Modified Chaplygin Gas and Bianchi type V cosmological model

8.1 Introduction

The study of Bianchi type V cosmological models create more interest as these models contain isotropic special cases and permit arbitrary small anisotropy levels at some instant of cosmic time. This property makes them suitable as a model of our universe. The homogeneous and isotropic Friedman–Robertson–Walker (FRW) cosmological models, which are used to describe standard cosmological models, are particular cases of Bianchi type I,V and IX universes, according to whether the constant curvature of the physical three-space, t = constant, is zero, negative or positive. These models will be interesting to construct cosmological models of the types which are of class one. Present cosmology is based on the FRW model which is completely homogeneous and isotropic. This is in agreement with observational data about the large scale structure of the universe. However, although homogeneous but anisotropic models are more restricted than the inhomogeneous models, they explain a number of observed phenomena quite satisfactorily. This stimulates the research for obtaining exact anisotropic solution for Einstein's field equations (EFEs) as cosmologically accepted physical models for the universe (at least in the early stages). Among different models Bianchi type-V Universes are the natural generalization of the open FRW model, which eventually become isotropic. Perfect fluid cosmological models in scalar-tensor theories of gravitation have been extensively studied using different techniques.

Solutions to the field equations may be generated by applying the law of variation for Hubble's parameter, as proposed by Berman which yields a constant value of the deceleration parameter. In general relativity, cosmological models are usually constructed under the assumption that the matter content is an idealized perfect fluid. This assumption may be a good approximation to the actual matter content of universe at the present epoch. Roy and Prasad have investigated Bianchi type V Universes which are locally rotationally symmetric and are of embedding class one filled with perfect fluid with heat conduction and radiation. Bianchi type V cosmological models have been studied by several researchers such as Farnsworth, Collins Maartens and Nel, Wainwright et al., Beesham, Maharaj and Beesham, Hewitt and wainwright, Camci et al., Meena and Bali, Pradhan et al., Aydogdu and Salti in different physical contexts. Christodoulakis et al. have studied un-tilted diffuse matter Bianchi V Universe with perfect fluid and scalar field coupled to perfect fluid sources obeying a general equation of state. Lyra introduced a gauge function into the structure-less manifold as a result of which a displacement field arises naturally. The energy momentum tensor is not conserved in Lyra's geometry. Several authors have studied cosmology in Lyra's geometry with a constant displacement field, which plays the same role as the cosmological constant in the standard general relativity. Halford studied Robertson-Walker models in Lyra's geometry for a time-independent gauge function. Several authors studied cosmological models based on Lyra's manifold with a constant displacement field vector d_i . However this restriction on the displacement field to be constant is only for convenience and there is no prior reasons for it. Many authors studied the cosmological models with constant and time dependent displacement field.

Singh studied the new models of string cloud and obtained some new exact solutions of the string cosmology with and without a source free-magnetic field in the context of Bianchi type-V space times in normal gauge for Lyra's geometry.

In this Chapter we consider the Bianchi type-V cosmological models with modified

Chaplygin gas within the framework of Lyra's geometry. Stability of the model have been investigated and statefinder parameters {r,s} has been adopted to characterize the different phases of the universe and also we discussed the geometrical and physical properties.

8.2 Metric and field equations:

Bianchi type-V cosmological model is

$$ds^2 = dt^2 - a_1^2 dx^2 - a_2^2 e^{-2mx} dy^2 - a_3^2 e^{-2mx} dz^2 \tag{8.1}$$

where $a_1(t)$, $a_2(t)$ and $a_3(t)$ are function of cosmic time t only and m is a constant.

The energy momentum tensor for the perfect fluid is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{8.2}$$

where ρ is the density and p is the pressure.

Also, $g_{ij}u^i u^j = 1$ The field equations in the normal gauge in Lyra manifold is

$$R_{ij} - \frac{1}{2}R g_{ij} + \frac{3}{2}\phi_i \phi_j - \frac{3}{4} g_{ij}\phi_k \phi^k = -8\pi T_{ij} \tag{8.3}$$

where ϕ_i is a displacement field and the other symbols have their usual meanings as in Riemannian geometry. The displacement field ϕ_i can be taken as

$$\phi_i = (0, 0, 0, \beta)$$

where β is either a constant or a function of t and T_{ij} is the stress energy tensor of the matter.

