Chapter 3

Higher dimensional phantom dark energy model ending at a de-Sitter phase

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3.1 Introduction

Since the profound discovery of dark energy (DE) (Riess et al. 1998; Perlmutter et al. 1999) in 1998, theoretical physicists and cosmologists consider it as one of the most important topics in modern cosmology due to its mystic nature with huge negative pressure responsible for the universe to expand at an expedited rate. This cryptic component is considered to be uniformly permeated and vary slowly or unchanged with time (Chan 2015b; Carroll 2001a, 2001b; Peebles & Ratra 2003). With a focus to investigate its nature and application to modern cosmology, cosmologists have utilized tremendous scientific efforts and are still scrabbling for a perfect answer. From literature and observations, DE is believed to dominate the massive universe. This qualifies DE as a complete irony of nature as the dominating component is also the least explored. Some worth mentioning studies on this enigmatic dark component of the universe in the last decade are briefly presented in the next paragraph.

Akarsu et al. (2020) presents the evolutionary nature of DE is presented. The discussion on the evolution of DE given the latest observational findings is presented by Wang et al. (2018). Martino (2018) studies the decaying nature of DE into photons. Josset et al. (2017) obtain DE from violation of energy conservation. The quantum-mechanical calculation of DE density is presented by Dikshit (2019). Clery (2017) predicts that galaxy clusters due to the stirring effect of DE. A theoretical investigation of DE on searching the solution of global warming is illustrated by Singh et al. (2017a). Hamilton et al. (2015) discuss the atom-interferometry constraints on DE. Chan (2015a) asserts that the presence of particles with imaginary energy density can lead us to the source of DE. Gutierrez (2015) reviews the status of the experimental data on DE. The need for DE with thermodynamic arguments is provided by Moradpour et al. (2017). Lastly, Hecht (2013) compares the speed of DE with that of the photon.

To precisely understand the underlying mechanism of the late time accelerated expansion of the universe, cosmologists have adopted two well-appreciated methods. Firstly, different possible forms of DE are developed. Secondly, modifying Einstein's theory of gravitation. Other than these two, many authors have successfully adapted other fascinating ways to explain the miraculous expanding phe nomenon. Racz et al. (2017) make a compelling attempt describing the expanding phenomenon in the absence of DE. Alfaro (2019) claims that acceleration is automatically induced by the Delta Gravity equations, other than DE. Freese (2003), Freese & Lewis (2002) and Dvali & Turner (2003) try out directly modifying the Friedmann equation empirically to explain the phenomenon. An approach is presented by Narain & Li (2018) in which the accelerating paradigm is explained by an Ultra Violet Complete Theory. Lastly, Berezhiani (2017) illustrates the expanding phenomenon by matter.

One of the possible forms of DE which has not escaped our attention is holographic dark energy (HDE), introduced by Gerard't Hooft (Hooft 2009). It is obtained by the application of holographic principle (Bousso 2002) to DE. Accordingly, all the physical quantities inside the universe including the energy density of DE can be illustrated by some quantities on the boundary of the universe (Wang et al. 2017). Models involving the interaction of HDE and dark matter (DM) or interacting holographic dark energy (IHDE) models are considered to be of paramount importance by many authors. A discussion on an expanding interacting HDE and DM model can be seen in the study of Adhav et al. (2014), where the DE component decays into pressureless DM. Nayak (2020) discusses an IHDE model asserting that, at present, the universe is dominated by quintessence DE and it will become phantom DE dominated in the near future. Kiran et al. (2014) study a minimally IHDE model in a scalar-tensor theory of gravitation experiencing cosmic re-collapse. Sarkar (2015) investigates an IHDE model undergoing accelerated expansion ending at the big rip singularity. Chirde & Shekh (2018) investigate a minimally interacting matter and HDE model with the discussion of singularity and predicting that their model universe expands with the fastest rate and the largest value of the Hubble's parameter. Umadevi & Ramesh (2015) consider an isotropic minimally IHDE model in Bianchi type-III universe exhibiting early inflation and late-time acceleration. Reddy et al. (2016a) discuss a minimally IHDE model in Brans–Dicke theory where the DE turns out to be of phantom type. In the study by Raju et al. (2016), we can witness an IHDE model expanding spatially with a constant overall density parameter. Reddy et al. (2016b) investigate an IHDE model free from initial singularity attaining isotropy at late times. In the last few years, strong arguments have been brought to light asserting that modified gravity can be explained by employing DE-DM interaction in Einstein frame (Felice & Tsujikawa 2010; He 2011; Zumalacarregui 2013; Kofinas 2016; Cai 2016). Due to the fascinating nature of such interacting models, many fundamental questions are arisen pointing out that there are a lot more physics still undiscovered.

