Generalized Cosmic Chaplygin Gas Model Interacting in Non-Flat Universe

5.1 Introduction

Latest cosmological observations made by different cosmologists states that our universe is expanding in a accelerated way and dark energy is the main reason for this accelerating expansion of the universe because dark energy models possesses negative pressure. Due to the negative pressure of these dark energy models our universe is expanding exponentially. The empirical results based on the data are $\Omega_m \approx 0.3$ while $\Omega_\Lambda \approx 0.7$. Many cosmologists have given several models of dark energy to describe this exponential expansion of the universe. Observations from Type Ia Supernovae, cosmic microwave background (CMB) radiation from Wilkinson Microwave Anisotropy Probe (WMAP), large scale structure (LSS) from Sloan Digital Sky Survey (SDSS) strongly indicate the accelerated expansion of our universe. When the strong energy condition is violated by the pressure p and energy density ρ of the universe then only the expansion of the universe will be accelerating i.e. only when the pressure is negative which corresponds to phantom fluid. At present there are a lot of theoretical models to describe dark energy which are based on dynamics of a scalar or multi-scalar fields. Cosmological constant with the equation of state

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$p = -\rho$ or $\omega = -1$ $(p = \omega\rho)$is the simplest model of dark energy corresponding to the ΛCDM model. The cosmological constant together with the cold dark matter (CDM) treated as the standard model which fits the current observational data sets consistently. Although it supports the observational data, the ΛCDM model face the fine tuning problem (FTP) and the cosmic coincidence problem (CCP) where the densities of dark energy and dark matter are comparable. The possible resolution to these problems is a model based on the interaction between dark energy and dark matter. To test the coincidence problem, the interacting dark energy model (IDE) was first introduced in which ρ_m could decrease with the expansion of our universe slower than R^{-3}. Besides cosmological constant the other dark energy models describing the accelerated expansion of the universe with negative pressure are for example quintessence, phantom, tachyon, holographic dark enrgy, K-essence, quintom model and Chaplygin gas models with various equation of state. Many authors have discussed the interaction between dark energy and dark matter. Chakraborty *et al.* [2007] studied the generalized cosmic Chaplygin gas as a unified model of dark energy and dark matter. Verma *et al.* [2010, 2012, 2013] did a work on the interaction between dark energy and matter of the tachyonic scalar field. Rudra *et al.* [2012] studied the dynamics of interacting generalized cosmic Chaplygin gas in Brane-World scenario. Amani *et al.* [2014] considered the interaction of closed string tachyon with generalized cosmic Chaplygin gas. Chowdhury *et al.* discussed the interaction between generalized cosmic Chaplygin gas in loop quantum cosmology. Naji *et al.* [2014] in flat FRW universe proposed the interaction between holographic dark energy density and generalized cosmic Chaplygin gas energy density. Jamil *et al.* [2008] investigated the interaction of dark energy and dark matter with inhomogeneous equation of state. The matter component in most of the dark energy models are considered as an invisible cosmic fluid. The Chaplygin gas is also used as an exotic type of fluid. The lifting force on a wing of an airplane in aerodynamics is taken as the base of the equation of state of the Chaplygin gas. The Chaplygin gas equation of state for a homogeneous model is given by

$$p = -A/\rho \tag{5.1}$$

where p and ρ are respectively pressure and energy density in commoving reference frame, with $\rho > 0$; $A$is a positive constant. The above equation is connected to string theory and can be achieved by the D-branes Nambu-Goto action which is moving in a (D+2)-dimensional space-time in the light-cone parametrization.

From the relativistic energy conservation equation using the equation of state (5.1) the density is given by

$$\rho_\Lambda = \sqrt{A + B/V^2} \tag{5.2}$$

where$B$is an integration constant.

Bento *et al.* [2002] generalized the equation of state (5.1) to

$$p = -A/\rho^\alpha, 0 \leq \alpha \leq 1 \tag{5.3}$$

which is known as generalized Chaplygin gas.

