Perfect fluid Bianchi Type I universe and Generalized Cosmic Chaplygin Gas with Large Scale Factor

2.1 Introduction

The astronomical observations like Type Ia Supernovae, cosmic microwave background (CMB) radiation, large scale structure (LSS) indicate that the expansion of our universe is accelerating and the matter content responsible for this expansion is referred to as dark energy. There are various candidates to play the role of dark energy. The simplest form of dark energy is the cosmological constant Λ which gave rise to the simplest model of the accelerating universe, the $\Lambda\mathrm{CDM}$ model. Dark energy associated with a scalar field is called quintessence, and provides an effective model of the accelerating universe. The Chaplygin gas with negative pressure behaves as pressure less fluid for small values of scale factor and as a cosmological constant for large values of scale factor which tends to accelerate the expansion. There exists several Chaplygin gas model such as Chaplygin gas, generalized Chaplygin model, modified Chaplygin gas model, generalized cosmic Chaplygin gas model etc.

In 2003 González-Diaz introduced the generalized cosmic Chaplygin gas which can be made stable and free from unphysical behaviors even when the vacuum fluid

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enters the phantom era. The equation of state of Generalized Cosmic Chaplygin gas is

$$p = -\rho^{-\alpha}\left[C + \left(\rho^{1+\alpha} - C\right)^{-\omega}\right], \tag{2.1}$$

where p and ρ are respectively the pressure and energy density, $C = \frac{A}{1+\omega} - 1$, with A being a constant that can take on both positive and negative values, and $-L < \omega < 0$,L being a positive definite constant, which can take on values larger than unity.

To discriminate between different dark energy models both quantitatively and qualitatively the statefinder parameters were introduced. The diagnostic pair is constructed by the scale factor *R(t)* and its derivatives as follows:

$$r = \frac{\dddot{R}}{RH^3} \quad \text{and} \quad s = \frac{r-1}{3\left(q - \frac{1}{2}\right)}$$

where a is the scale factor of the universe, *H* is the Hubble's parameter and *q* is the deceleration parameter.

Shri Ram et al. [2003] analyzed the Einstein's field equations for an anisotropic Bianchi type I space time filled with generalized Chaplygin gas. Chakraborty et al. [2007] studied the mixture of radiation and generalized cosmic Chaplygin gas with or without interaction. Chaubey [2009] studied modified Chaplygin gas in the framework of Bianchi type-I universe. Rudra [2013] have investigated the role played by dark energy in the form of Generalized cosmic Chaplygin gas in an accelerating universe described by FRW cosmology.

It is well established that anisotropic Bianchi type-I universe plays significant role in understanding the phenomenal formation of galaxies in early universe. In this paper we have studied the generalized cosmic Chaplygin gas in Bianchi type-I universe.

2.2 Field Equation

The Einstein field equation on account of the cosmological constant we write in the form

$$R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R = kT_\mu^\nu + \delta_\mu^\nu \Lambda \tag{2.2}$$

Here R_μ^ν is the Ricci tensor, R is the Ricci scalar and k is the Einstein gravitational constant, Λ is the cosmological constant.

We study the gravitational field given by an anisotropic Bianchi type I (BI) cosmological model and choose it in the form:

$$ds^2 = dt^2 - a_1^2 dx^2 - a_2^2 dy^2 - a_3^2 dz^2 \tag{2.3}$$

with the metric functions a_1, a_2, a_3 being the functions of time *t* only.