Saez-Ballester Theory (SBT), introduced by Saez & Ballester (1986), can be considered to be the right option to study DE and the accelerating universe. It is a member of the family of Scalar Tensor Theory (STT) of gravitation. In SBT, the metric potentials are coupled with a scalar field φ. Scalar fields are considered to play key roles in gravitation and cosmology as they can illustrate prodigies like DE, DM, etc. (Aditya et al. 2021). They can be regarded as a possible contributing factor in the late time acceleration of the universe (Kim 2005). STT of are direct generalization and extension of general relativity (Panotopoulos & Rincon 2018). STT can be considered as perfect candidates for DE (Mandal et al. 2018). STT also played a key role in getting rid of the graceful exit problem in the inflationary period (Piemental 1997). Linde (1982) asserts that a scaler field might be responsible for the inflation at the initial epoch. Currently, SBT and general relativity are held to align with observation.

Recently, there has been a growing interest among cosmologists to explore the DE-DM interaction in SBT setting. Ramesh & Umadevi (2016) study interacting HDE and DM model in SBT where the expanding model starts with a big bang. The construction of an interacting new HDE model in the framework of SBT can be found in presented by Aditya & D.R.K. Reddy (2018). Reddy et al. (2016) investigate an IHDE model in SBT where they use hybrid expansion law and predict a transitioning universe. Reddy (2017) investigates an IHDE model in SBT thereby obtaining three cosmological models. Rao et al. (2018a) observe an IHDE model in SBT obtaining a transitioning model due to cosmic-recollapse can be seen. Shaikh et al. (2019) discuss a model with matter and a modified holographic Ricci DE in SBT. Lastly, Rao et al. (2018b) investigate a modified holographic Ricci DE with matter in SBT predicting a quintom-like universe.

The possibility of space-time having more than 4D has fascinated many authors. Higher-dimensional cosmological model was introduced by Kaluza and Klein (Kaluza 1921; Klein 1926). Such models are useful to describe the late time expanding paradigm (Banik & K. Bhuyan 2017). The investigation on higher dimension can be considered as an important task as the universe might have encountered a higher dimensional phase during the early evolution (Singh et al. 2004). According to Alvax & Gavela (1983) and Guth (1981), the additional dimension might provide us an explanation for the flatness and horizon problem. Marciano (1984) discusses the evidence for the existence of the additional dimension. Lastly, Chakraborty & Debnath (2010) assert that the hidden extra dimension in 5D might correspond to the unknown DE and DM.

Keeping in mind the noteworthy studies mentioned above, we consider a DM-DE interaction in SBT considering a 5D spherically symmetric (SS) space-time. In this work, we present a detailed discussion on every cosmological parameter obtained. The definition of shear scalar and its physical significance are provided. The incompatibility of big rip singularity with HDE and its elimination in phantom DE scenario by de-Sitter phase is discussed. Additionally, we calculate the present values of the Hubble's parameter and the dark energy EoS parameter. To obtain realistic results, we make assumptions in concordance with present-day cosmology. The paper is divided into sections. After the introduction, in Sect. 3.2, we present problem formulations with solutions of the cosmological parameters. In Sect. 3.3, the solutions are discussed with graphs with the consideration of the recent findings. Lastly, as a summary, a concluding remark is provided in Sect. 3.4.

3.2 Formulation of problems with solutions

We start with the consideration of a SS metric in 5D (Samanta & Dhal 2013) as given below

$$ds^2 = dt^2 - e^{\mu}\left(dr^2 + r^2 d\Theta^2 + r^2 \sin^2\Theta d\phi^2\right) - e^{\delta} dy^2 \tag{3.2.1}$$

where μ and δ are cosmic scale factors which are functions of time only.

The Saez-Ballester field equations are given by

$$R_{ij} - \frac{1}{2}g_{ij}R - \omega_{SB}\varphi^n\left(\varphi_{,i}\varphi_{,j} - \frac{1}{2}g_{ij}\varphi_{,k}\varphi^{,k}\right) = -\left(T_{ij} + S_{ij}\right) \tag{3.2.2}$$

where T_{ij} and S_{ij} are the energy momentum tensors for matter and HDE respectively, R and R_{ij} are respectively the Ricci scalar and tensors, whereas the scalar field φ satisfies

$$2\varphi^n\varphi^{;i}_{;i} + n\varphi^{n-1}\varphi_{,k}\varphi^{,k} = 0 \tag{3.2.3}$$

where n is an arbitrary constant.

$T_{i\,j}$ and $S_{i\,j}$ are given by

$$T_{i\,j} = \rho_m u_i u_j \tag{3.2.4}$$

$$S_{i\,j} = (\rho_d + p_d)\, u_i u_j - g_{ij} p_d, \tag{3.2.5}$$

where ρ_m and ρ_d represent the energy density of matter and HDE respectively whereas p_d is pressure of the HDE.