Benaoum *et al.* [2012] within the framework of FRW introduced the modified Chaplygin gas whose equation of state is given by

$$p = A\rho - \frac{B}{\rho^\alpha},\ \alpha \geq 1 \tag{5.4}$$

where ρ and p are energy density and pressure respectively and A and B are positive constant.

Recent papers favoured a universe with spatial curvature and also based on some experimental data it corresponds that our universe is not perfectly flat universe. Setare et al. [2007] considering the generalized Chaplygin gas as a dark energy model studied the generalized Chaplygin gas to obtain the equation of state for the generalized Chaplygin gas energy density interacting in non-flat universe. In this chapter, we obtain the equation of state for interacting Chaplygin gas energy density in non-flat universe using Generalized Cosmic Chaplygin gas as dark energy model.

5.2 Generalized cosmic Chaplygin gas interacting dark energy

In this section we consider the interaction between the generalized cosmic Chaplygin gas energy density and a cold dark matter to obtain the generalized cosmic Chaplygin gas equation of state. The continuity equations for dark energy and cold dark matter are given by

$$\dot{\rho}_\Lambda + 3H(1 + w_\Lambda)\rho_\Lambda = -Q \tag{5.5}$$

$$\dot{\rho}_m + 3H\rho_m = Q \tag{5.6}$$

The quantity $Q = \Gamma\rho_\Lambda$ gives the interaction between the two, which is a decaying of the Chaplygin gas component with the decay rate Γ into cold dark matter.

Let us consider as usual $r = \rho_m/\rho_\Lambda$ which is the ratio of the two energy densities. Then the above equation leads to

$$\dot{r} = 3Hr\left[w_\Lambda + \frac{1+r}{r}\frac{\Gamma}{3H}\right] \tag{5.7}$$

After following Kim et a. [2006], if we consider

$$w_\Lambda^{eff} = w_\Lambda + \frac{\Gamma}{3H}, w_m^{eff} = -\frac{1}{r}\frac{\Gamma}{3H} \tag{5.8}$$

Then in the standard form the continuity equations can be written as

$$\dot{\rho}_\Lambda + 3H(1 + w_\Lambda^{eff})\rho_\Lambda = 0 \tag{5.9}$$

$$\dot{\rho}_m + 3H(1 + w_m^{eff})\rho_m = 0 \tag{5.10}$$

5.3 Non-flat FRW universe

The line element of the Friedmann-Robertson-Walker non-flat universe model is

$$ds^2 = -dt^2 + R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \tag{5.11}$$

where k denotes the curvature of space $k = 0, 1, -1$ for flat, closed and open universe respectively. With observations Bennett et al. [2006] the compatible universe is a closed universe with a small positive curvature ($\Omega_k = 0.01$). The energy density and the curvature of the universe are related by using the Friedmann equation. The first Friedmann equation is given by

$$H^2 + \frac{k}{R^2} = \frac{1}{3M_p^2}\left[\rho_\Lambda + \rho_m\right] \tag{5.12}$$

As usual we take

$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{\rho_m}{3M_p^2 H^2} \tag{5.13}$$

$$\Omega_m = \frac{\rho_\Lambda}{\rho_{cr}} = \frac{\rho_\Lambda}{3M_p^2 H^2} \tag{5.14}$$

$$\Omega_k = \frac{k}{R^2 H^2} \tag{5.15}$$

Friedmann equation can now be written as

$$\Omega_m + \Omega_\Lambda = 1 + \Omega_k \tag{5.16}$$

The relation r for ratio of energy densities from the above equations (5.13), (5.14), (5.15), (5.16) is obtained as

$$r = \frac{1 + \Omega_k - \Omega_\Lambda}{\Omega_\Lambda} \tag{5.17}$$

The equation of state of the Generalized Cosmic Chaplygin gas is

$$p_\Lambda = -\rho_\Lambda^{-\alpha}\left[C + \left(\rho_\Lambda^{1+\alpha} - C\right)^{-w}\right] \tag{5.18}$$

