Generalized Cosmic Chaplygin Gas in Bianchi Type-I Universe and Law of Variation of Hubble's Parameter

3.1 Introduction

The accelerating expansion of the present universe draws the special attention to the authors of the subject in cosmology. Bianchi type models have been studied by several authors in an attempt to understand better the observed small amount of anisotropy in the universe. In the last decades in understanding the Physics behind the accelerated expansion of the universe have taken considerable interest by the cosmologist. Recent observations like Type Ia Supernovae, cosmic microwave background (CMB) radiation, large scale structure (LSS) have strongly indicate that our universe is not only expanding but also going through a phase of accelerated expansion. This outstanding fact shows that the universe is spatially flat and the accelerated expansion is due to some mysterious form of energy called Dark Energy (DE), which is an exotic fluid with negative pressure with equation of state parameter (EoS) $p = w\rho$, where p is pressure and ρ is energy density and w is the equation of state parameter. For cosmic acceleration $w < -1/3$ is required. The simplest candidate for dark energy where $w = -1$ is the so called cosmological constant which is introduced by Einstein into his gravitational field equation. The region where EoS parameter $w < -1$, is typically

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referred to as a phantom dark energy universe. Phantom cosmologies often have the property that they end in a finite singularity in which the universe is destroyed in finite proper time by excessive expansion. Phantom energy can lead to a singularity in which scale factor and density become infinite at a finite time called as Big Rip or Cosmic Dooms Day. Cosmologist started making efforts to avoid this problem of Big Rip using $w < -1$. In the brane world scenario, Sahni and Shtanov has obtained well behaved expansion for the future universe without big rip problem with $w < -1$. Yadav have investigated FRW model for the future universe without big rip. Ghate and Patil have obtained models for future universe without big rip in Kaluza Klein and higher dimensional space times respectively. It is well established that anisotropic Bianchi type-I universe plays significant role in understanding the phenomenal formation of galaxies in early universe. Theoretical reasoning and recent observations of cosmic microwave background radiation (CMBR) subscribe to the existence of anisotropic phase that approaches an isotropic one. Being the straightforward generalization of the flat FRW model, Bianchi Type I is one of the simplest models of the anisotropic universe that describes a homogeneous and spatially flat universe. Bianchi Type I space-time has a different scale factor in each direction, thereby introducing an anisotropy to the system.

In the framework of Robertson Walker Cosmology Chaplygin Gas (CG) is also considered as a source of dark energy having negative pressure with equation of state given by $p = -A/\rho$, $(A > 0)$, where p,ρ are respectively the pressure and energy density $(\rho > 0)$and A is a positive constant . The Chaplygin gas has super symmetry generalization (Bento, Gorini). Bertolami et.al have found that generalized Chaplygin gas is better fit for latest Supernovae data.

In this chapter we consider the model, where dark energy behaves as Generalized Cosmic Chaplygin Gas as well as fluid with equation of state $p = w\rho$ simultaneously then Big Rip does not arise even for equation of state parameter $w < -1$ and the scale factor, obtained here, does not possess future singularity.

3.2 Metric and Field Equations

The Bianchi Type I line element is

$$ds^2 = -dt^2 + a_1^2(t)dx^2 + a_2^2(t)dy^2 + a_3^2(t)dz^2 \tag{3.1}$$

where the metric potentials a_1, a_2 and a_3 are functions of cosmic time t alone. This ensures that the model is spatially homogeneous.

The average scale factor R of Bianchi type I model is defined as

$$R = (a_1 a_2 a_3)^{\frac{1}{3}} \tag{3.2}$$

A volume scale factor $V$is given by

$$V = R^3 = a_1 a_2 a_3 \tag{3.3}$$

The generalized Hubble parameter H and deceleration parameter q in analogy with FRW universe defined as

$$H = \frac{\dot{R}}{R} = \frac{1}{3} = \left(\frac{\dot{a_1}}{a_1} + \frac{\dot{a_2}}{a_2} + \frac{\dot{a_3}}{a_3}\right) \tag{3.4}$$

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -\frac{\ddot{R}}{aH^2} \tag{3.5}$$

where an over dot denotes derivative with respect to the cosmic time t.