By conservation of energy, we have

$$T_{i\,j} + S_{i\,j} = 0 \tag{3.2.6}$$

Using co-moving coordinate system, the surviving field equations are obtained as

$$\frac{3}{4}\left(\dot{\mu}^2 + \dot{\mu}\dot{\delta}\right) + \frac{\omega_{_{SB}}}{2}\varphi^n\dot{\varphi}^2 = \rho \tag{3.2.7}$$

$$\ddot{\mu} + \frac{3}{4}\dot{\mu}^2 + \frac{\ddot{\delta}}{2} + \frac{\dot{\delta}^2}{4} + \frac{\dot{\mu}\dot{\delta}}{2} - \frac{\omega_{_{SB}}}{2}\varphi^n\dot{\varphi}^2 = -p_d \tag{3.2.8}$$

$$\frac{3}{2}\left(\ddot{\mu} + \dot{\mu}^2\right) - \frac{\omega_{_{SB}}}{2}\varphi^n\dot{\varphi}^2 = -p_d \tag{3.2.9}$$

From Eq. (3.2.6), we have

$$\ddot{\varphi} + \dot{\varphi}\left(\frac{3\dot{\mu} + \dot{\delta}}{2}\right) + \frac{n}{2}\dot{\varphi}^2\varphi^{-1} = 0 \tag{3.2.10}$$

where an overhead dot represents differentiation w.r.t. t.

We assume ω as the EoS parameter of the DE and hence, we have

$$p_d = \omega\rho_d \tag{3.2.11}$$

The conservation equation takes the obvious form as given by

$$\rho_m\left(\frac{3\dot{\mu} + \dot{\delta}}{2}\right) + \dot{\rho}_m + \dot{\rho}_d + \rho_d\,(1+\omega)\left(\frac{3\dot{\mu} + \dot{\delta}}{2}\right) = 0 \tag{3.2.12}$$

Due to their minimal interaction, HDE and matter conserve separately so that by Sarkar (2014a, 2014b), Eq. (3.2.12) can be written as

$$\rho_m\left(\frac{3\dot{\mu}+\dot{\delta}}{2}\right)+\dot{\rho}_m=0 \tag{3.2.13}$$

$$\rho_d\left(1+\omega\right)\left(\frac{3\dot{\mu}+\dot{\delta}}{2}\right)+\dot{\rho}_d=0 \tag{3.2.14}$$

Also, we have

$$(\rho+p)\left(\frac{3\dot{\mu}+\dot{\delta}}{2}\right)+\dot{\rho}=0 \tag{3.2.15}$$

From Eqs. (3.2.8) and (3.2.9), the expression for the cosmic scale factors are obtained as

$$\mu=l_1-\log\left(k-t\right)^{\frac{2}{3}} \tag{3.2.16}$$

$$\delta=m_1-\log\left(k-t\right)^{\frac{2}{3}} \tag{3.2.17}$$

where l_1, m_1 and k are arbitrary constants.

Now, from Eqs. (3.2.13), (3.2.14), (3.2.16) and (3.2.17), we have

$$\rho_m=l_0e^{-\frac{1}{2}(3l_1+m_1)}\left(k-t\right)^{\frac{4}{3}} \tag{3.2.18}$$

$$\rho_d=m_0e^{-\frac{1}{2}(1+\omega)(3l_1+m_1)}\left(k-t\right)^{\frac{4}{3}(1+\omega)} \tag{3.2.19}$$

so that the energy density of our universe is given by

$$\rho=\rho_m+\rho_d \tag{3.2.20}$$

where l_0 and m_0 are an arbitrary constants.

Again, using Eqs. (3.2.16), (3.2.17) and (3.2.20) in Eq. (3.2.15), the pressure of our universe is obtained as

$$p=\frac{1}{3}l_0e^{-\frac{1}{2}(3l_1+m_1)}(k-t)^{\frac{4}{3}}+m_0\left(\frac{4\omega+1}{3}\right)e^{-\frac{1}{2}(1+\omega)(3l_1+m_1)}(k-t)^{\frac{4}{3}(1+\omega)} \tag{3.2.21}$$

From Eqs. (3.2.11) and (3.2.19), the pressure of DE is given by

$$p_d = \omega\, m_0 e^{-\frac{1}{2}(3l_1+m_1)(1+\omega)}\, (k-t)^{\frac{4}{3}(1+\omega)} \tag{3.2.22}$$

At any time $t = t_0$, we can assume that $p = p_d$ so that from Eqs. (3.2.21) and (3.2.22), we have

$$l_0 e^{x}(k-t_0)^{\frac{4}{3}} + m_0(1+\omega)e^{(1+\omega)x}(k-t_0)^{\frac{4}{3}(1+\omega)} = 0 \tag{3.2.23}$$

where $x = -\frac{1}{2}(3l_1 + m_1)$

Eq. (3.2.23) will provide us the expression for EoS parameter ω.

