Universe Filled with Generalized Cosmic Chaplygin Gas and Barotropic Fluid

4.1 Introduction

Recent cosmological measurements obtained by SNe Ia, WMAP, SDSS and X-ray indicate that our universe is expanding and the expansion of the universe is accelerating. The notion known as dark energy (DE) with large negative pressure is proposed to explain this phenomena which marked the beginning of a new era in cosmology. At present there are a lot of theoretical models of dark energy. Cosmological constant is the simplest model of dark energy corresponding to the $\Lambda\mathrm{CDM}$ model. Besides cosmological constant the other dark energy models are for example quintessence, phantom, tachyon, holographic dark enrgy, K-essence and Chaplygin gas models with various equation of state. Our universe consists of about 70% dark energy, 25% dark matter and 5% normal matter (Cold matter and Baryons) and negligible radiation according to the cosmological measurements and analysis. For the accelerated expansion of the universe filled with fluids, the pressure p and the energy density ρ of the universe should violate the strong energy condition $\rho + 3p < 0$ i.e. pressure must be negative to accelerate the expansion of the universe.

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In Chaplygin gas (CG) the equation of state parameter for dark energy can be less than -1 as based on the observational data where the equation of state (EoS) is $p = -B/\rho$,$(B > 0)$. The Chaplygin gas is connected to string theory which can be obtained by the Nambu-Goto action moving in a $(d+2)$-dimensional space-time in the light-cone parametrization. The transition period can be described when the universe is filled with Chaplygin gas from a decelerated cosmological expansion to the present exponentially accelerated universe. A viable model was introduced by generalizing the above equation to the form $p = -B/\rho^{\alpha}$ with $0 \leq \alpha \leq 1$, which is known as generalized Chaplygin gas (GCG), consisting two free parameters B and α respectively. At high density the generalized Chaplygin gas corresponds to a dust $(p = 0)$universe which is not applicable to our universe. Therefore, modified Chaplygin gas (MCG) was introduced by Benaoum [2002] with the equation of state $p = A\rho - B/\rho^{\alpha}$, $(A > 0)$. This equation of state describes the evolution of the universe for small values of the cosmological scale factor corresponding to radiation era as well as for large values of the cosmological scale factor corresponding to $\Lambda\mathrm{CDM}$ model. The modified Chaplygin gas may be equivalently described in terms of a homogeneous minimally coupled scalar field ϕ in a Friedmann model.

Generalized cosmic Chaplygin gas was introduced in 2003 by P. F. Gonzalez-Diaz. This model can be made stable and free from unphysical behaviors even when the vacuum fluid satisfies the phantom energy condition, which is the striking factor of this model. The equation of state of Generalized cosmic Chaplygin gas is

$$p = -\rho^{-\alpha}\left[C + \left(\rho^{1+\alpha} - C\right)^{-\omega}\right] \tag{4.1}$$

where $C = \frac{A}{1+\omega} - 1$, with A being a constant that can take on both positive and negative values and $-L < \omega < 0$,L being a positive definite constant, which can take on values larger than unity.

W. Chakraborty and U. Debnath et al. [2007] has obtained the acceleration of the universe containing the modified Chaplygin gas and barotropic fluid. J. K. Singh et al.

[2015] within the framework of Lyra's geometry studied the modified Chaplygin gas with statefinder parameters. In this chapter we try to observe the state of the universe by using Generalized Cosmic Chaplygin gas and barotropic fluid and obtain the values of the statefinder parameters.

4.2 Field Equations and Their Solutions

The metric of a homogenous and isotropic universe in FRW model is

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] \tag{4.2}$$

where $R(t)$ is the scale factor and $k\ (=0, \pm 1)$ is the curvature scalar.