The field equations (8.3) with the help of (8.1) and (8.2) can be written as

$$\frac{m^2}{a_1^2} - \frac{\ddot{a}_2}{a_2} - \frac{\ddot{a}_3}{a_3} - \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} - \frac{3}{4}\beta^2 = 8\pi p \tag{8.4}$$

$$\frac{m^2}{a_1^2} - \frac{\ddot{a}_1}{a_1} - \frac{\ddot{a}_3}{a_3} - \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} - \frac{3}{4}\beta^2 = 8\pi p \tag{8.5}$$

$$\frac{m^2}{a_1^2} - \frac{\ddot{a}_1}{a_1} - \frac{\ddot{a}_2}{a_2} - \frac{\dot{a}_1\dot{a}_2}{a_1a_2} - \frac{3}{4}\beta^2 = 8\pi p \tag{8.6}$$

$$\frac{3m^2}{a_1^2} - \frac{\dot{a}_1\dot{a}_2}{a_1a_2} - \frac{\dot{a}_1\dot{a}_3}{a_1a_3} - \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{3}{4}\beta^2 = -8\pi p \tag{8.7}$$

$$2m\frac{\dot{a}_1}{a_1} - m\frac{\dot{a}_2}{a_2} - m\frac{\dot{a}_3}{a_3} = 0 \tag{8.8}$$

Here the dot denotes the differentiation with respect to time.

Equation (8.8) on integration gives

$$a_1^2 = c_1a_2a_3$$

Without loss of generality taking $c_1 = 1$ we get,

$$a_1^2 = a_2a_3 \tag{8.9}$$

The field equations (8.4), (8.5), (8.6) using (8.9) yields

$$\frac{\dot{a}_1}{a_1} = \frac{\dot{a}_2}{a_2} = \frac{\dot{a}_3}{a_3} \tag{8.10}$$

The conservation law for the energy momentum tensor gives

$$\dot{\rho} + \left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right)(\rho + p) = 0 \tag{8.11}$$

The average scale factor and the spatial volume are defined as

$$R = \sqrt[3]{a_1a_2a_3}, V = R^3 = a_1a_2a_3 \tag{8.12}$$

The generalized mean Hubble parameter H is given by

$H = \frac{1}{3}(H_x + H_y + H_z)$, where, $H_x = \frac{\dot{a}_1}{a_1}, H_y = \frac{\dot{a}_2}{a_2}, H_x = \frac{\dot{a}_3}{a_3}$

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\dot{R}}{R} = \frac{1}{3}(H_x + H_y + H_z) \tag{8.13}$$

Deceleration parameter in cosmology is the measure of the cosmic acceleration of the universe expansion and is defined as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} \tag{8.14}$$

The relation between average Hubble parameter H and average scale factor R given as

$$H = DR^{-m} \tag{8.15}$$

From (8.13) and (8.15) we get,

$$\dot{R} = DR^{-m+1} \tag{8.16}$$

Integrating equation (8.16) we get,

$$R = a_0 e^{Dt}, m = 0 \tag{8.17}$$

$$R = (mDt + l_1)^{\frac{1}{m}}, m \neq 0 \tag{8.18}$$

8.3 MCG model of the universe when $m = 0$

$$V = R^3 = a_1 a_2 a_3 = a_0^3 e^{3Dt} = c_1 e^{3Dt} \tag{8.19}$$

where $c_1 = a_0^3$

From (8.10) we get,

$$a_2 = \alpha_1 a_1, a_3 = \alpha_2 a_1 \tag{8.20}$$

From equations (8.19) and (8.20) we get,

$$a_1 = \left(\frac{c_1}{\alpha_1 \alpha_2}\right)^{\frac{1}{3}} e^{Dt} = c_2 e^{Dt} \tag{8.21}$$

where $c_2 = \left(\frac{c_1}{\alpha_1 \alpha_2}\right)^{\frac{1}{3}}$

Modified Chaplygin gas is given by

$$p = A\rho - \frac{B}{\rho^{\alpha}} \tag{8.22}$$

Using (8.11) we get,

$$\rho = \left[\frac{B}{1+A} + \frac{C}{V^{(1+A)(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} \tag{8.23}$$

Case (i) : For small values of the scale factors a_1, a_2, a_3 we get,

$$\rho \approx \frac{C^{\frac{1}{1+\alpha}}}{V^{(1+A)}} \tag{8.24}$$

which is very large and corresponds to the universe dominated by an equation of state $p = A\rho$i.e. radiation dominated universe for $A = \frac{1}{3}$.