Now, using Eqs. (3.2.16) and (3.2.17) in Eq. (3.2.10), the SB scalar field φ is obtained as below

$$\varphi = \left((6+3n)\,(k-t)^{\frac{7}{3}} - 14c_1\right)^{\frac{2}{2+n}} c_2 \tag{3.2.24}$$

where c_1 and c_2 are arbitrary constants.

Finally, the expressions of the different cosmological parameters are obtained as follows.

Spatial volume:

$$V = e^{\frac{3l_1+m_1}{2}}(k-t)^{-\frac{4}{3}} \tag{3.2.25}$$

Scalar expansion:

$$\theta = \frac{4}{3}\,(k-t)^{-1} \tag{3.2.26}$$

Hubble parameter:

$$H = \frac{1}{3}\,(k-t)^{-1} \tag{3.2.27}$$

Shear scalar:

$$\sigma^2 = \frac{2}{9}\left(\frac{1}{k-t} - 1\right)^2 \tag{3.2.28}$$

Anisotropic parameter:

$$A_h = 0 \tag{3.2.29}$$

Dark energy density parameter:

$$\Omega_d = \frac{\rho_d}{3H^2} = 3m_0 e^{-\frac{1}{2}(1+\omega)(3l_1+m_1)}(k-t)^{\frac{2}{3}(5+2\omega)} \tag{3.2.30}$$

Matter density parameter:

$$\Omega_m = \frac{\rho_m}{3H^2} = 3l_0 e^{-\frac{1}{2}(3l_1+m_1)}(k-t)^{\frac{10}{3}} \tag{3.2.31}$$

Overall density parameter:

$$\Omega = 3\left(l_0 e^{-\frac{1}{2}(3l_1+m_1)} + m_0 e^{-\frac{1}{2}(1+\omega)(3l_1+m_1)}(k-t)^{\frac{4\omega}{3}}\right)(k-t)^{\frac{10}{3}} \tag{3.2.32}$$

Jerk parameter:

$$j(t) = q + 2q^2 - \frac{\dot{q}}{H} = 28 \tag{3.2.33}$$

3.3 Discussion

For our convenience sake and to obtain realistic results, in this section, choose fixed values of the arbitrary constants appearing in the solutions i.e., $l_0 = l_1 = m_0 = m_1 = 1, k = 13.80497512437811$.

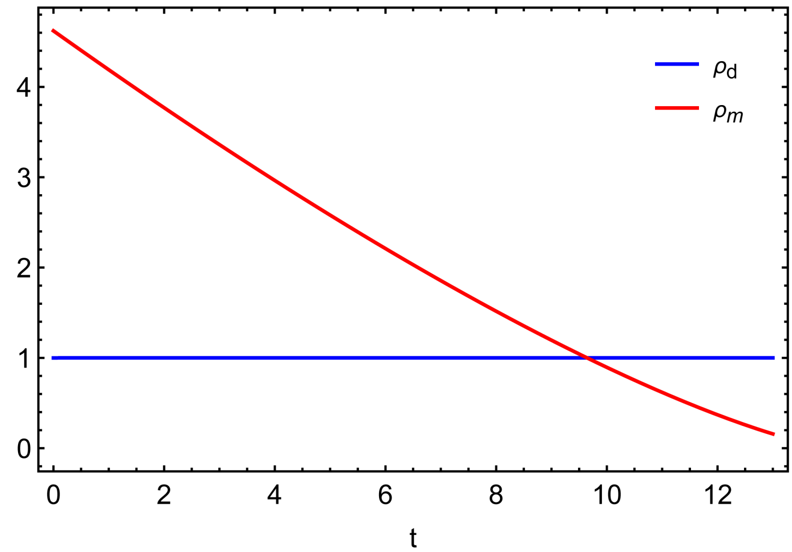

Figure 3.1: Energy densities of DE ρ_d and DM ρ_m with t when $l_0 = l_1 = m_0 = m_1 = 1, k = 13.80497512437811$.

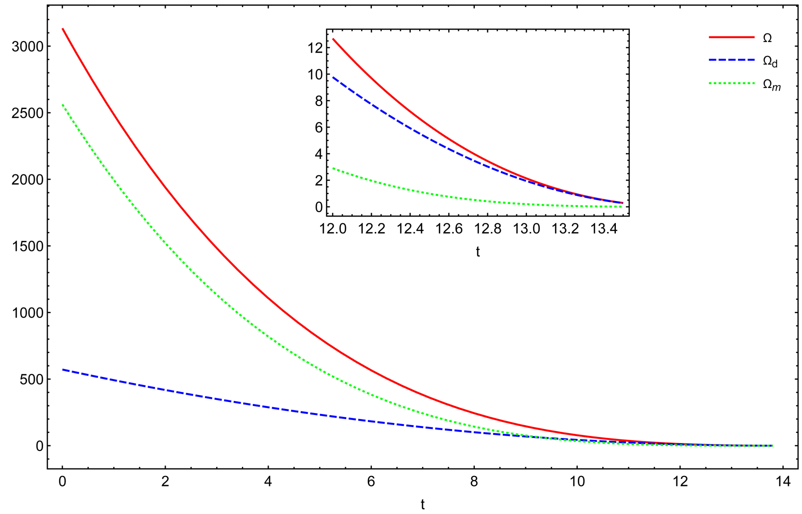

Figure 3.2: Overall density parameter Ω, DE density parameter Ω_d and DM density parameter Ω_m with t when $l_0 = l_1 = m_0 = m_1 = 1, k = 13.80497512437811$.