Also we have

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right) \tag{3.6}$$

where $H_1 = \frac{\dot{a_1}}{a_1}$, $H_2 = \frac{\dot{a_2}}{a_2}$ and $H_3 = \frac{\dot{a_3}}{a_3}$ are directional Hubble factors in the directions of x, y and z axes respectively.

When the cosmological model is dominated by a fluid obeying the equation of state $p = w\rho$ with p as isotropic pressure, ρ as energy density and $-1 \leq w < -1/3$

then it is supposed that the expansion of the universe is accelerating.

The equation of state of Generalized Cosmic Chaplygin gas is

$$p = -\rho^{-\alpha}\left[C + \left(\rho^{1+\alpha} - C\right)^{-\gamma}\right] \tag{3.7}$$

where p and ρ are respectively the pressure and energy density, $C = \frac{A}{1+\gamma} - 1$, with A being a constant that can take on both positive and negative values, and $-L < \gamma < 0$,L being a positive definite constant, which can take on values larger than unity.
The conservation equation for the homogeneous perfect fluid constituting the cosmic fluid is

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{3.8}$$

From equation (3.7) and (3.8) we get,

$$\rho^{\alpha+1} = C + \left[1 + \left(\frac{R_0}{R}\right)^{3(\alpha+1)(\gamma+1)}\left\{\left(\rho_0^{\alpha+1} - C\right)^{\gamma+1} - 1\right\}\right]^{\frac{1}{\gamma+1}} \tag{3.9}$$

where ρ_0 and R_0 are the present values of energy density and scale factor at the present time t_0.

In the present model, it is assumed that the dark energy behaves like Generalized Cosmic Chaplygin gas, obeying equation (3.7) as well as fluid with equation of state

$$p = w\rho \tag{3.10}$$

with $w < -1$ simultaneously.

Equation (3.7) and (3.10) yield as

$$\omega = -\frac{1}{\rho^{\alpha+1}}\left[C + \left(\rho^{\alpha+1} - C\right)^{-\gamma}\right] \tag{3.11}$$

At the present time t_0

$$C = \frac{-\omega_0\rho_0^{1+\alpha} - \rho_0^{-\gamma(1+\alpha)}}{1 + \gamma\rho_0^{-(1+\alpha)(1+\gamma)}} \tag{3.12}$$

From equations (3.9) and (3.12) we get,

$$\rho^{\alpha+1} = \frac{-\omega_0\rho_0^{1+\alpha} - \rho_0^{-\gamma(1+\alpha)}}{1+\gamma\rho_0^{-(1+\alpha)(1+\gamma)}}$$

$$+\left[1+\left(\frac{R_0}{R}\right)^{3(\alpha+1)(\gamma+1)}\left\{\left(\rho_0^{\alpha+1}+\frac{\omega_0\rho_0^{1+\alpha}+\rho_0^{-\gamma(1+\alpha)}}{1+\gamma\rho_0^{-(1+\alpha)(1+\gamma)}}\right)^{\gamma+1}-1\right\}\right]^{\frac{1}{\gamma+1}} \quad (3.13)$$

In the homogeneous model of the universe, a scalar field $\phi\,(t)$ with potential $V\,(\phi)$ has energy density

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi) \quad (3.14)$$

and pressure

$$p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi) \quad (3.15)$$

where $\dot{\phi}$ is differentiation of ϕ with respect to t.

From equations (3.14) and (3.15) it is obtained that

$$\dot{\phi}^2 = p_\phi + \rho_\phi \quad (3.16)$$

Using equations (3.7), (3.10), (3.12), equation (3.16) reduces to

$$\dot{\phi}^2 = \frac{\rho^{\alpha+1} - \frac{-\omega_0\rho_0^{1+\alpha}-\rho_0^{-\gamma(1+\alpha)}}{1+\gamma\rho_0^{-(1+\alpha)(1+\gamma)}}}{\rho^\alpha} - \frac{\left(\rho^{1+\alpha} - \frac{-\omega_0\rho_0^{1+\alpha}-\rho_0^{-\gamma(1+\alpha)}}{1+\gamma\rho_0^{-(1+\alpha)(1+\gamma)}}\right)^{-\gamma}}{\rho^\alpha} \quad (3.17)$$

