and as the time increases gradually they decreases and finally they become 0 at $t \to \infty$ (Fig. 2). Hence the model shows that with the increases of time the universe expands but the expansion rate becomes slower as time increases and at $t \to \infty$, the expansion stops. And since $\frac{dH}{dt} < 0$ which also tells us that our present universe is in the accelerated expanding mode.

vi. In equation (4.3) and Fig. 2. it seen that the value of the shear scalar $\sigma \to \infty$ at initial epoch and it decreases as the time increases and become zero at late universe showing that the universe obtained here is shear free in the late time.

vii. The mean anisotropy parameter $\Delta = \frac{3(n-1)^2}{(3n+1)^2} = constant \neq 0$ for $n \neq 1$ and $\Delta = 0$ for $n = 1$. Also as $t \to \infty$ the value of $\frac{\sigma^2}{\theta^2} = constant \neq 0$ for $n \neq 1$ and $\frac{\sigma^2}{\theta^2} = 0$ for $n = 1$. From both statements we can conclude that this model is anisotropic for large value of t when $n \neq 1$ but it is isotropic for $n = 1$.

6. CONCLUSION

In this article, we have attempted to present a new solution to the field equations obtained for Bianchi type-III universe with bulk viscosity by using the law of variation of H which yields constant DP. This variational law for H in equation (3.8) explicitly determine the values of the scale factors(R). So here, we have constructed a 5D Bianchi type-III string universe with bulk viscosity and constant DP in general relativity by the use of certain physically plausible assumptions, that agrees with the present day observational data in general relativity. The model is expanding, non shearing, anisotropic for $n \neq 1$ and isotropic for $n = 1$ in the late universe which is also in accordance to the present day observational data made by WMAP and COBE. The present universe starts at initial epoch $t = 0$ with 0 volume and then expand with accelerated motion and the expansion rate slows down with increase of time. The bulk viscosity coefficient plays a great role in the cosmological consequences. The tension density is negative quantity showing that the string phase disappears and present day universe is particle dominated.

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RESEARCH ARTICLE

Higher Dimensional LRS Bianchi Type-I String Cosmological Model with Bulk Viscosity in General Relativity

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Abstract

Objective: To present a new solution to the field equations obtained for higher dimensional LRS Bianchi type-I universe generated by means of a cloud of strings with particles connected to them with bulk viscosity in general relativity. **Methods:** To obtain the solutions of field equations of higher dimensional LRS Bianchi type-I universe we consider that the shear scalar of the model is proportional to the scalar expansion of the model ($\sigma\alpha\theta$), which leads to, $c = b^\wedge n$. The physical and geometrical behaviors of the model universe are studied by comparing with the present cosmological scenario and observations. **Findings:** It is observed that our model is anisotropic, expanding and decelerates at early stage and then accelerates in late universe giving the inflation model universe. **Novelty:** We obtained new solution to the field equations for higher dimensional LRS Bianchi type-I generated by means of a cloud of strings with bulk viscous fluid in general relativity.

Keywords: LRS Bianchi TypeI Metric; Bulk Viscous Fluid; Strings

1 Introduction

Still now it is an interesting area for the cosmologists to study and discover the unknown phenomenon of the universe that have yet to observe to study and explore its hidden knowledge. So cosmologists have taken considerable interest to understand the past evolution, present state and evolution in the future of the universe. Before the formation of particles, the strings took major role in the creation and evolution of the universe in early era. Authors in [1,2] initiated the general relativistic study of the strings, and they developed the classical theory of the geometric strings. Because of the major role of strings in describing the evolution of our universe in the early epoch, in recent times, many famous researchers are interested in cosmic strings in general relativity [3,4]. Strings give rise to density perturbations leading to the creation of Large Scale Structures (like galaxies) of the universe [5,6] which does not oppose present day observational findings of the universe and results in an anisotropy in the space-time though the strings are unobservable in the present day's universe. These strings possess stress energy and are named as geometric strings and massive strings. After the big-bang explosion there was symmetry breaking of the universe spontaneously during the transition of phase in the very early stages of the universe and these cosmic strings which are very important topological defect arose in the early universe as the cosmic temperature went down below some critical point temperatures according to grand unified theories[7–9].

The bulk viscosity assumes an extraordinary part in the development of the early universe. There are numerous events inside the development of the universe wherein the bulk viscosity could emerge. The size of the viscous stress comparative with the expansion is controlled by the coefficients of bulk viscosity. Spatially homogeneous and anisotropic Bianchi type-I models are attempted to comprehend the universe in its beginning phase of the evolution of the universe. The various pictures of the universe may show up at the beginning phase of the cosmological development of the universe because of the dissipative process brought about by viscosity which counteracts the cosmological breakdown (collapse). A few authors endeavored to find the specific solutions of field equations by considering viscous effects in general relativity in isotropic as well as anisotropic cosmological model universes. Authors in[10] have constructed Bianchi type-I cosmological models in presence of bulk viscosity. Bulk viscous fluid Bianchi Type-I string cosmological model in general relativity have been investigated by[11]. A new class of LRS Bianchi type-V cosmological model with string dust as a source of gravitational field was investigated in[12]. In[13] the authors have endeavored to introduce another solution for the field equations obtained for Bianchi type-III cosmological model in Lyra manifold by utilizing the law of variation of Hubble's Parameter (H), which yields constant DP.