From Fig. 3.1, we can witness the decreasing nature of ρ_m whereas ρ_d remains consistent all through. From Fig. 3.2, we can see that Ω and Ω_d tend to become constant after decreasing for a finite period, whereas Ω_m continue to decrease to a larger extent. It may be noted that due to the expansion, galaxies move apart from each other leading DM density to diminish gradually (Carroll 2001b), whereas DE varies slowly or unchanged with time (Chan 2015b; Carroll 2001a, 2001b; Peebles & Ratra 2003). From these, we have obtained a model which is DE dominated, similar to that predicted by Carroll (2001a), Adhav et al. (2014), Araujo (2005), Ray et al. (2013), Agrawal et al. (2018), Wu & Yu (2005), Straumann (2007) and Law (2020).

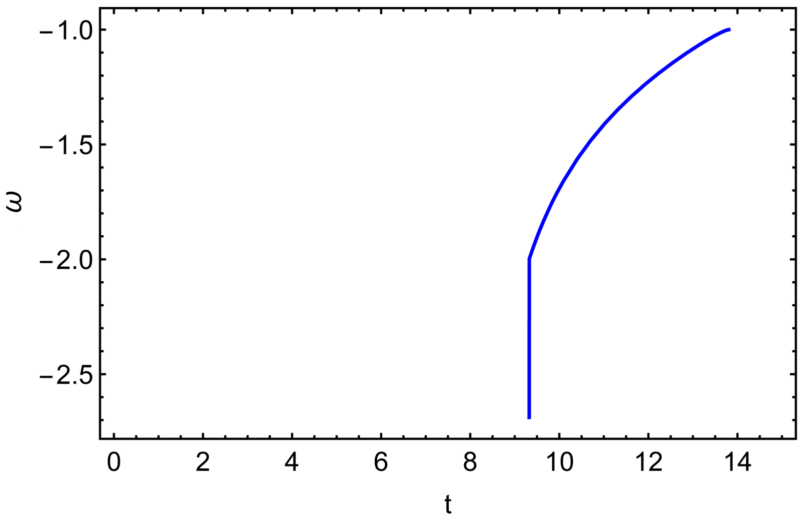

Figure 3.3: EoS parameter ω with t when $l_0 = l_1 = m_0 = m_1 = 1, k = 13.80497512437811$.

Fig. 3.3 shows the variation of time-dependent EoS parameter ω with cosmic time t. Here, it can be seen that ω starts evolving from the aggressive phantom region and tends to come very close to -1, which aligns with the recent studies (Amirhashchi 2017; V. Santhi et al. 2019). Similar observations of HDE with phantom-like nature can also be seen in the recent works (Belkacemi et al. 2020; Sharif & Ikram 2019). However, as ω appears to evolve due to time dependence, it attains the value $\omega = -1$ during evolution (Aditya et al. 2021; Basilakos & Sola 2014). Above all, a phantom model with $\omega < -1$ should reduce to $\omega = -1$ in the far future to ensure cosmological models bypass future singularity (big rip) thereby, ultimately, leading to the de-Sitter phase (Amirhashchi 2017; Carroll et al. 2003). It can also be noted that in HDE setting, the big rip singularity is not permitted, because the Planck scale excursion of UV cutoff in the effective field theory is forbidden so that the occurrence of the big rip would ruin the theoretical foundation of the HDE scenario (Zhang 2010). This issue can be solved by employing an extra dimension in HDE setting, and also the employment of an extra dimension makes HDE models more complete and consistent. The mechanism of replacing big rip singularity by de-Sitter phase with the employment of an extra dimension (higher dimension) in HDE setting can be seen in the study by Zhang (2010). Dymnikova (2019), Sakharov (1966) and Gliner (1966) also discuss on replacing big rip singularity by de-Sitter phase. According to the latest Planck 2018 result (Collaboration et al. 2020), the present age of the universe is 13.825 ± 0.037 Gyr. With $t_0 = 13.8$ Gyr and assuming $l_0 = l_1 = m_0 = m_1 = 1, k = 13.80497512437811$, from Eq. (3.2.23), the present value of EoS parameter is obtained to be $\omega = -1.00011$, which aligns with the value $\omega = -1.03 \pm 0.03$ of the of the latest Planck 2018 result (Collaboration et al. 2020).