Equation and lead to

$$\dot{\phi}^2 = \frac{\frac{-\omega_0\rho_0^{1+\alpha}-\rho_0^{-\gamma(1+\alpha)}}{1+\gamma\rho_0^{-(1+\alpha)(1+\gamma)}}+\left[1+\left(\frac{R_0}{R}\right)^{3(\alpha+1)(\gamma+1)}\left\{\left(\rho_0^{\alpha+1}+\frac{\omega_0\rho_0^{1+\alpha}+\rho_0^{-\gamma(1+\alpha)}}{1+\gamma\rho_0^{-(1+\alpha)(1+\gamma)}}\right)^{\gamma+1}-1\right\}\right]^{\frac{1}{\gamma+1}}-\frac{-\omega_0\rho_0^{1+\alpha}-\rho_0^{-\gamma(1+\alpha)}}{1+\gamma\rho_0^{-(1+\alpha)(1+\gamma)}}}{\left[\frac{-\omega_0\rho_0^{1+\alpha}-\rho_0^{-\gamma(1+\alpha)}}{1+\gamma\rho_0^{-(1+\alpha)(1+\gamma)}}+\left[1+\left(\frac{R_0}{R}\right)^{3(\alpha+1)(\gamma+1)}\left\{\left(\rho_0^{\alpha+1}+\frac{\omega_0\rho_0^{1+\alpha}+\rho_0^{-\gamma(1+\alpha)}}{1+\gamma\rho_0^{-(1+\alpha)(1+\gamma)}}\right)^{\gamma+1}-1\right\}\right]^{\frac{1}{\gamma+1}}\right]^{\frac{1}{\alpha}}} - \frac{\left(\frac{-\omega_0\rho_0^{1+\alpha}-\rho_0^{-\gamma(1+\alpha)}}{1+\gamma\rho_0^{-(1+\alpha)(1+\gamma)}}+\left[1+\left(\frac{R_0}{R}\right)^{3(\alpha+1)(\gamma+1)}\left\{\left(\rho_0^{\alpha+1}+\frac{\omega_0\rho_0^{1+\alpha}+\rho_0^{-\gamma(1+\alpha)}}{1+\gamma\rho_0^{-(1+\alpha)(1+\gamma)}}\right)^{\gamma+1}-1\right\}\right]^{\frac{1}{\gamma+1}}-\frac{-\omega_0\rho_0^{1+\alpha}-\rho_0^{-\gamma(1+\alpha)}}{1+\gamma\rho_0^{-(1+\alpha)(1+\gamma)}}\right)^{-\gamma}}{\left[\frac{-\omega_0\rho_0^{1+\alpha}-\rho_0^{-\gamma(1+\alpha)}}{1+\gamma\rho_0^{-(1+\alpha)(1+\gamma)}}+\left[1+\left(\frac{R_0}{R}\right)^{3(\alpha+1)(\gamma+1)}\left\{\left(\rho_0^{\alpha+1}+\frac{\omega_0\rho_0^{1+\alpha}+\rho_0^{-\gamma(1+\alpha)}}{1+\gamma\rho_0^{-(1+\alpha)(1+\gamma)}}\right)^{\gamma+1}-1\right\}\right]^{\frac{1}{\gamma+1}}\right]^{\frac{1}{\alpha}}} \tag{3.18}$$

From equation (3.18), it is observed that for $\omega_0 > -1$, $\dot{\phi}^2 > 0$, giving positive kinetic energy and for $\omega_0 < -1$, $\dot{\phi}^2 < 0$, giving negative kinetic energy. $\omega_0 > -1$and $\omega_0 < -1$are representing the case of quintessence and phantom fluid dominated universe respectively. Similar result are obtained by Hoyle and Narlikar in C -field with negative kinetic energy for steady state theory of universe. Thus the dual behavior of dark energy fluid, obeying equation (3.7) and (3.10) is possible for scalars, frequently used for cosmological dynamics. So, this assumption is not unrealistic.