Nowadays it is very interesting to study string cosmology in five-dimensional space-time in general relativity. The possibility of space-time having more than four dimensions (Extra dimensions) has fascinated many researchers. In the recent years, to study cosmological models, the higher dimensional space-time has been given more importance. Generally, the higher dimensional model was introduced by[14] and[15] in an effort to unify gravity with electromagnetism. Higher dimensional model can be regarded as a tool to illustrate the late time expedited expanding paradigm[16]. Investigation of higher dimensional space-time can be regarded as a task of paramount importance as the universe might have come across a higher dimensional era during the initial epoch[17]. Marciano[18] asserts that the detection of a time varying fundamental constants can possibly show us the proof for extra dimensions. According to[19] and[20], extra dimensions generate huge amount of entropy which gives possible solution to atness and horizon problem. Since we are living in a 4D space-time, the hidden extra dimension in 5D is highly likely to be associated with the invisible DM and DE[21]. Several authors have investigated various Bianchi type problems in the field of higher dimensional space-time. An in homogeneous higher dimensional cosmological model with massive string in general relativity was constructed by[22]. An LRS Bianchi type-I inflationary string Cosmological model with massive scalar field in general relativity in the field of five-dimensional space-time was constructed by[23] and found that, $\rho+\lambda=0$. In[24] the authors constructed some Bianchi type-III cosmological models with massive string as a source of gravitational field in five-dimensional space-time in general relativity. A Bianchi type-III cosmological models with string in general relativity with magnetic field is obtained by[25], where they used the condition $\rho+\lambda=0$ to solve the field equations. An LRS Bianchi type-I model in cosmology with bulk viscous fluid in Lyra geometry with the help of displacement vector depending upon time was constructed in[26] and found that the bulk viscosity decreases with time. In[27] a perfect fluid cosmological model in Lyra Geometry was studied by using constant deceleration parameter in five dimensional LRS Bianchi type-I Space-time.Considering reasonable cosmological assumptions within the limit of the present cosmological scenario, a spherically symmetric metric in five dimensional setting in the framework of Lyra geometry is analysed[28]. A cosmological model in 5D spherically symmetric space-time with energy momentum tensors of minimally interacting fields of dark matter and holographic dark energy in Brans-Dicke theory was constructed by[29]. Authors in[30–37] are the authors who studied different string cosmological models in general relativity in different context in various space-times.

Above discussion and investigations motivated us to study the five-dimensional LRS Bianchi type-I string model in cosmology with particles attached to them in general relativity. The paper is presented as: In Section 1, a brief presentation of strings, bulk viscosity and their importance are discussed. In Section 2, the five-dimensional LRS Bianchi type-I metric is introduced and the field equations in the framework of general relativity are determined; In Section 3, using some simplifying assumptions we find the determinate solutions of the survival field equations. In Section 4, we examined the geometrical and physical properties of our model universe with the help of graphs; In Section 5 conclusions are presented.

2 The Metric and Field Equations

We consider the 5-dimensional LRS Bianchi type-I metric as

$$ds^2=-dt^2+a^2dx^2+b^2\left(dy^2+dz^2\right)+c^2dm^2 \tag{1}$$

Here a, b and c are the metric functions of cosmic time t alone and the extra coordinate "m" is taken to be space-like.

For the above metric lets

$$x^1 = x, x^2 = y, x^3 = z, x^4 = m \text{ and } x^5 = t$$

In general relativity the Einstein's field equation is written as

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \tag{2}$$

For a cloud string the energy-momentum tensor is

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi \theta \left(u_i u_j + g_{ij} \right) \tag{3}$$

Where, ρ is the energy density and λ the tension density of the string and they are related as $\rho = \lambda + \rho_p$, where ρ_p is the particle density of matter, ξ is the coefficient of viscosity and θ is the expansion scalar. The co-ordinates are co-moving, x^i is the unit space-like vector indicating the direction of strings and u^i is the five velocity vector which satisfies the conditions-

$$u_i u^i = -1 = -x_i x^i \tag{4}$$

$$and\ u_i x^i = 0 \tag{5}$$

Here without loss of generality we can take

$$u^i = (0,0,0,0,1) \text{ and } x^i = \left(a^{-1}, 0,0,0,0 \right) \tag{6}$$

The spatial volume is given by

$$V = ab^2 c = R^4 \tag{7}$$

Where R(t) is the average scale factor of the universe.

The Scalar Expansion is given by

$$\theta = \frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} + \frac{\dot{c}}{c} \tag{8}$$

Hubble Parameter is given by

$$H = \frac{1}{4} \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} + \frac{\dot{c}}{c} \right) \tag{9}$$

Deceleration parameter is

$$q = -\frac{R\ddot{R}}{\dot{R}^2} \tag{10}$$

The shear scalar is given by

$$\sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} = \frac{1}{2} \left[\left(\frac{\dot{a}}{a} \right)^2 + 2 \left(\frac{\dot{b}}{b} \right)^2 + \left(\frac{\dot{c}}{c} \right)^2 \right] \tag{11}$$

And the mean anisotropy parameter is given by

$$\Delta = \frac{1}{4} \sum_{i=1}^{4} \left(\frac{H_i - H}{H} \right)^2 \tag{12}$$

Where, H$_i$ (i=1,2,3,4) represents the directional Hubble Parameters in the directions of x, y, z and m axes and are defined as $H_1 = \frac{\dot{a}}{a}$, $H_2 = H_3 = \frac{\dot{b}}{b}$ and $H_4 = \frac{\dot{c}}{c}$ for the metric (1).

Using the equations (3)-(6), the field equation (2) takes the form

$$2\frac{\ddot{b}}{b}+\frac{\ddot{c}}{c}+\frac{\dot{b}^2}{b^2}+2\frac{\dot{b}\dot{c}}{bc}=\lambda+\xi\theta \tag{13}$$

$$\frac{\ddot{a}}{a}+\frac{\ddot{b}}{b}+\frac{\ddot{c}}{c}+\frac{\dot{a}\dot{b}}{ab}+\frac{\dot{a}\dot{c}}{ac}+\frac{\dot{b}\dot{c}}{bc}=\xi\theta \tag{14}$$

$$\frac{\ddot{a}}{a}+2\frac{\ddot{b}}{b}+\frac{\dot{b}^2}{b^2}+2\frac{\dot{a}\dot{b}}{ab}=\xi\theta \tag{15}$$

$$2\frac{\dot{a}\dot{b}}{ab}+\frac{\dot{a}\dot{c}}{ac}+2\frac{\dot{b}\dot{c}}{bc}+\frac{\dot{b}^2}{b^2}=\rho \tag{16}$$

Here, the overhead dots mean differentiation with time 't'.