The density evolution from the above equation of state is

$$\rho_\Lambda = \left[C + \left\{1 + \frac{B}{R^{3(1+\alpha)(1+w)}}\right\}^{\frac{1}{1+w}}\right]^{\frac{1}{1+\alpha}} \tag{5.19}$$

With respect to cosmic time we take derivatives on both sides of the above equation and obtain

$$\begin{aligned}\dot{\rho}_\Lambda = -3BHR^{-3(1+\alpha)(1+w)} &\left[C + \left\{1 + \tfrac{B}{R^{3(1+\alpha)(1+w)}}\right\}^{\frac{1}{1+w}}\right]^{-\frac{\alpha}{1+\alpha}} \\ &\left[1 + \tfrac{B}{R^{3(1+\alpha)(1+w)}}\right]^{-\frac{w}{1+w}}\end{aligned} \tag{5.20}$$

Substituting this relation in equation (5.5) and using the definition $Q = \Gamma\rho_\Lambda$, we obtain

$$\begin{aligned}w_\Lambda = \tfrac{B}{R^{3(1+\alpha)(1+w)}} &\left[1 + \tfrac{B}{R^{\cdot(1+\alpha)(1+w)}}\right]^{-\frac{w}{1+w}} \\ &\left[C + \left\{1 + \tfrac{B}{R^{3(1+\alpha)(1+w)}}\right\}^{\frac{1}{1+w}}\right]^{-1} - \tfrac{\Gamma}{3H} - 1\end{aligned} \tag{5.21}$$

As mentioned in the we assume the decay rate given by the relation

$$\Gamma = 3b^2\left(1 + r\right)H \tag{5.22}$$

with coupling constant b^2. Using equation (5.17) the above decay rate takes the following form

$$\Gamma = 3b^2 H\left(\frac{1 + \Omega_k}{\Omega_\Lambda}\right) \tag{5.23}$$

Substituting the value of this in equation (5.21), then the generalized cosmic Chaplygin gas energy equation of state is given by

$$\begin{aligned}w_\Lambda = \tfrac{B}{R^{3(1+\alpha)(1+w)}} &\left[1 + \tfrac{B}{R^{3(1+\alpha)(1+w)}}\right]^{-\frac{w}{1+w}} \\ &\left[C + \left\{1 + \tfrac{B}{R^{3(1+\alpha)(1+w)}}\right\}^{\frac{1}{1+w}}\right]^{-1} - b^2\left(\tfrac{1+\Omega_k}{\Omega_\Lambda}\right) - 1\end{aligned} \tag{5.24}$$

Now using the definition of generalized cosmic Chaplygin gas energy ρ_Λ and using Ω_Λ the above equation can be written as

$$\begin{aligned}w_\Lambda = \tfrac{B}{\left[3M_p^2 H^2 R^{3(1+w)}\Omega_\Lambda\right]^{1+\alpha}} &\left[1 + \tfrac{B}{R^{3(1+\alpha)(1+w)}}\right]^{-\frac{w}{1+w}} \\ &- b^2\left(\tfrac{1+\Omega_k}{\Omega_\Lambda}\right) - 1\end{aligned} \tag{5.25}$$

From equations (5.8), (5.23) and (5.25) the effective equation of state is given by

$$w_\Lambda^{eff} = \frac{B}{\left[3M_p^2 H^2 R^{3(1+w)}\Omega_\Lambda\right]^{1+\alpha}}\left[1+\frac{B}{R^{3(1+\alpha)(1+w)}}\right]^{-\frac{w}{1+w}} - 1 \tag{5.26}$$

Now when the value of B is negative we get $w_\Lambda^{eff} < -1$ which corresponds to a phantom energy dominated universe and it corresponds to the effective parameter of state of Chaplygin gas for $\alpha = 1$and $\omega = 0$. The term under the square root in the equation (5.19) for energy density should be positive for $\alpha = 1$, i.e. $R < \left[\frac{B}{-1+(-C)^{1+w}}\right]^{\frac{1}{6(1+w)}}$ and for $\alpha = 1$and $\omega = 0$ we get $R < \left[-\frac{B}{1+C}\right]^{\frac{1}{6}}$ the minimal value of the scale factor is given by $R_{\min} = \left[-\frac{B}{1+C}\right]^{\frac{1}{6}}$ and we get a bouncing universe according to this model.