The Einstein field equations (2.2) for the BI space-time in presence of the Λ term now we write in the form

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3} = kT_1^1 + \Lambda \tag{2.4}$$

$$\frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_3}{a_3}\frac{\dot{a}_1}{a_1} = kT_2^2 + \Lambda \tag{2.5}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} = kT_3^3 + \Lambda \tag{2.6}$$

$$\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} + \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3} + \frac{\dot{a}_3}{a_3}\frac{\dot{a}_1}{a_1} = kT_0^0 + \Lambda \tag{2.7}$$

Here over-dot means differentiation with respect to *t*. The energy-momentum tensor of the source is given by

$$T_\mu^\nu = (\rho + p)u_\mu u^\nu - p\delta_\mu^\nu \tag{2.8}$$

where u^ν is the flow vector satisfying

$$g_{\mu\nu}u^\mu u^\nu = 1 \tag{2.9}$$

Here ρ is the total energy density of a perfect fluid, while p is the corresponding pressure. p and ρ are related by an equation of state which will be studied below in detail. In a co-moving system of coordinates from (2.8) one finds

$$T_0^0 = \rho, \qquad T_1^1 = T_2^2 = T_3^3 = -p \tag{2.10}$$

In view of (2.10) from (2.4) to (2.7) one immediately obtains (Saha,2001b)

$$a_1(t) = D_1 V^{\frac{1}{3}} \exp\left[X_1 \int \frac{dt}{V(t)}\right] \tag{2.11}$$

$$a_2(t) = D_2 V^{\frac{1}{3}} \exp\left[X_2 \int \frac{dt}{V(t)}\right] \tag{2.12}$$

$$a_3(t) = D_3 V^{\frac{1}{3}} \exp\left[X_3 \int \frac{dt}{V(t)}\right] \tag{2.13}$$

Here D_i and X_i are some arbitrary constants obeying

$$D_1 D_2 D_3 = 1, X_1 + X_2 + X_3 = 0$$

Here *V* is the volume scale factor of the BI Universe, i.e.,

$$V = \sqrt{-g} = a_1 a_2 a_3 \tag{2.14}$$

Let us define

$$\frac{\dot{V}}{V} = \frac{\dot{a_1}}{a_1} + \frac{\dot{a_2}}{a_2} + \frac{\dot{a_3}}{a_3} = 3H$$

From equations $(2.4) - (2.7)$ for *V* one finds

$$\frac{\ddot{V}}{V} = \frac{3}{2}(\rho - p) \tag{2.15}$$

Using the energy conservation equation we get,

$$\rho = \left[C + \left\{1 + \frac{B}{V^{(1+\alpha)(1+\omega)}}\right\}^{\frac{1}{1+\omega}}\right]^{\frac{1}{1+\alpha}} \tag{2.16}$$

where *B* is an arbitrary integration constant.

Now for large values of the scale factors we have

$$\rho \cong (C+1)^{\frac{1}{(1+\alpha)}} \tag{2.17}$$

For this value of ρ, the value of p is $p = -\rho$.

Then from equation (2.15) we get,

$$\frac{\ddot{V}}{V} = 3\,(C+1)^{\frac{1}{(1+\alpha)}} \tag{2.18}$$

which gives

$$V = \sqrt{\frac{C_1}{3\,(C+1)^{\frac{1}{(1+\alpha)}}}} \sinh\left[t\sqrt{3\,(C+1)^{\frac{1}{(1+\alpha)}}} + C_2\right] \tag{2.19}$$

$$R=\left[\frac{C_1}{3\left(C+1\right)^{\frac{1}{(1+\alpha)}}}\right]^{\frac{1}{6}}\sinh^{\frac{1}{3}}\left[t\sqrt{3\left(C+1\right)^{\frac{1}{(1+\alpha)}}}+C_2\right] \tag{2.20}$$

The physical quantities of observational interest in cosmology are the expansion scalar θ, the mean anisotropy Δ, the shear scalar σ^2 and the deceleration parameter q are defined as

$$\theta=3H \tag{2.21}$$

$$\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \Delta H_i=H_i-H \tag{2.22}$$

$$\sigma^2=\frac{1}{2}\left(\sum_{i=1}^{3}H_i^2-3H^2\right)=\frac{3}{2}\Delta H^2 \tag{2.23}$$