3 Solutions of the Field Equations

In this part, we intend to derive the solutions of the four highly non-linear independent equations (13)-(16) with 6 unknown variables a, b, c, ξ, λ and ρ. For deterministic solution we considered the following physical plausible conditions:

We consider that the shear scalar and expansion scalar are proportional, which leads to

$$c=b^n, \tag{17}$$

Here,

The reason of assuming the above condition depends on observations of the velocity red-shift relation for extragalactic sources recommended that the Hubble expansion of the universe is isotropic today to within 30% [38–40]. If H is Hubble constant and σ is the shear then the red-shift studies limit $\frac{\sigma}{H}\leq 0.30$. If θ is expansion scalar, the normal to the spatially homogeneous metric is congruence to the homogeneous hyper-surface that satisfies the condition $\frac{\sigma}{\theta}$ is constant [41].

From (14),(15) by using (17), we get

$$\frac{\dot{b}}{b}\left[\frac{\ddot{b}}{b}(n+1)\frac{\dot{b}}{b}+\frac{\dot{a}}{a}\right]=0 \tag{18}$$

Using (17) in (14) and comparing with (15), we get n=1.

From (17) we get

$$b=c \tag{19}$$

Equation (18) yields the following cases:

$$CaseI:\frac{\ddot{b}}{b}+2\frac{\dot{b}}{b}+\frac{\dot{a}}{a}=0,\ and\ CaseII:\ \frac{\dot{b}}{b}=0 \tag{20}$$

We intend to determine the cosmological models for the above two cases separately.

CaseI: $\frac{\ddot{b}}{b}+2\frac{\dot{b}}{b}+\frac{\dot{a}}{a}=0$

Solving we get,

$$b(t)=\left[3\left(\int\frac{K}{a(t)}dt+K_1\right)\right]^{\frac{1}{3}} \tag{21}$$

Clearly, the solution are not unique because b(t) can be obtained for any given a(t). So for further studies here we consider [42]

$$\frac{\ddot{b}}{b}+2\frac{\dot{b}}{b}=-\frac{\dot{a}}{a}=k\;(Cons\tan t) \tag{22}$$

Now solving (22) and using (19) we get,

$$a=k_1e^{-kt} \tag{23}$$

$$b=\left[3\left(\frac{k_2}{k}e^{kt}+k_3\right]^{\frac{1}{3}}\right. \tag{24}$$

$$c=\left[3\left(\frac{k_2}{k}e^{kt}+k_3\right]^{\frac{1}{3}}\right. \tag{25}$$

Here, $k_1(\neq 0), k_2$ and k_3 are constants of integrations.

The following metric describes the geometry of the model

$$ds^2=-dt^2+k_1^2e^{-2kt}dx^2+\left[3\left(\frac{k_2}{k}e^{kt}+k_3\right]^{\frac{2}{3}}\left(dy^2+dz^2+dm^2\right)\right. \tag{26}$$

The tension density of the string is obtained as

$$\lambda=\frac{kk_2e^{kt}}{\left(\frac{k_2}{k}e^{kt}+k_3\right)}-k^2 \tag{27}$$

The energy density of the string is

$$\rho=\frac{k_2^2e^{2kt}}{3\left(\frac{k_2}{k}e^{kt}+k_3\right)^2}-\frac{kk_2e^{kt}}{\left(\frac{k_2}{k}e^{kt}+k_3\right)} \tag{28}$$

The particle density is given by

$$\rho_p=\frac{k_2^2e^{2kt}}{3\left(\frac{k_2}{k}e^{kt}+k_3\right)^2}-\frac{2kk_2e^{kt}}{\left(\frac{k_2}{k}e^{kt}+k_3\right)}+k^2 \tag{29}$$

From (15) we get

$$\xi\theta=k^2-\frac{k_2^2e^{2kt}}{3\left(\frac{k_2}{k}e^{kt}+k_3\right)^2} \tag{30}$$

The spatial volume is obtained as

$$V=3k_1e^{-kt}\left(\frac{k_2}{k}e^{kt}+k_3\right) \tag{31}$$

The scalar expansion is obtained as

$$\theta = \frac{k_2 e^{kt}}{\left(\frac{k_2}{k}e^{kt} + k_3\right)} - k \tag{32}$$

Using (32) in (30) we get

$$\xi = \frac{-k_2^2 e^{2kt}}{3kk_3\left(\frac{k_2}{k}e^{kt} + k_3\right)^2} - \frac{k_2}{k_3}e^{kt} - k \tag{33}$$

The Hubble parameter is obtained as

$$H = \frac{k_2 e^{kt}}{4\left(\frac{k_2}{k}e^{kt} + k_3\right)} - \frac{k}{4} \tag{34}$$

The deceleration parameter is

$$q = -\left[\frac{4k_2 e^{kt}}{kk_3} + 1\right] \tag{35}$$

The shear scalar of the model is

$$\sigma^2 = \frac{1}{8}\left[3k^2 + \frac{k_2^2 e^{2kt}}{3\left(\frac{k_2}{k}e^{kt} + k_3\right)^2} + \frac{2kk_2 e^{kt}}{\left(\frac{k_2}{k}e^{kt} + k_3\right)}\right] \tag{36}$$

3.2 CaseII: $\frac{\dot{b}}{b} = 0$

Solving it we get,

$$b = k_4 \tag{37}$$

From the equation (19) together with (37) we have,

$$c = k_4 \tag{38}$$

Now using equations (37)-(38) in the field equations (13)-(16) we get,

$$\lambda + \xi\theta = 0 \tag{39}$$

$$\frac{\ddot{a}}{a} = \xi\theta \tag{40}$$

$$\rho = 0 \tag{41}$$

Here ξ, λ, ρ and a are four unknowns involved in three equation (39)-(41). To obtain a determinate solution we have assumed following different plausible conditions of equations of state as Geometric String

$$\rho = \lambda \text{ (Geometric String)} \tag{42}$$

$$\text{And } \rho = (1+\omega)\lambda \text{ (p-string)} \tag{43}$$

Where, $\omega > 0$ is a constant.

3.2.1 $\rho = \lambda$ (Geometric String)
This yields,

$$\rho = \lambda = 0 \tag{44}$$

$$\xi = 0 \tag{45}$$

$$\text{and } a = lt + m \tag{46}$$

Where, $l \neq 0$ and m are integrating constants.