From (8.19) and (8.24) we get,

$$\rho \approx C^{\frac{1}{1+\alpha}} \left[c_1 e^{3Dt}\right]^{-\frac{4}{3}} \tag{8.25}$$

$$p \approx \frac{1}{3} C^{\frac{1}{1+\alpha}} \left[c_1 e^{3Dt}\right]^{-\frac{4}{3}} \tag{8.26}$$

Mean anisotropy parameter is

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = 0 \tag{8.27}$$

Shear scalar is

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right) = \frac{3}{2}\Delta H^2 = 0 \tag{8.28}$$

From equations (8.4), (8.21) and (8.26) Gauge function is

$$\beta^2 = \frac{4}{3}\left[\frac{m^2}{c_2^2 e^{2Dt}} - 3D^2 - \frac{8\pi}{3} C^{\frac{1}{1+\alpha}} \left(c_1 e^{3Dt}\right)^{-\frac{4}{3}}\right] \tag{8.29}$$

Case(ii): For large values of the scale factors a_1, a_2, a_3 we get,

$$\rho \approx \left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}} \tag{8.30}$$

and

$$p \approx -\left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}} \tag{8.31}$$

From equations (8.4), (8.21) and (8.29) we get,

$$\beta^2 = \frac{4}{3}\left[\frac{m^2}{c_2^2 e^{2Dt}} - 3D^2 + 8\pi\left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}}\right] \tag{8.32}$$

8.4 MCG model of the universe when $m \neq 0$

We discuss the model of the universe when $m \neq 0$. Average scale factor for this model of the universe is $R = (mDt + l_1)^{\frac{1}{m}}$.

Volume scale factor is

$$V = R^3 = a_1 a_2 a_3 = (mDt + l_1)^{\frac{1}{m}} \tag{8.33}$$

From equations (8.20) and (8.33) we get,

$$a_1 = k_2\,(mDt + l_1)^{\frac{1}{m}} \tag{8.34}$$

where $k_2 = \left(\frac{1}{\alpha_1\alpha_2}\right)^{\frac{1}{3}}$

Case(i): For small values of the scale factors a_1, a_2, a_3 we get,

$$\rho \approx \frac{C^{\frac{1}{1+\alpha}}}{V^{(1+A)}} \tag{8.35}$$

which is very large and corresponds to the universe dominated by an equation of state $p = A\rho$ (i.e. radiation dominated universe for $A = \frac{1}{3}$.

Using equations (8.33) and (8.35) we get,

$$\rho \approx C^{\frac{1}{1+\alpha}}\,[mDt + l_1]^{-\frac{4}{3m}} \tag{8.36}$$

$$p \approx \frac{1}{3}C^{\frac{1}{1+\alpha}}\,[mDt + l_1]^{-\frac{4}{3m}} \tag{8.37}$$

The physical parameters are

$$\Delta = 0 \tag{8.38}$$

$$\sigma^2 = 0 \tag{8.39}$$

From equations (8.4), (8.34), (8.37) Gauge function is

$$\beta^2 = \frac{4}{3}\left[\frac{m}{k_2^2\left(mDt+l_1\right)^{\frac{2}{m}}} - \frac{2D^2(1-m)+D^2}{\left(mDt+l_1\right)^2} - \frac{8\pi}{3}\frac{C^{\frac{1}{1+\alpha}}}{\left(mDt+l_1\right)^{\frac{4}{3m}}}\right] \tag{8.40}$$

Case(ii): For large values of the scale factors a_1, a_2, a_3 we get,

$$\rho \approx \left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}} \tag{8.41}$$

and

$$p \approx -\left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}} \tag{8.42}$$

The gauge function β is given as

$$\beta^2 = \frac{4}{3}\left[\frac{m}{k_2^2\left(mDt+l_1\right)^{\frac{2}{m}}} - \frac{2D^2(1-m)+D^2}{\left(mDt+l_1\right)^2} + 8\pi\left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}}\right] \tag{8.43}$$

The expansion scalar θ mean anisotropy parameter Δ and shear scalar σ^2 remain same.

8.5 Stability analysis:

We consider the reality of sound speed to investigate the stability of the model. The sound speed is given by

$$C_s^2 = \frac{dp}{d\rho} \tag{8.44}$$

When $C_s^2 \geq 0$, the model becomes physically acceptable. To obtain C_s^2, for modified Chaplygin gas model of the universe corresponding to $m = 0$ we use equations (8.19), (8.22) and (8.23) for the model of the universe corresponding to $m \neq 0$ we use equations (8.22), (8.23) and (8.33). We plotted the resulting C_s^2 in terms of time as shown in fig. 1 and fig. 2 and for both the models we get, $0 \leq C_s^2 \leq 1$. Thus we can say that the modified Chaplygin gas model of the universe within the framework of Lyra's geometry are satable.