The Einstein field equations are $(\text{for } 8\pi G = c = 1)$

$$\frac{\dot{R}^2}{R^2} + \frac{k}{a^2} = \frac{1}{3}\rho \tag{4.3}$$

and

$$\frac{\ddot{R}}{R} = -\frac{1}{6}(\rho + 3p) \tag{4.4}$$

The energy conservation equation is

$$\dot{\rho} + 3\frac{\dot{R}}{R}(\rho + p) = 0 \tag{4.5}$$

For Generalized Cosmic Chaplygin gas, equation (4.5) yields

$$\rho = \left[C + \left\{1 + \frac{B}{R^{3(1+\alpha)(1+\omega)}}\right\}^{\frac{1}{1+\omega}}\right]^{\frac{1}{1+\alpha}} \tag{4.6}$$

where B is an arbitrary integration constant.

We consider two fluid cosmological model containing a component of generalized cosmic Chaplygin gas, with equation of state (4.6) and also a component of barotropic fluid with equation of state $p_1 = \omega\rho_1$. ω satisfies $-1 \leq \omega \leq 1$ for accelerating universe normally. But ω satisfies $-1.6 \leq \omega \leq 1$ according to observations i.e.

$\omega < -1$ corresponds to phantom model. The right hand side of equations (4.3) and (4.4), i.e. ρ and p should be replaced by $\rho + \rho_1$ and $p + p_1$ for these two fluid components respectively. We assumed that the two fluids are conserved separately. For generalized cosmic Chaplygin gas the expression for the density is given by equation (4.6). For another fluid, from equation (4.5) the expression for density is given by

$$\rho_1 = \frac{d}{a^{3(1+\omega)}} \tag{4.7}$$

where d is an integration constant.

Now to derive the expression for the potential, the following Lagrangian is considered

$$L_\phi = \frac{\dot{\phi}^2}{2} - V(\phi) \tag{4.8}$$

The analogous energy density ρ_ϕ and pressure p_ϕ corresponding scalar field ϕ having a self interacting potential $V(\phi)$ are given by

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi) = \rho + \rho_1 = \left[C + \left\{1 + \frac{B}{R^{3(1+\alpha)(1+\omega)}}\right\}^{\frac{1}{1+\omega}}\right]^{\frac{1}{1+\alpha}} + \frac{d}{R^{3(1+\omega)}} \tag{4.9}$$

$$p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi) = p + p_1 = -\left[C + \left\{1 + \frac{B}{R^{3(1+\alpha)(1+\omega)}}\right\}^{\frac{1}{1+\omega}}\right]^{-\frac{\alpha}{1+\omega}} \left[C + \left\{1 + \frac{B}{R^{3(1+\alpha)(1+\omega)}}\right\}^{-\frac{\omega}{1+\omega}}\right] + \frac{\omega d}{R^{3(1+\omega)}} \tag{4.10}$$

Now for flat universe ($k = 0$) we have the expression for ϕ and $V(\phi)$ as follows

$$\phi = \int \left[Y^{\frac{1}{1+\alpha}} - Y^{-\frac{\alpha}{1+\alpha}}\left[C + (Y - C)^{-\omega}\right] + \frac{(1 + C)\, d}{R^{3(1+\gamma)}}\right]^{\frac{1}{2}} dt \tag{4.11}$$

$$V(\phi) = \frac{1}{2}Y^{\frac{1}{1+\alpha}} + \frac{1}{2}Y^{-\frac{\alpha}{1+\alpha}}\left[C + (Y - C)^{-\omega}\right] + \frac{(1 - C)\, d}{R^{3(1+\gamma)}} \tag{4.12}$$

where $Y = C + \left\{1 + \frac{B}{R^{3(1+\alpha)(1+\omega)}}\right\}^{\frac{1}{1+\omega}}$.

4.3 The statefinder parameters

It was of utmost necessity to devise a method that would both qualitatively and quantitatively discriminate between various dark energy models, as numerous dark energy models began appearing. Sahni et. al [2003] proposed a pair of parameters $\{r, s\}$, called statefinder parameters in this context. In a independent manner these parameters are able to discriminate between different dark energy models. The statefinder diagnostic pair has the following form

$$r = \frac{\dddot{R}}{RH^3}, \quad \text{and} \quad s = \frac{r-1}{3(q-1/2)} \tag{4.13}$$

where $H\left(=\frac{\dot{R}}{R}\right)$ and $q\left(=\frac{R\ddot{R}}{\dot{R}^2}\right)$ are the Hubble parameter and the deceleration parameter respectively. These parameters allow us to characterize the properties of dark energy as these are dimensionless. Corresponding to different cosmological models, trajectories in the $\{r, s\}$ plane corresponds to the fixed point $s = 0, r = 1$.