This case leads to the five-dimensional LRS Bianchi type-I vacuum model universe in Einstein's theory of relativity.

The following metric described the geometry of our model,

$$ds^2 = -dt^2 + (lt+m)^2 dx^2 + k_4^2\left(dy^2 + dz^2 + dm^2\right) \tag{47}$$

3.2.2 $\rho = (1+\omega)\lambda$ (p-string)
Using (41) in (43) we get,

$$(1+\omega)\lambda = 0 \tag{48}$$

Which yields either $\omega = -1$ or $\lambda = 0$. But $\omega = -1$ is not acceptable as $\omega > 0$.

Since, $\lambda = 0$ the model in this case also reduces to the model already obtained in above case.

4 Interpretations of the Results

In case I we have obtained a five-dimensional LRS Bianchi type-I cosmological model universe with string in general relativity given by (26). The variation of parameters with time for this model are shown below by taking, $k = -1, k_1 = k_2 = k_3 = 1$ and the physical and geometrical behavior of the model are discussed as

- At initial epoch t=0, the metric (model) (26) becomes flat.
- It is seen that at time t=0 the evolution of energy density ρ is infinite and it decreases gradually as the time t increases and become constant after some finite time (Figure 1). This model satisfies the conditions $\rho \geq 0$ and $\rho_p \geq 0$ (Known as energy density conditions).

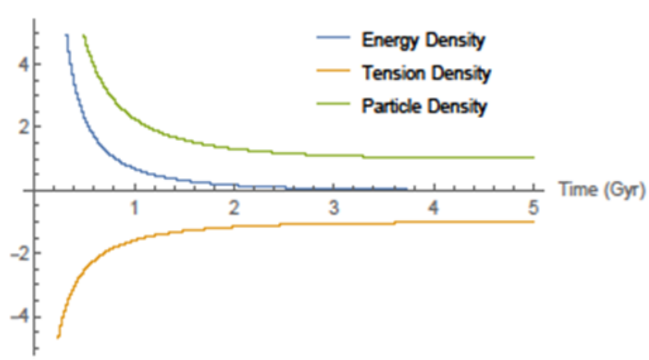

Fig 1. Variation of ρ, λ, ρ_p Vs. time t

- It is also seen from Figure 1 that $\rho > 0, \lambda < 0$ and $\rho_p > 0$ showing that at early era particles exists with positive ρ_p but strings exist with negative λ . The particle density ρ_p has a large value during the time of big-bang when $t = 0$ and as the evolution of time it decreases and moves to a finite value (constant) at $t \to \infty$, which corresponds to total constant finite number of particles in the late universe. This may correspond to the matter dominated era. From the comparative study of tension density λ and particle density $\rho_p$with time as shown in Figure 1, it is seen that $|\lambda| < |\rho_p|$ which describes that in the late time the string vanishes, leaving particles only.
- Initially at $t = 0$ the expansion scalar θ is finite and very large and as the time progresses gradually it decreases and it becomes a small constant after some finite time (as shown in the Figure 2) explaining the Big-Bang scenario of the universe. So, this model shows that the universe is expanding with the increase of time but the rate of expansion becomes slower than time increases and the expansion becomes finite at $t \to \infty$.

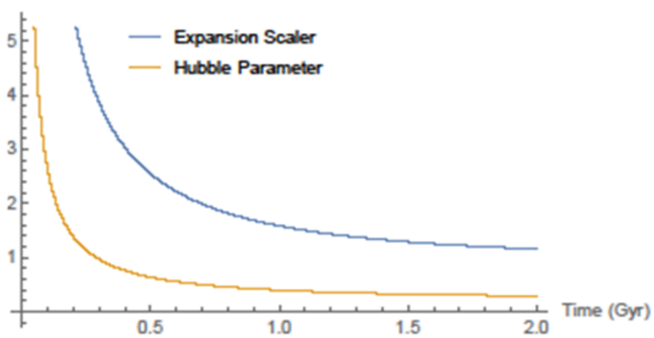

Fig 2. Variation of θ, H Vs. time t

- The average Hubble parameter H is a decreasing function of time. It is large constant when $t \to 0$ but as the time progresses gradually it also decreases and reached to a small constant value after some finite time as shown by Figure 2 and at $t \to \infty$, the value of $\frac{dH}{dt} \to 0$,which also shows the expanding universe of our model.
- Since, $\frac{\sigma^2}{\theta^2} = \text{Constant}(\neq 0)$ as $t \to \infty$ and hence our model universe obtained here is anisotropic one. Though the anisotropy is included, it does not make any contradiction with the present day observational findings that the universe is isotropic one. This is due to the reason that during the process of evolution of our universe, the initial anisotropy disappears after some epoch and approaches to the isotropy in late time universe.
- From the Figure 3 it is seen that the spatial volume V is finite at initial epoch $t = 0$ and it increases exponentially as t increases and becomes infinite as $t \to \infty$, which shows the expanding universe of our model.

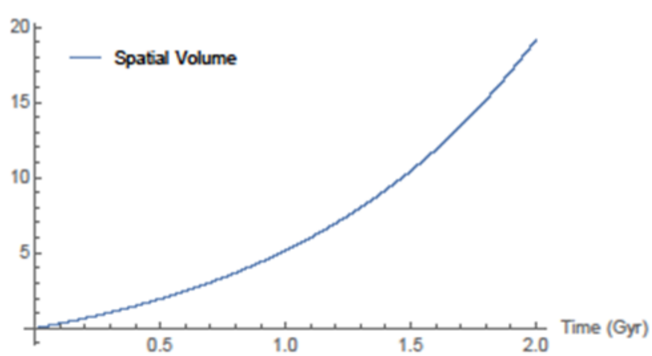

Fig 3. Variation of Volume (V) Vs. time t.

- The DP "q" in the model is decelerating $q > 0$ at $t = 0$ and it decreases as time increases and becomes accelerating $q < 0$ after some finite time as shown in Figure 4. Also $q \to -1$ at $t \to \infty$, which indicates that the present model universe has a transition from decelerating phase to accelerating phase, indicating the inflation in the model after an epoch of deceleration.