3.3 Law of variation of Hubble's parameter

By applying a special law of variation of Hubble's parameter that yield a constant value of deceleration parameter (DP) S. Kumar and C. P. Singh have investigated a spatially homogenous and anisotropic Bianchi Type I model. The law of variation of Hubble's parameter is

$$H = DR^{-n} = D(a_1a_2a_3)^{-n} \tag{3.19}$$

where D and n are positive constant.

The deceleration parameter (q) is given by

$$q = -\frac{R\ddot{R}}{\dot{R}^2} \tag{3.20}$$

From equations (3.4) and (3.19) we get,

$$\frac{\dot{R}}{R} = Da^{-n} \tag{3.21}$$

Integrating equation (3.21) we get

$$R = (nDt + c_0)^{1/n} \tag{3.22}$$

where c_0 is the constant of integration.

Equation (3.22) represents accelerated expansion of the universe with $R(t) \to \infty$ as $t \to \infty$ and observations of Supernova Ia, WMAP also supported this result.
In this case, Hubble's distance is given by

$$H^{-1} = nt + D_0 \tag{3.23}$$

where $D_0 = \frac{c_0}{D}$ is constant.

From equation (3.23), when $t \to 0, H^{-1} \to D_0$ and $t \to \infty, H^{-1} \to \infty$ showing the growth of Hubble's distance with time. Thus in the present case galaxies will not disappear when $t \to \infty$. It is observed that, if phantom fluid behaves like Generalized Cosmic Chaplygin Gas and fluid with $p = \omega\rho$ simultaneously then the accelerated growth of scale factor of future universe for time $t_0 < t < \infty$ with no future singularity can be obtained.

The horizon distance for this case is

$$d_H = R(t) \int_0^t \frac{dt'}{a(t')} \tag{3.24}$$

From equation (3.22) and (3.24) we get,

$$d_H = \frac{1}{D(n-1)}\left[(nDt + c_0) - D'(nDt + c_0)^{1/n}\right] \tag{3.25}$$

Equation (3.25) shows that

$$d_H > R(t) \text{ for } t > \frac{(1+D')^{\frac{n}{n-1}} - c_0}{nD}$$

So the universe will be colder and darker since the horizon grows more rapidly than scale factor. With the dominance of dark energy the universe is like flat or open.

Using equation (3.22) and (3.13) the energy density is given by

$$\rho^{\alpha+1} = \frac{-\omega_0\rho_0^{1+\alpha} - \rho_0^{-\gamma(1+\alpha)}}{1+\gamma\rho_0^{-(1+\alpha)(1+\gamma)}}$$

$$+\left[1+\left(\frac{R_0}{(nDt+c_0)^{1/n}}\right)^{3(\alpha+1)(\gamma+1)}\left\{\left(\rho_0^{\alpha+1}+\frac{\omega_0\rho_0^{1+\alpha}+\rho_0^{-\gamma(1+\alpha)}}{1+\gamma\rho_0^{-(1+\alpha)(1+\gamma)}}\right)^{\gamma+1}-1\right\}\right]^{\frac{1}{\gamma+1}} \tag{3.26}$$

3.4 Conclusion

It is found that dark energy behaves simultaneously like a fluid with equation of state $p = \omega\rho$; $\omega < -1$ as well as Generalized Cosmic Chaplygin Gas with equation of state $p = -\rho^{-\alpha}\left[C + (\rho^{1+\alpha} - C)^{-\gamma}\right]$. From equation (3.20), the energy density increases with time since $\omega_0 < -1$ and also pressure increases with time. Singh et. al have estimated ω_0 for model in the range $-2.4 < \omega_0 < -1.74$ up to 95% confidence level based on the data of Supernovae Ia. Escaping from cosmic doomsday is demonstrated using quantum correction in field equations near $t = t_s$, for models with future singularity. A model for phantom cosmology, with accelerated expansion, using classical approach is explored with is free from catastrophic situations. From Bianchi Type I space time this model is derived using effective role of Generalized Cosmic Chaplygin Gas behavior in natural way.