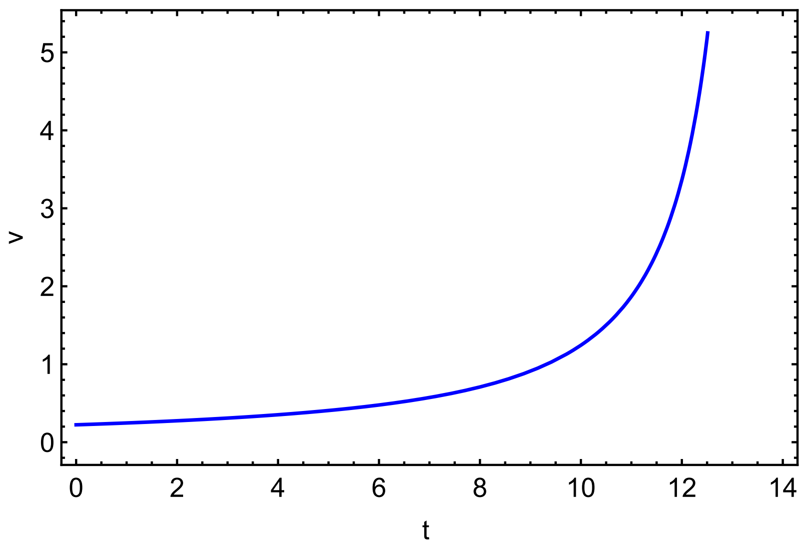

Figure 3.4: Spatial volume V with t when $l_1 = m_1 = 1, k = 13.80497512437811$.

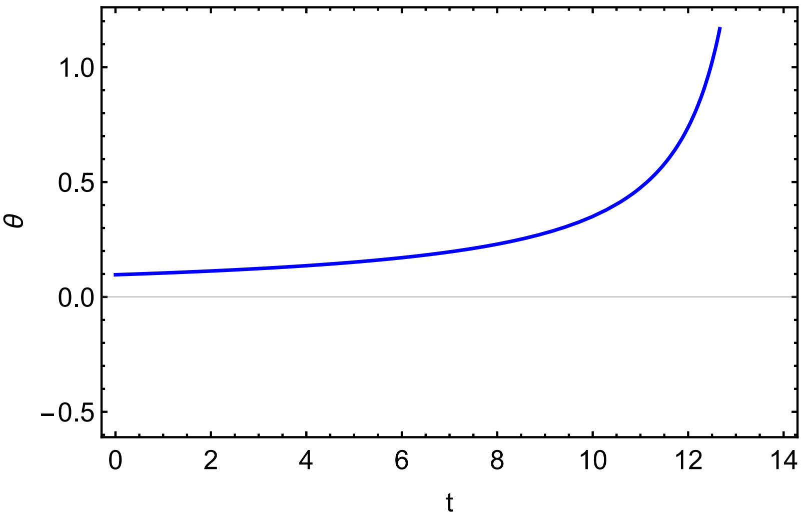

Figure 3.5: Scalar expansion θ with t when $k = 13.80497512437811$.

Fig. 3.4 and Fig. 3.5 can be considered as the perfect pieces of evidence for the spatial expansion of the universe at an expedited rate. At $t = 0$, V and other related parameters are constant which indicates that the model doesn't evolve from an initial singularity. Whereas, as discussed before, the future big rip singularity is replaced by the de-Sitter phase.

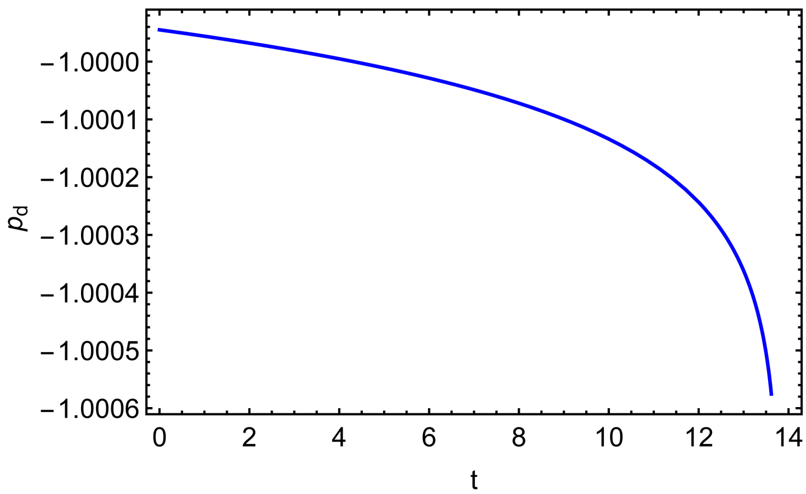

Figure 3.6: DE pressure p_d with t when $l_1 = m_0 = m_1 = 1, k = 13.80497512437811$.