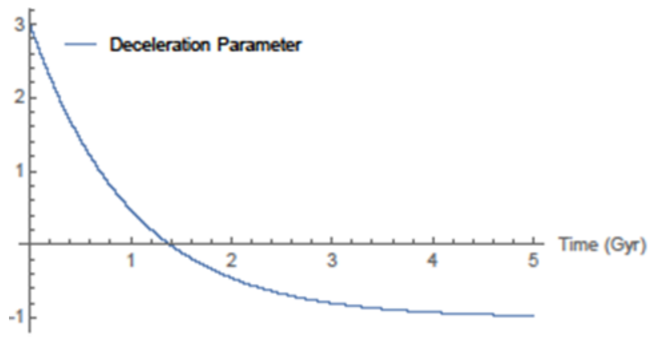

Fig 4. Variation of DP (q) Vs. time t.

- The bulk viscosity for this model is decreasing function of time. The function of the bulk viscosity is to retard the expansion of the universe and since bulk viscosity deceases with the time so retardness also decreases which supports in the expansion in faster rate in the late time universe. From the above discussion it can be seen that the bulk viscosity plays a great function in the evolution of the universe.

The case II, leads to LRS Bianchi type-I vacuum model in Einstein's theory of relativity in five-dimensional space-time represented by which is not realistic because strings don't survive for this model.

5 Conclusions

In this study, we have investigated a five-dimensional LRS Bianchi type-I String cosmological model in general theory of relativity in presence of bulk viscous fluid given by (26) which is an inflationary model. The model universe obtained here is anisotropic, accelerating and expanding. The DP "q" obtained here is decelerating at initial stage and accelerates after some finite time, indicating inflation in the model after an epoch of deceleration which is in accordance with the present day observational scenario of the accelerated expansion of our universe as type la supernovae[(43,44)]. The model universe obtained here is anisotropic one in the early epoch but the recent observations tell us that there is disparity in the measurement of intensities of microwaves coming from the sky in different directions, which urge us to study about the universe with LRS Bianchi type-I metric to describe the universe in more reasonable circumstances. During the inflation, shear decreases with time and then it turns into an isotropic universe with very small shear. As expected our model satisfies the energy conditions $\rho \geq 0$ and $\rho_p \geq 0$ which in turn imply that our derived models are physically realistic as the present day observational data. The tension density and the particle density are comparable and the model represents a matter dominated universe in the late time which agrees with the present-day observational findings. Also, the model represents an exponentially expanding universe that starts with big-bang at cosmic time $t = 0$ giving inflationary model. The bulk viscosity is the decreasing function of time (t). Thus, our model universe has cosmological importance since it is clarifying the early universe and it should be a sensible representation of the universe at early age.Finally, we may conclude that all the above solutions and results presented in the paper are new, are in good agreement with the present-day cosmological observations and useful for a better understanding of the evolution of the universe in Bianchi type-I space-time with bulk viscous fluid. This study is likely to be useful for the analysis of different kinds of Bianchi Models in various space-times. The model needs further deep study in higher dimensional space-time considering all the observational findings, which will be our upcoming work.

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HIGHER DIMENSIONAL PERFECT FLUID COSMOLOGICAL MODEL IN GENERAL RELATIVITY WITH QUADRATIC EQUATION OF STATE (EoS)

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Abstract: Higher dimensional Bianchi type-V cosmological model in the general theory of relativity with quadratic equation of state interacting with perfect fluid has been studied. For higher dimensional Bianchi type-V space time, the general solutions of the Einstein's field equations have been obtained under the assumption of quadratic equation of state (EoS) $p = \alpha\rho^2 - \rho$, where $\alpha \neq 0$ is arbitrary constant. The physical and geometrical aspects of the model are discussed.

Keywords: higher dimensions; Bianchi type-V; perfect fluid; quadratic EoS.

2010 AMS Subject Classification: 85A40, 58D30, 83F05.

1. INTRODUCTION

It is now almost proved from theoretical and astronomical observations such as type Ia supernovae (Riess et al. [1], Perlmutter et al. [2], Riess et al. [3]) that the universe is expanding

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with acceleration from the big-bang till today. However, no one can guarantee for forever expansion because there is no final conclusion about the expansion or contraction of universe till today and represents an open question for theoretical physicists which inspired numerous cosmologists and physicists for similarly research within the area of studies on this field to resolve this problem including the modified theory of gravity and possible existence of dark energy (DE). From various literatures and opinions it can be belief that the acceleration of present universe may be accompanied by way of deceleration. In the recent past years, several models in cosmology has been proposed by different authors in order to explain the hidden reasons of expansion of the existing universe with the acceleration in the framework of general theory of relativity and modified theories. Spatially homogeneous and anisotropic Bianchi type cosmological model plays a great role to describe the large-scale behavior of the universe. Modern cosmology revolves around the study about the past and present state of the universe and how it will evolve in future.

In the study of universe, among different models of Bianchi types, Bianchi type-V cosmological models is the natural generalization of the open Friedmann-Robertson-Walker(FRW) model which plays an important role in the description of universe and more interesting is that these models contain isotropic special cases and it may permit small anisotropy levels at some of time. There are theoretical arguments from the recent experimental data which support the existence of an anisotropic phase approaching to isotropic phase leading to the models of the universe with anisotropic background. Cosmological models which are spatially homogeneous and anisotropic play significant roles in the description of the universe at its early stages of evolution. Some authors such as Maartens and Nel [4], Meena and Bali [5] have studied Bianchi type-V models in different contexts. Coley [6] have investigated Bianchi type-V spatially homogeneous universe model with perfect fluid cosmological model which contains both viscosity and heat flow. In recent years, the solution of Einstein's field equation for homogeneous and anisotropic Bianchi type models have been studied by several authors such as Hajj-Boutros [7], Shri Ram [8,9], Pradhan and Kumar [10] by using different generating techniques. Ananda and Bruni [11] discussed the cosmological models by considering different form and non-linear quadratic equation of state. A few models that describe an anisotropic space time and generate particular interest are among Lorenz and Pestzold [12], Singh and Agrawal[13], Marsha [14], Socorro and Medina [15]. Furthermore, from several kinds of literature and findings one can actually locate that the anisotropic model had been taken as possible models to initiate the expansion of the universe. Banerjee and Sanyal [16] have considered Bianchi type-V cosmological models with bulk viscosity and heat flow. Conformally flat tilted Bianchi type-V cosmological models in the presence of a bulk viscous fluid are investigated by Pradhan and Raj [17]. Kandalkar et al. [18] discussed the variation law of Hubble's parameter, average scale factor in spatially homogeneous anisotropic Bianchi type-V space time filled with viscous fluid where the universe exhibits power law and exponential expansion.