These statefinder parameters for one fluid model are given by

$$r = 1 + \frac{9}{2}\left(1 + \frac{p}{\rho}\right)\frac{\partial p}{\partial \rho} \tag{4.14}$$

$$s = \left(1 + \frac{\rho}{p}\right)\frac{\partial p}{\partial \rho} \tag{4.15}$$

Equations (4.14) and (4.15) takes the following form for the two fluid component,

$$r = 1 + \frac{9}{2\left(\rho + \rho_1\right)}\left[\frac{\partial p}{\partial \rho}\left(\rho + p\right) + \frac{\partial p_1}{\partial \rho_1}\left(\rho_1 + p_1\right)\right] \tag{4.16}$$

$$s = \frac{1}{\left(p + p_1\right)}\left[\frac{\partial p}{\partial \rho}\left(\rho + p\right) + \frac{\partial p_1}{\partial \rho_1}\left(\rho_1 + p_1\right)\right] \tag{4.17}$$

The deceleration parameter q has the form

$$q = -\frac{\ddot{R}}{RH^2} = \frac{1}{2} + \frac{3}{2}\left(\frac{p + p_1}{\rho + \rho_1}\right) \tag{4.18}$$

For generalized cosmic Chaplygin gas and barotropic equation of state,

$$x = \frac{p}{\rho} = -\frac{1}{\rho^{1+\alpha}} \left[C + \left(\rho^{1+\alpha} - C \right)^{-\omega} \right] \tag{4.19}$$

$$y = \frac{\rho_1}{\rho} = \frac{\frac{d}{R^{3(1+\omega)}}}{\left[C + \left\{ 1 + \frac{B}{R^{3(1+\alpha)(1+\omega)}} \right\}^{\frac{1}{1+\omega}} \right]^{\frac{1}{1+\alpha}}} \tag{4.20}$$

Thus equations (4.16) and (4.17) can be written as

$$r = 1 + \frac{9s}{2} \left(\frac{x + \omega y}{1 + y} \right) \tag{4.21}$$

$$s = \frac{(1+x)\left\{ \omega\left(\alpha+1\right) - x\alpha\rho^{-2\alpha} \right\} + \omega(1+\omega)y}{x + \omega y} \tag{4.22}$$

Now $q < 0$ for cosmic acceleration and since $y > 0$ we get $x + \omega < -1/3$. The ratio between energy density of the barotropic fluid to that of Generalized cosmic Chaplygin gas is denoted by y and the ratios of fluid pressure to energy density for barotropic fluid and Generalized cosmic Chaplygin gas is denoted by x, ω respectively. We can assign different values to the barotropic index, since ω is constant. But at least one of the fluids must generate negative pressure for cosmic acceleration.

For different values of ω we get different cases which are as follows :

Case(i): For $\omega = 1/3$, we get $x < -2/3$ which corresponds to a universe having cosmic acceleration but unable to violate the strong energy condition as it does not contain barotropic fluid.

Case(ii): For $\omega = 0$ and $x < -1/3$ it also corresponds to cosmic acceleration and here Chaplygin gas violates the strong energy condition. The barotropic fluid represents the dust.

Case(iii): For $\omega = -1$ we get cosmic acceleration and here the Chaplygin gas does not violate the strong energy condition.

Case(iv): For $\omega < -1$ the Chaplygin gas represents the dark energy or the dark matter depending upon the values of x and the barotropic fluid represents the phantom model.

4.4 Conclusion

In this chapter we have discussed the universe filled with generalized cosmic Chaplygin gas and barotropic fluid to observe the phases of the universe when it undergoes acceleration using the statefinder parameters and obtained different situations depending upon the values of ω where both the fluids represent dark energy and sometimes dark matter.