$$q=\frac{d}{dt}\left(\frac{1}{H}\right)-1 \tag{2.24}$$

From the equations we get,

$$\theta=K_2\coth\left[K_2t+C_2\right] \tag{2.25}$$

$$\Delta=\frac{3X^2}{K_1^2K_2^2}\tanh^2\left[K_2t+C_2\right] \tag{2.26}$$

$$\sigma^2=\frac{X^2}{2K_1^2}\csc h^2\left[K_2t+C_2\right] \tag{2.27}$$

$$q=\frac{3}{\cosh^2\left[K_2t+C_2\right]}-1 \tag{2.28}$$

where $X^2\equiv X_1^2+X_2^2+X_3^2$

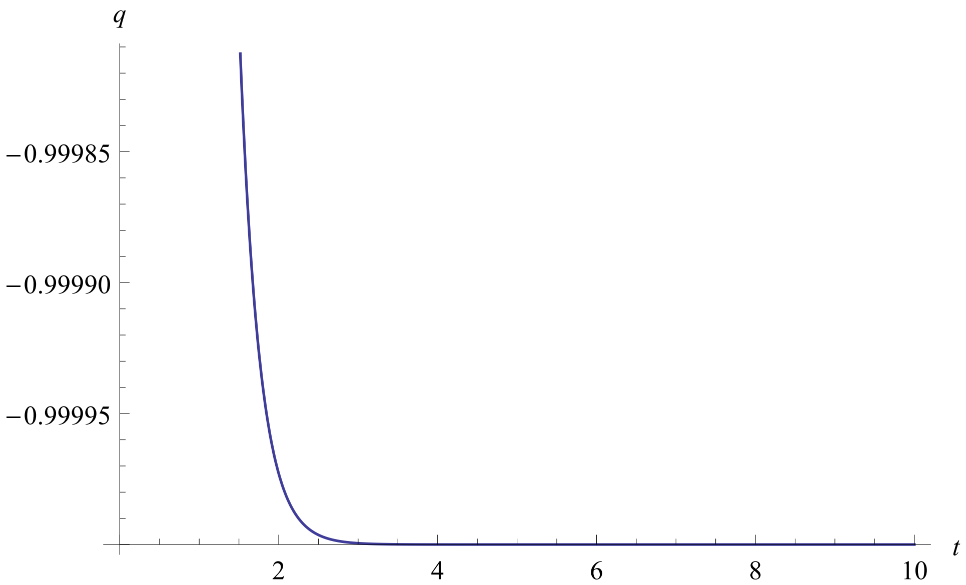

Figure-1 : Variation of q vs. t

The statefinder parameters are given by

$$r = 27\tanh^2[K_2t + C_2] \tag{2.29}$$

$$s = \frac{54\tanh^2[K_2t + C_2] - 2}{6\sec h^2[K_2t + C_2] - 3} \tag{2.30}$$

2.3 Discussion:

Here the pressure of this model universe is negative; it helps in the accelerated expansion of the universe, because a negative pressure stimulates a repulsive gravity and inflates the universe by over-whelming the usual gravitational effects of matter. Also the scalar expansion is found to be an increasing function of time which agrees with accelerated expansion of the universe.

On the other hand the pressure and density of this model universe are found to have finite value at finite time. In the limit when $t \to \infty$ the value of σ^2 tends to zero, the shear scalar is decreasing function of the cosmic time corresponding to the isotropic limit. We observe that at late time when $t \to \infty,\ \Delta$ tends to finite quantity which shows that our model has transition from initial anisotropy to isotropy at present epoch. Also we seen that the universe has a decelerating expansion in early phase of evolution which is crucial for the successful nucleosynthesis as well as structure formation. In the later phase the universe expands with acceleration since for large values of the scale factor the deceleration parameter is negative, so our model universe is an accelerating one which supports the present observational findings that, due to the dark energy component, the present universe is accelerating.

The statefinder is a ‘geometrical’ diagnostic in the sense that it depends upon the expansion factor. Equation (2.28) shows that the trajectories in the $\{r, s\}$ plane, corresponding to different cosmological models, for example model diagrams, correspond to the fixed point $s = 0$ for $r = 1$. From these values of state finder parameter given in equation (2.27), (2.28) we seen that the statefinder diagnostic along with future SNAP observation may perhaps be used to discriminate between dark energy model.