In relativity and cosmology, the equation of state is nothing but the relationship among combined matter, pressure, temperature, energy and energy density for any region of space, plays an important role in the study about universe. For the study of dark energy and general relativistic dynamics in different cosmological models, the quadratic EoS plays an important role. Many researchers like Ivanov [19], Sharma and Maharaj [20], Thirukkanesh and Maharaj [21], Varela et al. [22] etc. studied cosmological models with linear and non-linear equation of states. Various authors like Nojiri and Odintsov [23,24,25], Nojiri et al. [26], Capozziello et al. [27], Bamba et al. [28] already discussed dark energy universe with different equations of state(EoS).

The general form of quadratic equation of state(EoS)

$$p = \rho_0 + \alpha\rho + \beta\rho^2$$

is nothing but the first term of the Taylor's expansion of an equation of state of the form $p = p(\rho)$ about the $\alpha = 0$, where ρ_0, α and β are the parameters.

Ananda and Bruni[29] investigated the general relativistic dynamics of Robertson-Walker models considering a non-linear equation of state(EoS) in the form of a quadratic equation $p = \rho_0 + \alpha\rho + \beta\rho^2$. They have shown that in general relativistic theory setting, the anisotropic behavior at the singularity obtained in the Brane Scenario can be reproduced. In the general theory of relativity, they have also discussed the anisotropic homogeneous and inhomogeneous cosmological models with the consideration of quadratic equation of state of the form $p = \alpha\rho + \frac{\rho^2}{\rho_c}$ and attempted to isotropize the model universe in the initial stages when the initial singularity is approached. In this paper, we have considered the quadratic equation of state of the form $p = \alpha\rho^2 - \rho$, where $\alpha \neq 0$ is a constant quantity, here we take $\rho_0 = 0$ to make our calculations easier without affecting the quadratic nature of the equation of state.

Chavanis[30] investigated a four-dimensional Friedmann-Lemaitre-Roberston-Walker (FLRW) cosmological model unifying radiation, vacuum energy and dark energy by considering an equation of state in quadratic nature. Again, by using a quadratic form of equation of state, Chavanis[31] formulated a cosmological model that describes the early inflation, the intermediate decelerating expansion, and the late-time accelerating expansion of the universe. Many authors like Maharaj et al.[32], Rahaman et al.[33], Feroze and Siddiqui[34] have investigated cosmological models on the basis of EoS in quadratic form under different circumstances. Recently, V. U. M. Rao et al.[35], Reddy et al.[36], Adhav et al.[37], MR Mollah and KP Singh[38], MR Mollah et al.[39], Beesham et al.[40] studied different space-time cosmological models with a quadratic EoS in general and modified theories of relativity.

A cosmological model in higher-dimensions performs a crucial role in different aspects of the early phases of the cosmological evolution of the universe and are not observable in real universe. It is not possible to unify the gravitational forces in nature in typical four-dimensional space-times. So the theory in higher dimensions may be applicable in the early evolution. The study on higher-dimensional space-time gives us an important idea about the universe that our universe was much smaller at initial epoch than the universe observed in these days. Due to these reasons studies in higher dimensions inspired and motivated many researchers to enter into such a field of study to explore the hidden knowledge of the present universe. Subsequently, many researchers have already investigated various cosmological models in five dimensional space-time with various Bianchi type models in different aspects [35,38,41,42,43,44].

Inspired by the above studies, here in this article we have investigated the higher dimensional cosmological model in Bianchi type-V space–time considering quadratic equation of state(EoS) interacting with perfect fluid . In this paper, Sec. 2 describes the formulation of problem. Sec.3. gives the solutions of the cosmological problems. In sec.4, some of the important physical and geometrical parameters are derived. The results found are discussed in Sec.5. Finally, in Sec.6, concluding points are provided.

2. THE METRIC AND FIELD EQUATION

We consider the higher dimensional Bianchi type-V cosmological model in the form

$$ds^2 = -dt^2 + a^2dx^2 + e^{-2x}(b^2dy^2 + c^2dz^2 + D^2dm^2) \quad (1)$$

Where a, b, c and D are functions of time only and m is the extra dimension (Fifth dimension) which is space-like.

The energy momentum tensor for the perfect fluid is given by

$$T_{ij} = (p + \rho)u_iu_j + pg_{ij} \quad (2)$$

Where ρ is the energy density, p is the pressure and $u^i = (0,0,0,0,1)$ is the five velocity vector of particles.

Also

$$g_{ij}u^iu^j = 1 \quad (3)$$

We have assumed an equation of state in the general form $p = p(\rho)$ for the matter distribution. In this case, we consider the quadratic form as

$$p = \alpha\rho^2 - \rho \quad (4)$$

Where $\alpha \neq 0$ is arbitrary constant.

The Einstein's field equations in general relativity is given by

$$R_{ij} - \frac{1}{2}g_{ij}R = -T_{ij} \quad (5)$$

If R(t) be the average scale factor, then the spatial volume is given by

$$V = abcD = R^4 \quad (6)$$

Where R(t) is given by

(7) $$R(t) = (abcD)^{\frac{1}{4}}$$

The expansion scalar is defined as

(8) $$\theta = \frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c} + \frac{\dot{D}}{D}$$

The Hubble's parameter is defined as

(9) $$H = \frac{\dot{R}}{R} = \frac{1}{4}\theta = \frac{1}{4}\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c} + \frac{\dot{D}}{D}\right)$$

Also we have

(10) $$H = \frac{1}{4}(H_1 + H_2 + H_3 + H_4)$$

Here, $H_1 = \frac{\dot{a}}{a}$, $H_2 = \frac{\dot{b}}{b}$, $H_3 = \frac{\dot{c}}{c}$ and $H_4 = \frac{\dot{D}}{D}$ are directional Hubble's factors in the directions of x, y, z and m respectively.