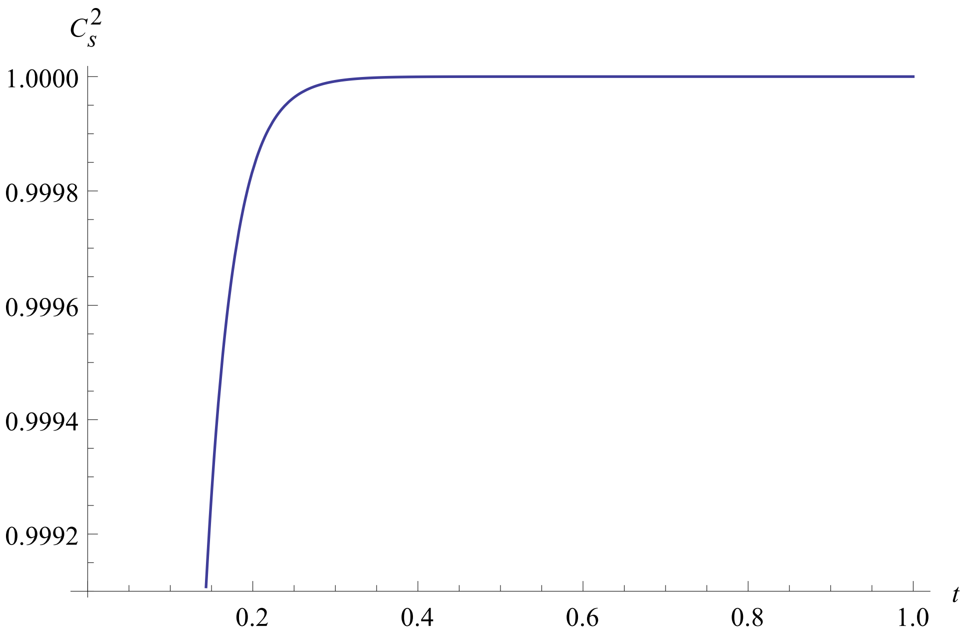

Figure-1 : Variation of C_s^2 vs. time t when $m = 0$

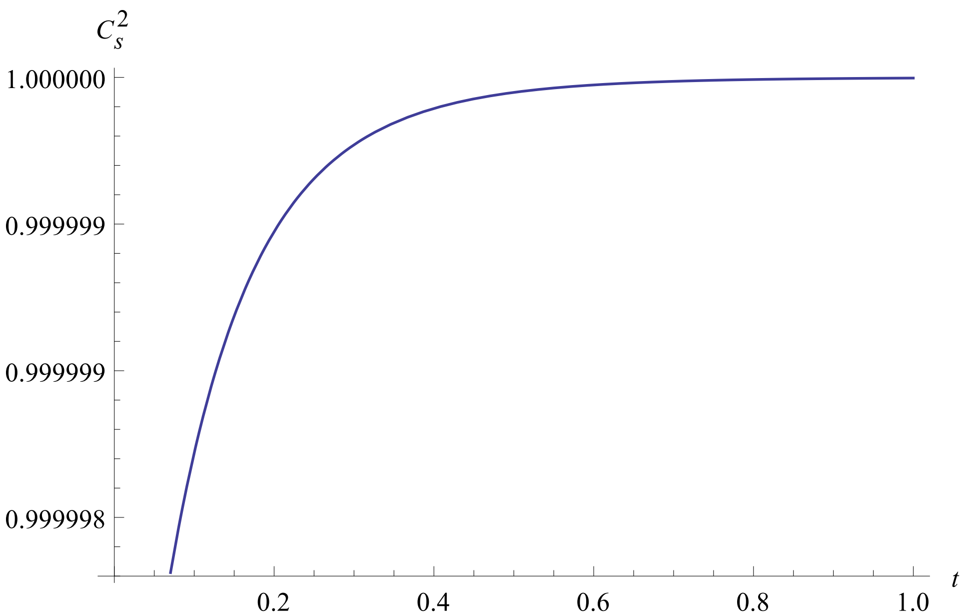

Figure-2 : Variation of C_s^2 vs. time t when $m \neq 0$

8.6 Conclusion:

In this paper, within the framework of Lyra's geometry using the relation between average Hubble parameter H and average scale factor a, we explore the cosmological solutions of Bianchi type-V universe filled with modified Chaplygin gas. The assumption of constant deceleration parameter leads to the exponential and power law model. In case of mofified Chaplygin gas model of the universe i.e. the exponential model of the universe with small values of the scale factor corresponding to $m = 0$, the model is non-singular because exponential is never zero and hence it does not exist any physical singularity. The model is isotropic as the values of mean anisotropy parameter Δ and shear scalar σ^2 are zero. For large values of the scale factors, the physical quantities ρ, p and β remain constant and other parameters are same as well for small values of the scale. In case of power law model of the universe corresponding to $m \neq 0$ with small values of scale factors, the universe exhibits initial singularity type of the point-type at $t = \left(-\frac{a_1}{mD}\right)$. The physical parameters ρ, p and β tend to zero as $t \to \infty$. In this case also the values of mean anisotropy parameter Δ and shear scalar σ^2 are zero which correspond to the isotropic universe. In both the cases, $0 \leq C_s^2 \leq 1$ from which we can conclude that modified Chaplygin gas models of the universe within the framework of Lyra's geometry are completely satble.