From equation (5.19) it is observed that the value of cosmic scale factor lies in the interval $R_{i\ \min} < R_i < \infty$(for $i = 1, 2, 3$)which yields $0 < \rho < (C+1)^{\frac{1}{1+\alpha}}$, where

$$R_{\min} = \left[-\frac{B}{1+C}\right]^{\frac{1}{3(1+\alpha)(1+\omega)}} \tag{5.27}$$

From equation (5.2) we observe that the Chaplygin gas interpolates between dust and cosmological model for small and large values of the scale factor a_i respectively. If we consider a homogenous scalar field and potential field following to describe the Chaplygin Cosmology, then

$$\begin{aligned}\dot{\phi}^2 = &\left[C+\left\{1+\tfrac{B}{R^{3(1+\alpha)(1+\omega)}}\right\}^{\frac{1}{1+\omega}}\right]^{\frac{1}{1+\alpha}}\\ &\left[\begin{array}{l} 1-\left\{C+\left\{1+\tfrac{B}{R^{3(1+\alpha)(1+\omega)}}\right\}^{\frac{1}{1+\omega}}\right\}^{-1} \\ \left\{C+\left\{1+\tfrac{B}{R^{3(1+\alpha)(1+\omega)}}\right\}^{\frac{-\omega}{1+\omega}}\right\}\end{array}\right]\end{aligned} \tag{5.28}$$

The Lagrangian of scalar field $\phi(t)$ in this case is given by

$$L = \frac{1}{2}\dot{\phi}^2 - V(\phi) = -\frac{1}{2}\dot{\psi}^2 - V(i\psi) \tag{5.29}$$

Corresponding to the scalar field $\psi$the energy density and the pressure are respectively

$$\rho_\psi = -\frac{1}{2}\dot{\psi}^2 + V(i\psi) \tag{5.30}$$

$$p_\psi = -\frac{1}{2}\dot{\psi}^2 - V(i\psi) \tag{5.31}$$

Therefore, ψ the scalar field is a phantom field. Thus interacting generalized cosmic Chaplygin gas dark energy model in non-flat universe generates equation of state which corresponds to phantom energy.

5.4 Conclusion

Among the different dark energy models given by different cosmologists to explain the accelerated expansion of the universe, Chaplygin gas has emerged as a possible dark energy and dark matter unification as it acts as a pressure less fluid at an early stage of the universe and as a cosmological constant at a later stage. So it interpolates different phases of the evolution of the universe, including the present day accelerated expansion of the universe. As the order of energy density of dark energy and dark matter is same in the present universe it suggests that there may be some relation between them. Thakur et al. has obtained the results for the best-fit values of the parameters for CDM and unified dark matter energy models in agreement with union compilation data. The best-fit values of the models are $A_s = 0.99$, $B = 0.01$, $\alpha = 0.01$ for CDM and $A_s = 0.8$, $B = 0.06$, $\alpha = 0.11$ for unified dark matter and energy model. Sharif et al. determined a physically acceptable range of the scalar-tensor ratio $r < 0.36$ which represents the expanding universe. Paul et al. studied the observational constraints on modified Chaplygin gas in Hrava-Lifshitz gravity with dark radiation. Liao et al. given the constraints on the new generalized Chaplygin gas using latest observational data. Ranjit et al. also investigated the constraining parameters of generalized cosmic Chaplygin gas in loop quantum cosmology and obtained that the model is in agreement with the Supernovae Type Ia sample data.

In this paper we considered the interaction between the generalized cosmic Chaplygin gas energy density and cold dark matter (CDM) and obtained the equation of state for the interacting generalized cosmic Chaplygin energy density with cold dark matter in spatially non-flat universe. By considering $B < 0$ we get $w_{\Lambda}^{eff} < -1$ which can describe the phantom field interacting generalized cosmic Chaplygin gas dark energy and also supports the latest observational data.