Deceleration parameter q is defined as

(11) $$q = -\frac{R\ddot{R}}{R^2}$$

The mean anisotropy parameter is defined as

(12) $$\Delta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2$$

The shear scalar is defined as

(13) $$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left(\frac{\dot{a}^2}{a^2} + \frac{\dot{b}^2}{b^2} + \frac{\dot{c}^2}{c^2} + \frac{\dot{D}^2}{D^2} - \frac{\theta^2}{4}\right)$$

Using the equations (1)-(3) and (5) we obtain

(14) $$\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\ddot{D}}{D} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{b}\dot{D}}{bD} + \frac{\dot{c}\dot{D}}{cD} - \frac{3}{a^2} = -p$$

(15) $$\frac{\ddot{a}}{a} + \frac{\ddot{c}}{c} + \frac{\ddot{D}}{D} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{c}\dot{D}}{cD} - \frac{3}{a^2} = -p$$

(16) $$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{D}}{D} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{b}\dot{D}}{bD} - \frac{3}{a^2} = -p$$

(17) $$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{b}\dot{c}}{bc} - \frac{3}{a^2} = -p$$

(18) $$\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{b}\dot{D}}{bD} + \frac{\dot{c}\dot{D}}{cD} - \frac{6}{a^2} = \rho$$

(19) $$3\frac{\dot{a}}{a} = \frac{\dot{b}}{b} + \frac{\dot{c}}{c} + \frac{\dot{D}}{D}$$

The overhead dots here denote the order of differentiation with respect to time `t’.

The conservational law of the energy momentum tensor is

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c} + \frac{\dot{D}}{D}\right) = 0 \tag{20}$$

3. Cosmological Solution

From Equation (19) we get

$$a^3 = kbcD \tag{21}$$

Here, k is the constant of integration. Without loss of generality we can choose $k = 1$ and so we get

$$a^3 = bcD \tag{22}$$

Subtracting (14) from (15), we get

$$\frac{\ddot{a}}{a} - \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{c}}{ac} - \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{a}\dot{D}}{aD} - \frac{\dot{b}\dot{D}}{bD} = 0 \tag{23}$$

Subtracting (15) from (16), we get

$$\frac{\ddot{b}}{b} - \frac{\ddot{c}}{c} + \frac{\dot{a}\dot{b}}{ab} - \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{b}\dot{D}}{bD} - \frac{\dot{c}\dot{D}}{cD} = 0 \tag{24}$$

Subtracting (16) from (17), we get

$$\frac{\ddot{c}}{c} - \frac{\ddot{D}}{D} + \frac{\dot{a}\dot{c}}{ac} - \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{b}\dot{c}}{bc} - \frac{\dot{b}\dot{D}}{bD} = 0 \tag{25}$$

Subtracting (17) from (14), we get

$$\frac{\ddot{D}}{D} - \frac{\ddot{a}}{a} + \frac{\dot{b}\dot{D}}{bD} - \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{c}\dot{D}}{cD} - \frac{\dot{a}\dot{c}}{ac} = 0 \tag{26}$$

From the equations (23)-(26) the following equations are obtained

$$\frac{\dot{a}}{a} - \frac{\dot{b}}{b} = \frac{k_1}{abcD} \tag{27}$$

$$\frac{\dot{b}}{b} - \frac{\dot{c}}{c} = \frac{k_2}{abcD} \tag{28}$$

$$\frac{\dot{c}}{c} - \frac{\dot{D}}{D} = \frac{k_3}{abcD} \tag{29}$$

$$\frac{\dot{D}}{D} - \frac{\dot{a}}{a} = \frac{k_4}{abcD} \tag{30}$$

Here $k_1, k_2,$ k_3 and k_4 are the constant of integration. Without loss of generality, we can choose $k_1 = k_2 = k_3 = k_4 = k$(say)

The Equations (27)-(30) yields

$$a = b = c = D \tag{31}$$

We consider negative constant deceleration parameter model defined by

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = \text{constant} \tag{32}$$

The solution of above equation is

$$R = (Lt + M)^{\frac{1}{1+q}} \tag{33}$$

Where $L \neq 0, M$ are constants and $q + 1 \neq 0$

Using (7), (22), (31) and (33) we obtain

$$a = b = c = D = (Lt + M)^{\frac{1}{1+q}} \tag{34}$$

The model is described by the following metric

$$ds^2 = -dt^2 + (Lt + M)^{\frac{2}{1+q}} dx^2 + e^{-2x}(Lt + M)^{\frac{2}{1+q}}(dy^2 + dz^2 + dm^2 \tag{35}$$

4. COSMOLOGICAL PARAMETERS

Some of the important Physical and Geometrical Parameters are obtained bellow:

Volume element of the model is

$$V = (Lt + M)^{\frac{4}{1+q}} \tag{36}$$

The Hubble's Parameter is given by

$$H = \frac{L}{(q+1)(Lt+M)} \tag{37}$$

The expansion scalar (θ) is given by

$$\theta = \frac{4L}{(q+1)(Lt+M)} \tag{38}$$

Using equation (4), (34) the equation (20) yields

$$\rho = \frac{q+1}{4\alpha \log(Lt+M) + (q+1)k_5} \tag{39}$$

Where k are arbitrary constant.

From equations (4) and (39) becomes

(40) $$p = -\frac{(q+1)[3\alpha\log(Lt+M)-(q+1)(k_5+\alpha)]}{[3\alpha\log(Lt+M)+(q+1)k_5]^2}$$

The shear scalar is

(41) $$\sigma^2 = 0$$

The average anisotropy parameter is

(42) $$\Delta = 0$$

5. DISCUSSION

The variation of some of the important cosmological parameters (Physical and Geometrical) with respect to time for the model are shown below by taking, $q = -0.5$, $L = M = 1$, $k_5 = 2$ and $\alpha = 1$.