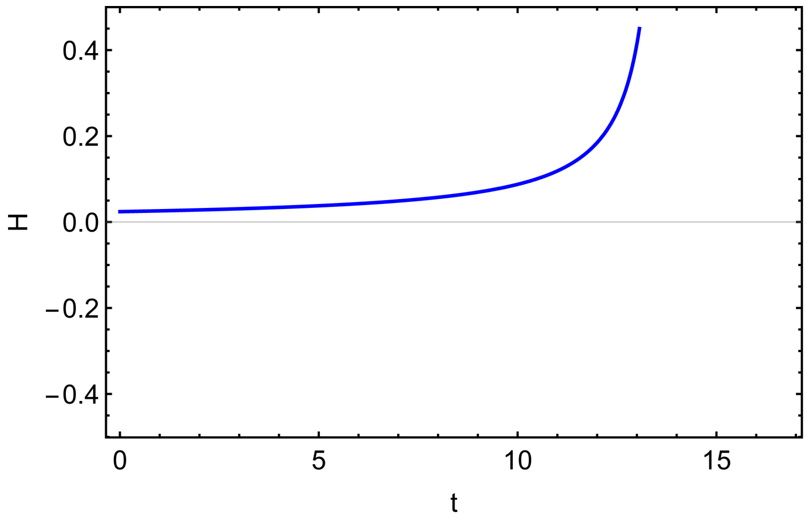

Figure 3.7: Hubble parameter H with t when $k = 13.80497512437811$.

From Fig. 3.6, it is obvious that the graph of the pressure of DE p_d lies in the negative plane during the entire course of evolution, which aligns with the ambiguous property of DE, which accounts for the accelerated expansion. From Fig. 3.7, it is clear that the Hubble's parameter H of the model universe tends to remain almost constant during the early evolution so that the model was in an inflationary epoch experiencing rapid exponential expansion (Kremer et al. 2019). The latest Planck 2018 result (Collaboration et al. 2020), estimates the present age of the universe to be 13.825 ± 0.037 Gyr. Assuming $t = 13.8$ and $k = 13.80497512437811$, from Eq. (3.2.27), the value of Hubble parameter is measured to be $H = 67$, approximately equal to the value $H_0 = 67.36 \pm 0.54$ kms^{-1} Mpc^{-1} of the latest Planck 2018 result (Collaboration et al. 2020).

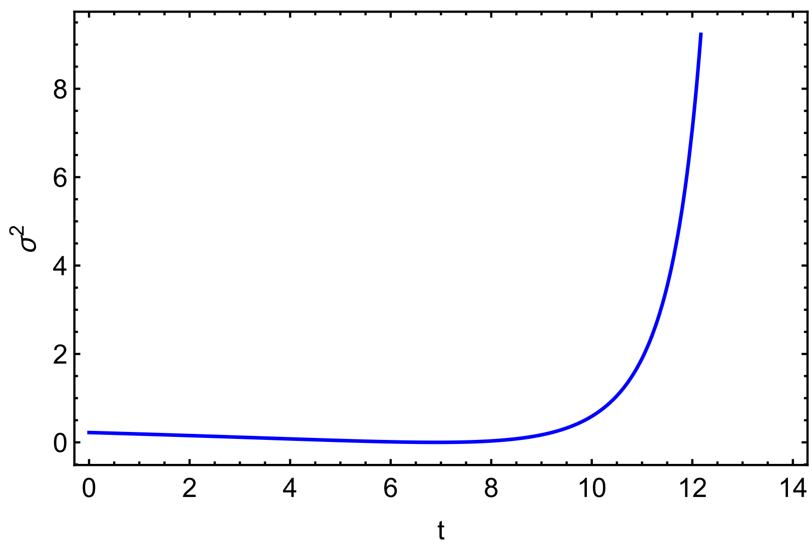

Figure 3.8: Variation of the shear scalar σ^2 with t when $k = 13.80497512437811$.

Fig. 3.8 shows us the variation of σ^2 with cosmic time t. Initially, σ^2 appears to decrease negligibly, and then, it tends to diverge. σ^2 shows us the rate of deformation of the matter flow within the massive cosmos (Ellis & Elst 1999). From Eq. (3.2.29), the anisotropic parameter $A_h = 0$. So, we can sum up that the universe is isotropic and expands with a slow and uniform change of size in the early evolution, whereas the change tends to become faster at late times. This is in agreement with the present observation of the accelerated expansion of the universe.