From the equation (36) it is seen that the spatial volume V is finite when time, $t = 0$. With the increases of the time the spatial volume V also increases and it becomes infinitely large as $t \to \infty$, so spatial volume expands as shown in fig.1. This shows that the universe is expanding with the increases of time.

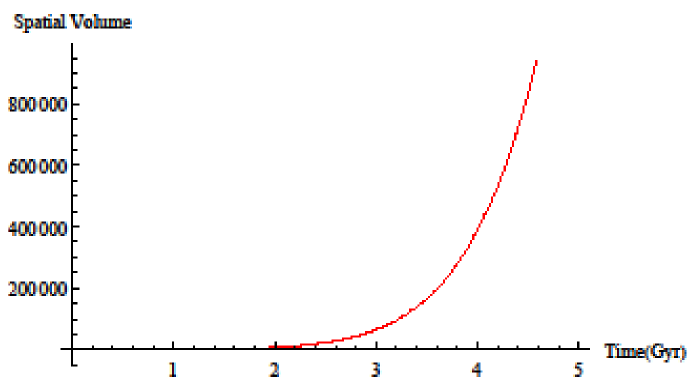

Fig1.Variation of Volume 'V' Vs. Time t.

From the equations (37) and (38) we observed that the quantities like Hubble's parameter H and expansion scalar θ are decreasing functions of time and they are finite when, $t = 0$ and are vanishing for infinitely large value of $'t'$ which are shown in the fig2 bellow. Also, since $\frac{dH}{dt} < 0$ which also tells us that our present universe is in the accelerated expanding mode.

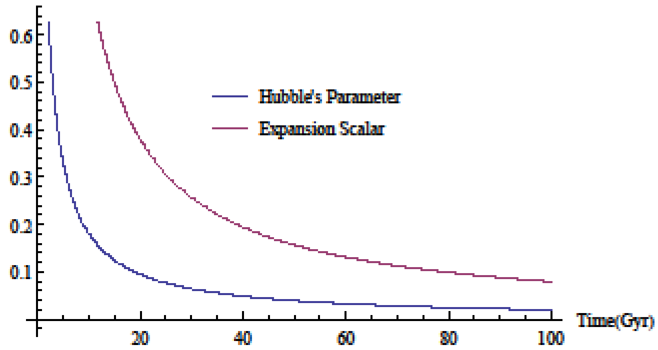

Fig2.Variation of H, θ Vs. Time ‘t’.

We observe from fig.3 or from equation (39), that the evolution of the energy density ρ is large constant at the time $t = 0$ and with increases of time the energy density decreases and finally when time $t \to \infty$ the energy density ρ becomes zero. This shows that the universe is expanding with time.

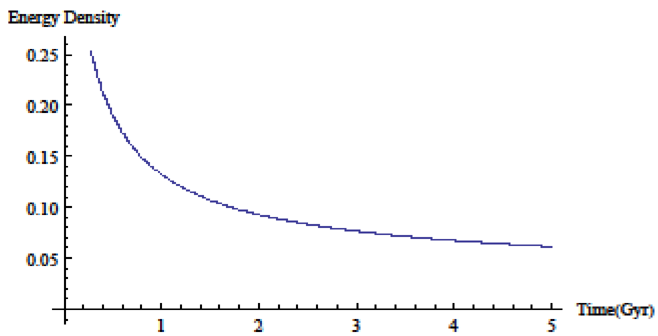

Fig3.Variation of energy density ‘ρ’ Vs. Time ‘t’.

While from the equation (40) it is observed that the isotropic pressure p is a small positive constant at initial epoch, $t = 0$ and it decreases with progresses of time and changes sign from positive to negative as shown in the fig4. Thus the present model universe passes through the transition from matter dominated era to inflationary era.

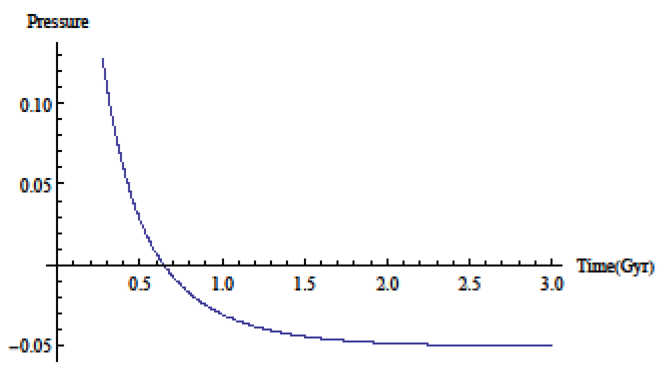

Fig4.Variation of p Vs. Time t.

For this model the value of shear scalar σ is zero throughout the evolution as seen from the equation (41), so the model is shear free. The ratio $\frac{\sigma^2}{\theta^2}$ tends to zero as time tends to infinity, showing that the model approaches isotropy at the late time universe, which agrees with Collins and Hawking [45].

From the equation (42) it is seen that average anisotropy parameter, $\Delta = 0$, so the model is isotropic one and this model also shows the late time acceleration of the universe.

It is observed that the spatial volume, all the three scale factors and all other physical and kinematical parameters are constant at initial epoch, $t = 0$. This shows that the present model is free from the initial singularity.

6. CONCLUSION

Here in this paper we studied a Bianchi type-V cosmological model in the context of general theory of relativity with quadratic EoS interacting with perfect fluid in five-dimensional space-time. The general solutions of the Einstein's field equations have been obtained under the assumption of quadratic EoS, $p = \alpha\rho^2 - \rho$, where $\alpha \neq 0$ is arbitrary constant considering the deceleration parameter as a constant quantity. The model obtained here is expanding with accelerated motion which agrees the recent observational data. The model is isotropic throughout the evolution, non-sharing and free from the initial singularity. We observe from eqn. (39) that the evolution of the energy density ρ is constant at the time $t = 0$ and when time $t \rightarrow \infty$ the energy density ρ decreases to a finite constant value. While eqn. (40) indicates that the isotropic pressure (p) is a negative quantity and it decreases when time $t \rightarrow \infty$. Such type of solution is consistent with recent observational data like SNe1a. The negative pressure may be a possible cause of the accelerated expansion of the universe. Also, generally the negative pressure can resist the attractive gravity of matter.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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