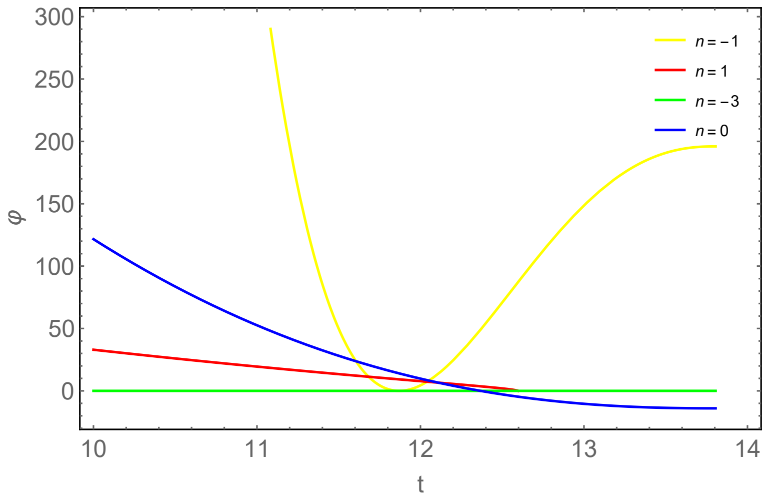

Figure 3.9: Variation of φ with t when $c_1 = c_2 = 1$.

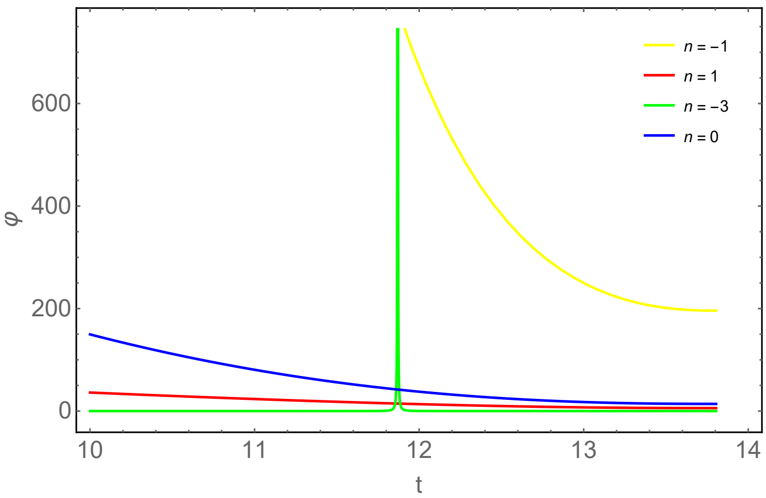

Figure 3.10: Variation of φ with t when $c_1 = -1, c_2 = 1$.

Fig. 3.9 shows the variation the SB scalar field φ with cosmic time t when $c_1 = c_2 = 1$ (both $c_1, c_2 > 0$) whereas Fig. 3.10 shows the variation when $c_1 = -1, c_2 = 1$ ($c_1 < 0, c_2 >$

0). In both cases, the real value of φ can't be obtained for $n = -2$. In Fig. 3.9, we can see the decreasing nature of φ. However, when $n = -1$, it decreases up to a minimum value and increases to attain a constant positive value. It decreases to become negative when $n = 0$. In Fig. 3.10, φ decreases and is positive all through. When $n = -1$, it tends to attain a constant positive value after decreasing for a finite time. When $n = -3$, it attains its maximum and minimum values during the evolution. Hence, in both cases, when $n = -1$, φ tends to attain almost the same large positive constant, which might be the reason for the phantom-like nature of the DE at present. This observation is somewhat similar to that obtained by Naidu et al. (2019), where after both increasing and decreasing, the scalar field tends to attain a positive constant value.

Lastly, from Eq. (3.2.33), the value of the jerk parameter is obtained to be $j(t) = 28$. It can be used as a tool to describe the closeness of models to the standard ΛCDM model. Its value for the standard ΛCDM model is $j(t) = 1$.

3.4 Conclusions

In this chapter, we have investigated an interacting model of HDE and matter in a SS space-time in 5D setting within the framework of SBT. We have obtained an accelerating model where HDE with phantom-like nature dominates the universe. The model doesn't evolve from an initial singularity. To preserve the theoretical foundation of HDE scenario, an extra dimension is employed. In the far future, the DE departs from phantom-like nature to cosmological constant thereby bypassing future singularity and ultimately leading to the de-Sitter phase. The universe is predicted to be isotropic. At $t = 13.8$ Gyr, the approximate present age of the universe, the values of Hubble parameter and DE EoS parameter are measured to be $H = 67$ and $\omega = -1.00011$, which agree with the respective values $H_0 = 67.36 \pm 0.54$ kms^{-1} Mpc^{-1} and $\omega = -1.03 \pm 0.03$ of the latest Planck 2018 result (Collaboration et al. 2020). It is predicted that the model expands with a slow and uniform change of size in the early evolution whereas the change tends to become faster at late times. We observe that when $n = -1$, the SB scalar field φ tends to attain a positive constant value in the course of evolution.